Mathematics Education Journal
ISSN: 2549-1040 (e) | 1978-0044 (p)
https://doi.org/10.22342/jpm.v19i3.pp413-436

Students' Communication Skill and Algebraic Thinking through Commognitive Framework in Algebra Learning

Weni Dwi Pratiwi, Zulkardi*, Ratu Ilma Indra Putri, Cecil Hiltrimartin

Department of Mathematics Education, Universitas Sriwijaya, Palembang, Indonesia
*Email: zulkardi@unsri.ac.id

Abstract

Mathematical communication is a skill to articulate mathematical ideas through written and verbal expressions. Despite its importance, students often exhibit difficulties in initiating and organizing mathematical ideas, particularly in written form. Prior research highlights the critical role of communication in fostering early algebraic thinking. This study aims to: (1) examine students' mathematical communication skills and explore how it reflects early algebraic thinking, using the commognitive framework, which encompasses word use, visual mediators, narratives, and routines, and (2) describe statistically the relationship between algebraic thinking and oral communication skills. A descriptive research design with mixed-methods approach was employed, involving 29 eighth-grade students as participants. Data was collected through written tests, and the results were analyzed descriptively. The findings indicate that four students demonstrated excellent communication skills, nine students were categorized as good, 11 students as fair, three students as poor, and two students as very poor. Further analysis showed that students with stronger communication skills more effectively conveyed algebraic ideas. The results also underscore the commognitive framework practical application in enhancing students' conceptual understanding. This study contributes to a deeper understanding of how students communicate, reason algebraically, and demonstrates the potential of the commognitive framework to support instructional design and skill development in mathematics education.

Keywords: Algebra Learning, Algebraic Thinking, Basic Education, Commognitive Framework, Communication Skills

How to Cite: Pratiwi, W. D., Zulkardi, Putri, R. I. I., & Hiltrimartin, C. (2025). Students' communication skill and algebraic thinking through commognitive framework in algebra learning. *Mathematics Education Journal, 19*(3), 413-436. https://doi.org/10.22342/jpm.v19i3.pp413-436

INTRODUCTION

One key facet of mathematical communication is the skill to express mathematical ideas clearly and effectively. Khaidir et al. (2024) emphasize that mathematical communication skills include understanding various forms of communication, such as writing and interpreting mathematical information, which allows students to convey their thought processes and engage in collaborative problem-solving effectively. Similarly, Marsitin & Sesanti (2018) argue that successful mathematics education requires students to communicate their reasoning through structured communication, which is essential for effective problem-solving. In addition, Lu et al. (2022) highlight that mathematical communication abilities are a fundamental component of mathematics learning. Zulu & Mudaly (2023) further emphasize that mathematical communication plays a critical role in the identification and development of ideas and concepts during the learning process, while also supporting the understanding of abstract concepts and symbolic language, particularly in written communication.

Written mathematical communication refers to the skill to express mathematical language through images, graphs, tables, diagrams, and other visual representations (Lu et al., 2023). Indicators of written mathematical communication skills include writing (written tests), drawing (visual

Received June 10, 2024; Revised August 03, 2024; Accepted July 11, 2025

representations), and mathematical expressions. Writing entails providing logical and accurate mathematical explanations, drawing involves representing data through tables, diagrams, and graphs, while mathematical expressions refer to performing calculations accurately and comprehensively (Setyowati et al., 2022).

However, despite the recognized importance of mathematical communication, students' skill to effectively convey mathematical ideas—particularly in written form—remains relatively low (Pinto & Cooper, 2023; Rahmawati et al., 2023). This challenge becomes evident when students are faced with unfamiliar problems that differ from standard examples, often leaving them uncertain about how to proceed. Furthermore, students find it challenging to explain each response they receive due to the teacher-led, dominant learning process. Mathematical communication, in this context, is not merely about expressing solutions but also about reading, interpreting, and engaging with mathematical problems meaningfully. It encompasses the articulation of mathematical reasoning and the skill to establish logical connections between concepts (Kristiani et al., 2024). As highlighted by recent educational frameworks, enhancing communication skills is fundamental to the learning objectives outlined in curricular guidelines (Rahmawati & Soekarta, 2021). Importantly, the role of communication in mathematics extends beyond the subject itself, it contributes significantly to students' cognitive development and fosters deeper mathematical thinking.

Mathematical communication skills function as a crucial bridge between students' cognitive processes and effective mathematics learning. The way students articulate their mathematical thinking—whether through written, oral, or visual means—directly reflects and shapes how they process mathematical information cognitively, which in turn influences their overall performance in mathematics (Sukasno et al., 2024). The significance of communication in mathematics is critical. The National Council of Teachers of Mathematics (NCTM) asserts that communication is not only a component of mathematics learning but a foundational element of mathematical understanding itself (Shinno & Fujita, 2021). This view is reinforced by contemporary educational frameworks that place communication skills at the core of students' mathematical development. Through effective communication, students are better able to express ideas, justify reasoning, and demonstrate comprehension—key competencies for mastering mathematical concepts. Therefore, fostering students' skill to communicate clearly, logically, and convincingly—both in written and oral forms—is essential, as it emphasizes explanation, precision, and argumentation as integral aspects of mathematical literacy (Nauli et al., 2024).

The interaction between communication skills and cognitive processes in mathematics education is both significant and multifaceted. The commognitive framework offers a meaningful perspective for analyzing and enhancing students' mathematical communication abilities (Ching et al., 2020). By implementing strategies that emphasize precise language use, visual representations, contextual narratives, and consistent problem-solving routines, educators can significantly improve students' capacity to express and comprehend mathematical ideas (Makgakga, 2023). Such approaches not only support individual cognitive development but also contribute to more collaborative and engaging learning environments that are essential for students' academic and professional readiness in mathematics (Chan et al., 2022; Matthews, 2024).

One area where students often struggle to demonstrate effective mathematical communication is in learning the System of Linear Equations in Two Variables (SLETV). This topic involves two equations with two distinct variables, requiring students to integrate symbolic, graphical, and procedural understanding. However, studies have shown that students' written mathematical communication in this context remains relatively weak. For example, Handayani & Wandini (2020) reported low performance across three key indicators: written explanations (33.00%), visual representations such as graphs or diagrams (36.41%), and mathematical expressions (14.44%). These findings highlight the ongoing challenge of fostering students' written communication skills, particularly when working with abstract or multi-representational mathematical content like SLETV.

To address these challenges, the commognitive framework provides a meaningful theoretical approach by integrating cognitive and communicative aspects of mathematical thinking (Setyowati et al., 2022). This framework consists of four key components: word use, visual mediators, narratives, and routines (Chan et al., 2022; Matthews, 2024). Word use refers to students' skill to express known and unknown elements using precise mathematical language. Visual mediators include the use of graphs, tables, and diagrams to represent mathematical relationships. Narratives involve the structured explanation of mathematical facts, such as theorems or properties, while routines represent the procedures and problem-solving steps used to arrive at a solution. As supported by Zayyadi et al. (2019) applying the commognitive framework can enhance students' skill to solve mathematical problems.

Commognitive-Based Learning

The commognitive framework, developed by Anna Sfard, provides an insightful perspective on algebra learning, especially regarding discourse and interaction. This framework asserts that mathematical learning is fundamentally a discursive activity, involving both communication practices and cognitive processes (Sfard, 2012). A notable application of this theory is found in studies examining the teaching and learning of algebraic concepts. For instance, the research by Daher & Swidan (2019) highlights how low-achieving students used interactive technological tools to engage with mathematical equality, underscoring the critical role of discourse in developing conceptual understanding through a commognitive perspective. The incorporation of dynamic media supports students' discursive practices, enhancing engagement with algebraic principles.

Moreover, Zayyadi et al. (2020) findings reveal how prospective teachers' content and pedagogical knowledge can be analyzed through a commognitive lens, demonstrating how specific discursive interactions between teachers and students, such as the use of visual mediators, can enhance algebra learning. This aligns with the assertion by Rahmatina et al. (2022) that commognitive frameworks can positively influence students' problem-solving capabilities in mathematical contexts, including algebra. Similarly, Makgakga (2023) demonstrated that applying problem-solving strategies grounded in the commognitive framework resulted in notable improvements in students' performance in algebraic tasks. His findings emphasize the importance of teacher-led discourse and pedagogical strategies in implementing commognitive principles effectively in the classroom.

In addition to algebra, the commognitive framework has also been employed to examine mathematical discourse related to functions in higher education. Research by Rabin et al. (2013), Venegas-Thayer (2019), & Viirman (2014) demonstrate how teaching and learning activities involving functions can be understood discursively—integrating communicative and cognitive skills in what Sfard (2012) refers to as the process of recognition. Sfard (2012) makes a distinction between literacy-based talks that are typically spontaneous in ordinary life and conversations that possess the traits of mathematical discourse, which necessitates a learning process to engage in.

This framework comprises the following cognitive elements: (1) word use; (2) visual mediators; (3) routines; and (4) endorsed narratives (Heyd-Metzuyanim & Sfard, 2012; Venegas-Thayer, 2019). It is not sufficient to merely produce representations when it comes to the skill to represent functions in equations, arrow diagrams, and graphs; mathematical language must also support these skills. It is highly probable that the abstract idea of function will be employed as a discursive object. Students get the chance to verbally explain their answers to instructional activities in the form of explanations using mathematical language related to the concept of function during student-teacher interactions.

The term commognitive, a blend of "communication" and "cognitive," underscores this dual focus. As Ching et al. (2020) note, the commognitive framework encourages students to engage in discussions about concepts they are familiar with, thus fostering deeper understanding through language. When facilitated by teachers, such discourse not only promotes the development of mathematical thinking but also cultivates a sociocultural learning environment, where knowledge is co-constructed through shared communication practices.

To guide the implementation of commognitive-based learning in classrooms, Sfard (2012) proposed a set of practical indicators, as outlined in the Table 1. Indicator of commognitive-based learning:

Table 1. Indicator of commognitive-based learning

Commognitive-Based Learning Components	Indicators
Word Uses	• Understand relevant information and utilize mathematical terminology while writing it. • Define the relationship between the known and unknown data, then enter it into a mathematical model.
Visual Mediators	• Identify the tables, graphs, and figures that can be used to depict mathematical concepts.

Commognitive-Based Learning Components	Indicators
Narratives	• Explain information derived from visual mediators and word usage. • Give an explanation of the procedure plan.
Routines	• Execute the procedures for problem-solving and apply the corresponding mathematical formulas. • Verify the word use, visual mediator, routine, and narrative in order to comprehend issues, devise procedures, and execute the plan.

These indicators in Table 1 were used in this study to design the steps of teaching and learning activities that support students' development in written and oral mathematical communication as well as algebraic thinking. The commognitive framework serves as a pedagogical foundation in structuring lesson plans for teaching mathematical concepts. By emphasizing the interplay between communication and cognitive processes, this framework enables students to articulate their mathematical reasoning more effectively while deepening their conceptual understanding (Weingarden & Buchbinder, 2023). Specifically, the framework consists of four interrelated components—word use, visual mediators, narratives, and routines—each playing a critical role in supporting mathematical discourse and cognitive development (Gallego-Sánchez et al., 2023). When integrated into classroom practice, these components enhance student engagement by encouraging learners to express mathematical ideas both verbally and in writing, thus fostering meaningful communication in mathematics.

In this study, the commognitive framework was employed to examine students' algebraic thinking and written mathematical communication skills in solving problems involving systems of linear equations in two variables (SLETV). More specifically, the study aimed to: (1) describe statistically the relationship between algebraic thinking and oral communication skill, and (2) investigate students' written mathematical communication skills and algebraic thinking when solving SLETV problems using a learning approach based on the commognitive framework.

METHODS

This study employs a mixed-methods approach using both descriptive qualitative and quantitative methods (Denscombe, 2014), with the aim of analyzing students' algebraic thinking and mathematical communication skills within a commognitive-based learning framework, specifically on the topic of Systems of Linear Equations in Two Variables (SLETV). The stages of the research are summarized in Figure 1, which presents the structure of the study. The research process was organized into three interconnected phases: preparation, data collection, and data analysis and interpretation. During the initial phase, essential instruments were prepared, and research participants were determined. The data collection phase included classroom-based activities followed by the administration of assessments and interviews. The final phase focused on analyzing the collected data using appropriate quantitative and qualitative techniques, followed by integrating the findings to generate a comprehensive understanding of students' performance and discourse in the context of commognitive-based instruction.

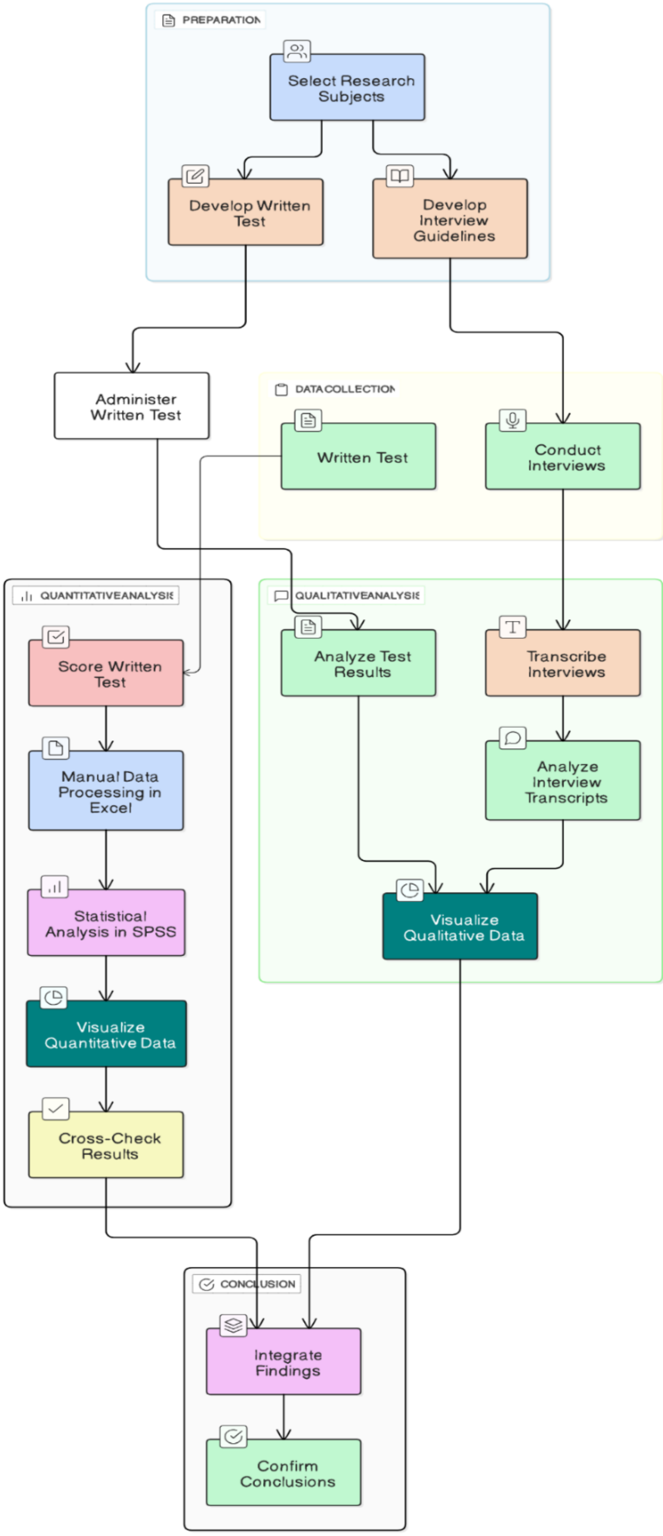


Figure 1. The research procedure flowchart

Hypotheses

To examine the relationship between the two main variables—algebraic thinking skill (X) and oral mathematical communication skill (Y)—the following hypotheses were formulated:

- H_0 (*Null Hypothesis*): There is no relationship between algebraic thinking abilities and students' oral mathematical communication abilities in the context of commognitive-based learning.
- H_a (*Alternative Hypothesis*): There is a significant relationship between algebraic thinking abilities and students' oral mathematical communication abilities in the context of commognitive-based learning.

Research Subjects and Sampling

The research subjects were 29 students from Class VIII.8 of Junior High School 24 Palembang, selected using purposive sampling. This non-probability sampling technique was used to ensure participants had already mastered prerequisite mathematical concepts from the curriculum, allowing them to better engage with the SLETV topic.

Research Instruments

Data in this study were collected through two main instruments. The first instrument was a written test consisting of three open-ended questions, which were developed based on indicators of both written mathematical communication and algebraic thinking. The indicators of algebraic thinking were adapted from three key dimensions: general activities, transformational activities, and global meta-level mathematical activities. These dimensions provided a comprehensive framework for evaluating how students represent, manipulate, and reflect on algebraic problems.

The second instrument was a semi-structured interview, conducted with selected students after the completion of the written test. The interview aimed to obtain more in-depth insights into the students' oral mathematical communication skills during the problem-solving process, particularly in relation to the learning activities guided by the commognitive framework. The interviews allowed for the exploration of how students verbalized mathematical reasoning and engaged in discursive practices relevant to algebraic thinking.

Data Collection Technique

The written test was administered to all participants after they completed learning activities related to SLETV. Students' responses were collected and categorized based on a scoring rubric that classifies written communication skills into five levels: excellent, good, fair, poor, and very poor. Subsequently, interviews were conducted with selected students representing various levels of performance to further explore their oral mathematical communication and how they expressed algebraic thinking through discourse.

Data Analysis Procedures

The quantitative analysis aimed to investigate the statistical relationship between students' algebraic thinking (X) and their oral mathematical communication skills (Y), with both variables treated as ratio data. Regression analysis was employed to obtain the coefficient of determination and the correlation coefficient, indicating the strength and significance of the relationship between the two constructs. The analysis process was carried out using both Microsoft Excel and IBM SPSS Statistics 24 to ensure accuracy and transparency. In addition, visual representations such as tables and graphs were generated using these tools to support the interpretation of results and enhance the clarity of the findings.

The qualitative data, derived from students' written responses and semi-structured interviews, were analysed descriptively using the commognitive framework. This analysis focused on identifying discursive patterns in students' mathematical reasoning and communication by examining four core components of the framework: word use, visual mediators, narratives, and routines. These elements were used to interpret how students constructed and conveyed mathematical understanding, particularly in the context of solving SLETV problems. Through this approach, the study gained in-depth insights into students' mathematical communication processes and the development of their algebraic thinking within commognitive-based learning activities.

RESULTS AND DISCUSSION

The implementation of commognitive-based learning revealed several significant aspects of students' algebraic thinking and oral mathematics communication skills. By applying their understanding of algebraic concepts for – such as equations – students can be guided to address real-world problems when learning the system of linear equation through this framework.

Commognitive-based learning offers students opportunities to explore and articulate the outcomes of their discussions through its structured approach. The collaborative dialogues that occur students work together to solve mathematical problems are closely related to their written and oral communication within the group activities guided by worksheets. Through their discussion, students can share ideas, clarify calculation methods and algebraic concepts utilized in problem solving, and deepen their understanding through reflection and peer feedback. When presenting the result of group discussions to the class, students can explain the concepts they employed and the graphs they constructed.

The information provided above plays a significant role in developing students' algebraic thinking and oral mathematical communication skills, both of which are fostered through commognitive-based learning. This indicates that commognitive-based learning serves as a bridge between these two skills. Through this approach, data were collected from 29 students to examine the relationship between algebraic thinking capabilities and spoken mathematical communication abilities.

The researcher began the study by implementing a learning process using research instruments, namely the lesson plan and the worksheet with commognitive framework learning approach. This approach was intended to observe the relationship between two abilities through the application of the learning. The learning process conducted by researchers revealed several important aspects related to students' algebraic thinking abilities and oral mathematical communication abilities. Learning the SLETV through commognitive approach provides students with the opportunity to be guided in solving real-life problems using algebraic knowledge, particularly concepts such as straight-line equations, which they can apply prior to learning the SLETV.

Algebraic Thinking Skills and Oral Communication Skills

Data were collected from 29 students to examine the relationship between algebraic thinking skills and oral communication skills, as facilitated through a commognitive-based learning. Each variable was assessed using appropriate instruments: algebraic thinking was measured through written test items, while oral mathematical communication was explored through semi-structured interviews. These instruments were designed to ensure alignment with the learning objectives and to capture both the cognitive and communicative dimensions of students' mathematical engagement.

The data from these two variables were subjected to Shapiro-Wilk normality test to determine whether they originated from a normally distributed population. The Figure 2 is following graphic displays the result of the SPSS normality test.

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Aljabar	.101	29	.200*	.967	29	.473

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Komunikasi	.130	29	.200*	.945	29	.138

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Figure 2. Normality test of variable X and Y using Shapiro-wilk

Figure 2 demonstrates that the significance values for the two variables are 0.473 and 0.138, both of which are higher than the significance level of $\alpha = 0.05$. Consequently, the results indicate that the data of both variables originates from a normally distributed population. This result is further supported by the visual representation of the normality test through the Q-Q plot and histogram shown below in Figure 3a and Figure 3b.

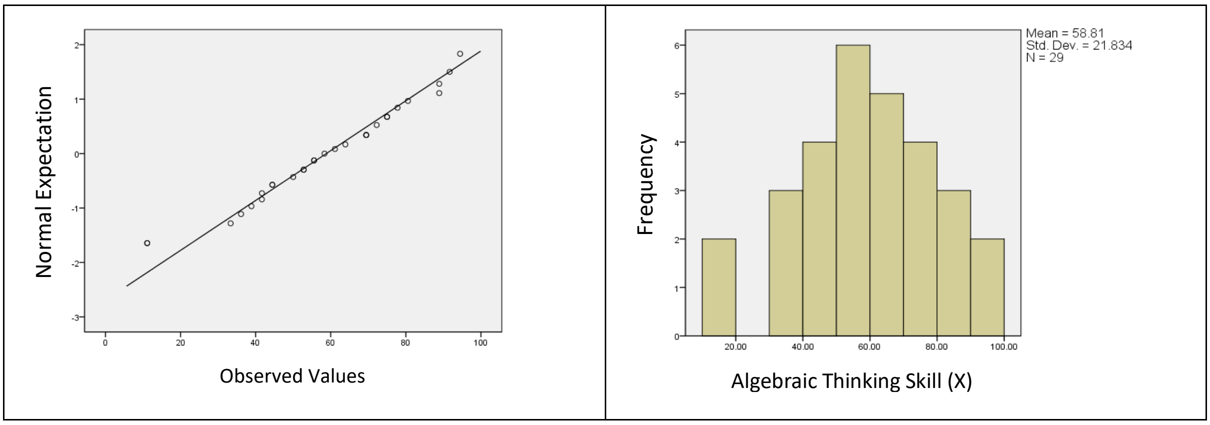


Figure 3a. Visualization of normality test of variable X with Q-Q Plot and histogram

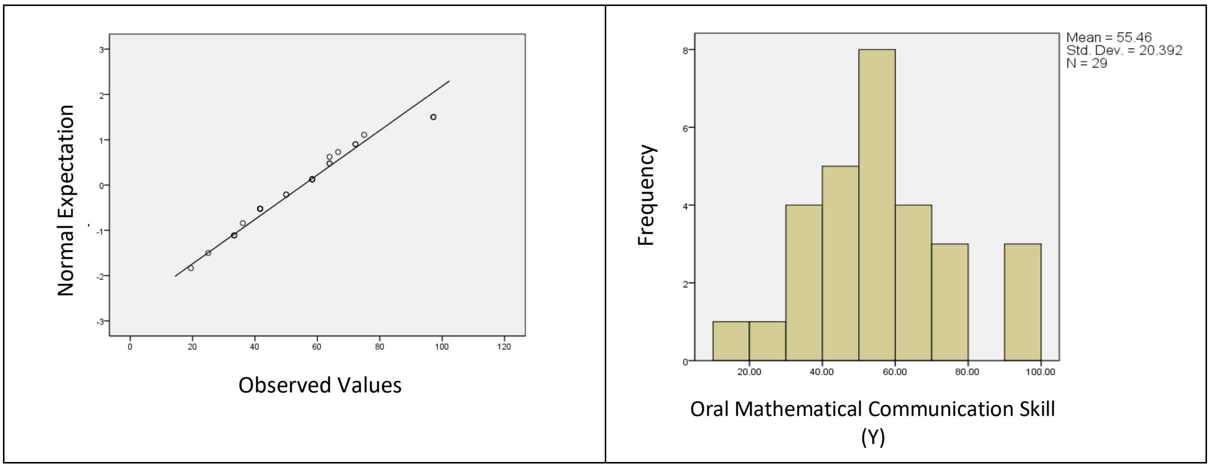


Figure 3b. Visualization of normality test of variable Y with Q-Q Plot and histogram

Figure 3a and Figure 3b showed the visualization of normality tests. The Q-Q plots show minimal deviation of the data points from the reference lines, indicating the quantiles of the X and Y variables closely follow a normal distribution. The two histograms also display a normal curve form. This visualization is consistent with the results of the SPSS normality test for both variables. Upon confirming that the data originates from a normally distributed population, the subsequent regression equation analysis can be conducted. Figure 4 is a scattering plot of algebraic thinking skills and oral mathematical communication skills.

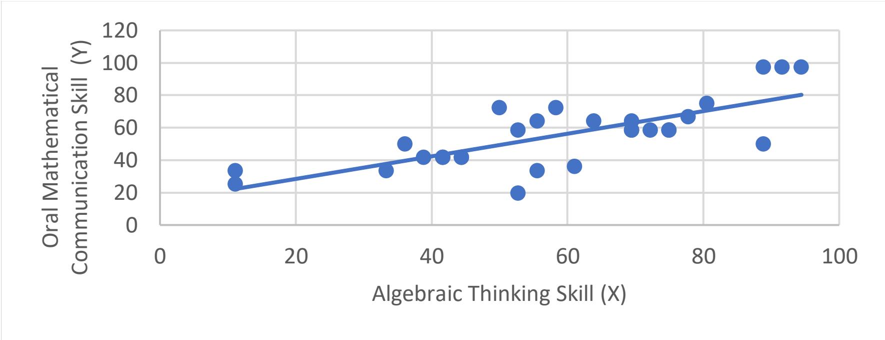


Figure 4. Scatter plot of algebraic thinking skills and oral mathematical communication skills

A blue linear line, known as a trend line or regression equation line, is visible in the scatter plot above (See Figure 4). The regression equation can be derived by using Excel to calculate the necessary constant values.

$$\hat{Y} = 14.49 + 0.7X \qquad (1)$$

The scatter plot's trendline, or regression equation has a gradient of 0.7 and a y-intercept at 14.49. The anticipated value of variable Y, which used to compute the coefficient of determination, can be derived from this regression equation. Excel calculation yielded a coefficient of determination of 0.56. The correlation coefficient was also calculated by taking the square root of the coefficient of determination (0.75). The following Figure 5 was SPSS tests can also verify this calculation.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.746[a]	.556	.540	13.83178

a. Predictors: (Constant), Aljabar

Figure 5. The results of linear regression algebraic thinking skills and oral mathematical communication skills

Figure 5 shows the evident that the correlation and determination coefficient values we compute match the R and R squared values provided by SPSS. To determine whether these values obtained can be generalized to the population, the significance of the correlation coefficient and coefficient of determination must be assessed. Figure 6 is the T test that was used for this analysis.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	14.488	7.494		1.933	.064
	Aljabar	.697	.120	.746	5.819	.000

a. Dependent Variable: Komunikasi

Figure 6. T test result of algebraic thinking skills and oral mathematical communication skills

The significant value, or p-value, of the t test above is 0.000, which is less than the significance level of α=0.05 (See Figure 6). This value will now be analyzed to draw conclusions from the developed hypothesis. According to the data analysis results, the correlation coefficient is 0.75, the coefficient of determination is 0.56, the t-value is 5.82, and the p-value is 0.000003. The correlation coefficient interpretation table from Olive et al. (2010) will be used to interpret the correlation coefficient value. Table 2 were presented with the interpretation of coefficient correlation (R) value.

Table 2. Coefficient correlation interval

Correlation Coefficient Interval	Level of Correlation
$-0.200 \leq R \leq 0.200$	Very low
$-0.400 \leq R \leq -0.200, 0.200 \leq R \leq 0.400$	Low
$-0.600 \leq R \leq -0.400, 0.400 \leq R \leq 0.600$	Average
$-0.800 \leq R \leq -0.600, 0.600 \leq R \leq 0.800$	Strong
$-1.000 \leq R \leq -0.800, 0.800 \leq R \leq 1.000$	Very Strong

Figure 5 depicts the obtained correlation coefficient value indicates a strong relationship, falling within the interval range $0.600 < 0.75 \leq 0.800$, which is categorized as strong (See Table 2). Additionally, this positive correlation coefficient value suggests a one-way relationship, indicating that as variable X increases, variable Y also increases, and vice versa, for the two variables under study. The coefficient of determination value indicates the extent to which one variable influences the other. In this case, the computed coefficient of determination value is 0.56.

Figure 6 demonstrates that variable X has a considerable influence on variable Y. The result of the t-test yielded a p-value of 0.000003, which is considerably small and well below the $\alpha = 0.05$ threshold. This indicates that the obtained data is highly unlikely to have occurred by chance. Therefore, it can be concluded that the data is significantly different from other data with higher p-values, which suggests that H_0 can be rejected, and a statistically significant difference exists.

The results of this study indicate a statistically significant and strong positive correlation between students' algebraic thinking skills and their oral mathematical communication abilities. The regression analysis revealed a correlation coefficient ($r = 0.75$) and a coefficient of determination ($R^2 = 0.56$), signifying that approximately 56% of the variance in students' oral mathematical communication skills can be explained by their algebraic thinking capabilities. This finding aligns with previous studies emphasizing the interconnected nature of mathematical thinking and communication, particularly in algebraic contexts (Sfard, 2012; Blanton et al., 2015). The commognitive-based learning environment employed in this research provided a structured yet collaborative space that promoted both the development and articulation of algebraic concepts, enabling students to link symbolic reasoning with verbal explanation effectively.

Written Communication Skills through Commognitive Learning

The data used in this study were obtained from students' written responses to test items involving SLETV, designed within a commognitive-based learning framework. The aim was to examine the emergence of written mathematical communication indicators, such as the use of symbolic language, algebraic manipulation, representation of real-world contexts using mathematical models, and the interpretation of graphical solutions. Students' written work, such as the construction of equations, substitution or elimination methods, graph plotting, and justification of answers, were carefully analysed.

The students' responses were categorized into five levels: excellent, good, fair, poor, and very poor. These categories were based on the accuracy, completeness, and clarity of mathematical expressions and explanations. For instance, as shown in the sample answer (Figure 7), the student correctly translated the context into two linear equations, applied substitution to find the intersection point (600, 500), and used a labelled graph to reinforce the solution. Subjects were then selected based on these performance levels and further validated through recommendations from their mathematics teachers.

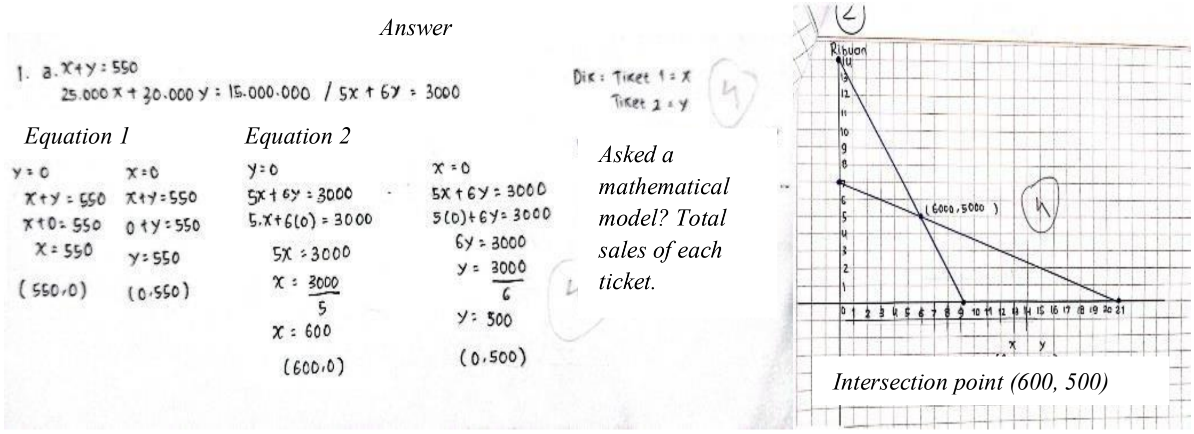
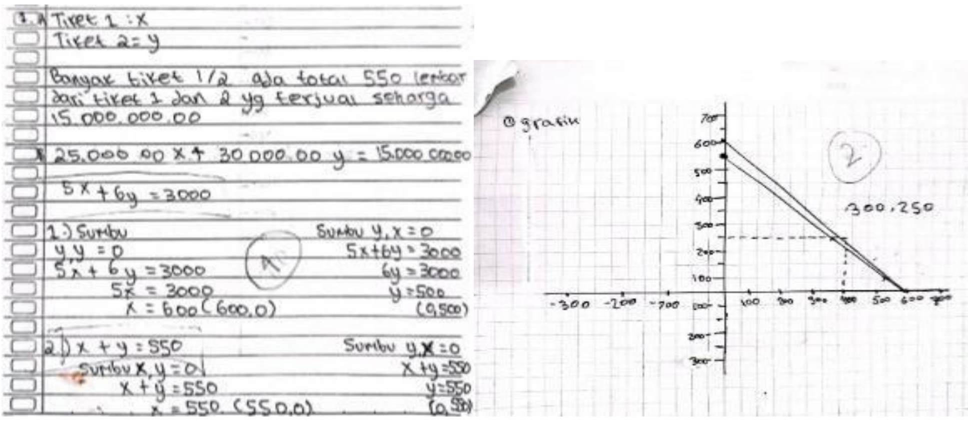


Figure 7. S1's solution in the written test (excellent level)

Based on Figure 7, S1 can ascertain what is known, what is asked, and develop mathematical models. The student demonstrates the ability to identify known information, interpret the problem context, and formulate two appropriate linear equations representing the total ticket sales. S1 proceeds to solve the equations algebraically—using substitution or elimination—and finds the solution as the point of intersection, (600, 500), which is interpreted in the context of the problem.

Furthermore, S1 constructs a coordinate graph with properly labelled axes and plots the two equations accurately, indicating the intersection point as the solution. This demonstrates proficiency in mathematical representation and interpretation. In the other hand, S1 shows strong written mathematical communication skills through three key indicators: (1) translating contextual problems into mathematical models, (2) performing accurate algebraic procedures, and (3) representing and interpreting solutions graphically.

To further explore students' written mathematical communication skills, responses from Subject S2 were analyzed based on the same problem-solving task. Figure 8 presents Subject S2's written response to the task.



English version:

Ticket 1 = x, ticket 2 = y

Many tickets 1 and 2 there are a total of 550 pieces. from tickets 1 and 2 sold for 15,000,000

Figure 8. S2 student's solution in the written test (good level)

Based on Figure 8, S2 demonstrates the ability to respond to the test questions appropriately. The written work shows that the student can identify the known information, determine what is being asked, and construct relevant mathematical models. This is also supported by the following interview excerpt:

R : "While working on the problem did you have any difficulties in solving it?"
S2 : "No, Mam."
R : "How did you solve it?"
S2 : "From the information in the problem."
R : "What is the information?"
S2 : ""There are 1 ticket and 550 tickets, and the price of ticket 1 is 25,000 and the price of ticket 2 is 30,000. Then the total sales are 15,000,000."
R : "What is asked from the question?"
S2 : ""Find the mathematical model and price of ticket 1 and ticket 2."
R : ""How to determine the math model?"
S2 : "By assuming ticket 1 = x and ticket 2 = y. Because the sum of all tickets 1 and ticket 2, then the equation x + y = 550."

This dialogue indicates that S2 understands the context of the problem and can translate verbal information into mathematical form. Furthermore, S2 represents the situation graphically in the Cartesian plane, although with a slight error in determining the intersection point of the two lines. In terms of mathematical expression, S2 can carry out the necessary computations accurately and completely. Overall, all three indicators of written mathematical communication—understanding the problem, expressing mathematical ideas, and visual representation—emerged clearly in S2's performance, supported by both written work and verbal explanation.

The following Figure 9 presents Subject S3's written response to the test, which illustrates how the student applied commognitive-based learning in solving a SLETV. This response will be analyzed to identify the emergence of written mathematical communication indicators.

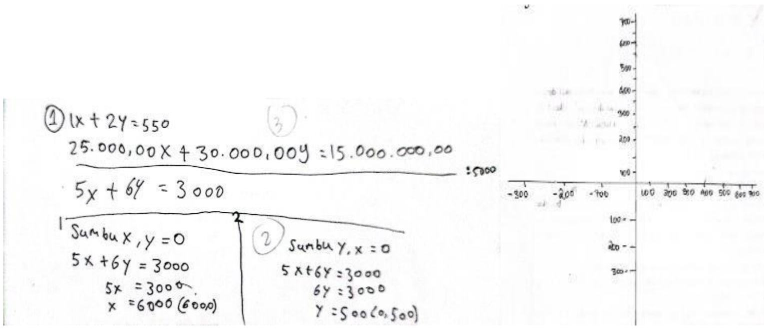
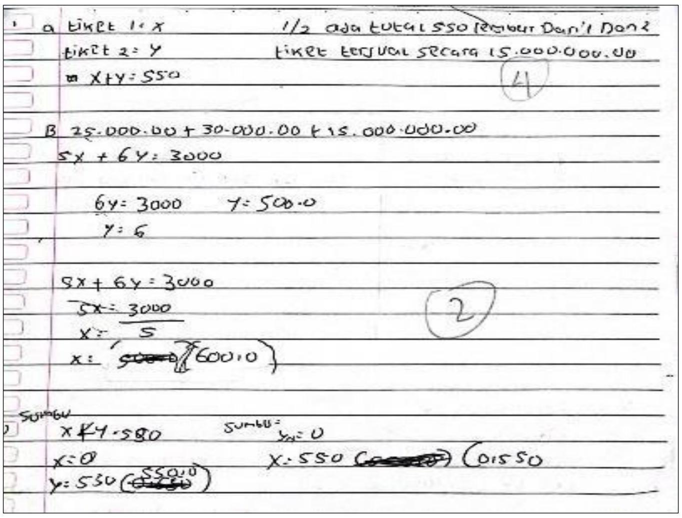


Figure 9. S3 student's solution in the written test (fair level)

Figure 9 indicates that S3 demonstrates fair (moderate) skill in written exam indicators, although the response remains incomplete. In other words, S3 tends to proceed directly to solving mathematical models without fully articulating the problem-solving process in writing. In the drawing section, S3 is only able to define cartesian coordinates and is unable to determine the coordinate points. However, in terms of mathematical expressions, S3 can calculate equation coordinates accurately. Like S2, S3 demonstrates the emergence of all three indicators of written mathematical communication skills; however, in the drawing portion, the graph representation is incomplete, with only the Cartesian coordinates being shown. Meanwhile, the response from a student categorized as poor can be seen in Figure 10.



English version:
1 or 2 there are a total of 550 pieces, and 1 and 2 tickets are sold in 15,000,000.

Figure 10. S4 student's solution in the written test (poor level)

Figure 10 illustrates that S4 demonstrates limited skill in correctly answering the questions. While S4 performs reasonably well in identifying the known information in the written exam indicator, this occurs despite the question not being explicitly presented. However, the mathematical model developed by S4 contains several errors. In terms of the drawing indicator, S4 does not produce any visual representation and is therefore unable to demonstrate this aspect. Regarding the mathematical expression indicator, S4 is still unable to provide a fully accurate, step-by-step answer and appears unaware of the correct procedure to follow. Consequently, only two of the three indicators of written mathematical communication skill, the written test and mathematical expression—are partially demonstrated by the S4, with notable errors still present in the mathematical expression. Meanwhile, the response from a student categorized as very poor can be seen in Figure 11.

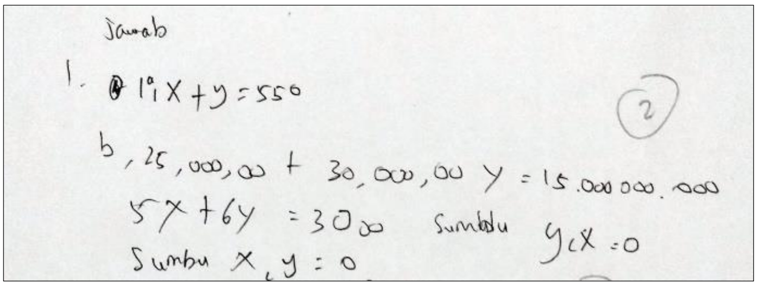


Figure 11. S5 student's solution in the written test (very poor level)

Figure 11 illustrates that subject S5 did not accurately complete the written exam indicator. The subject only wrote the mathematical model of the problem without identifying what was known or what was being asked. Furthermore, the drawing and mathematical expression indicators were not demonstrated at all. Based on the data analysis, the written test indicators of mathematical communication skills are the most frequently observed during the problem-solving process. In general, students can construct mathematical models and identify key information from the questions, such as known values and the specific questions being asked.

Algebraic Thinking Skill and Oral Mathematical Communication Skill of Students

The skill to think algebraically is described by Kontorovich (2021) as the use of algebraic "tools" to solve real-world problems. By examining the indicators used to assess students' algebraic thinking abilities, it becomes clear that students' skills in using algebra itself are central to solving real-world problems. This includes how students generalize real-world problems into mathematical language models, select and apply correct and appropriate algebraic tools to process these models, and finally interpret the results in the context of the real world. All of these components are encompassed in algebraic thinking skills (Sfard, 2012; Venegas-Thayer, 2019).

This algebraic thinking skill is theoretically a continuation of arithmetic thinking skill (Amalia et al., 2023). It is undeniable that the "tools" students use to solve algebraic problems are abstract in nature. Mastery of these tools requires further adjustment, particularly for students transitioning from arithmetic studies to algebra. In addressing this abstraction, students cannot rely solely on themselves; there must be communication between them and their environment to support their understanding.

Algebraic Thinking Skills and Oral Communication Skills

In practice, students' understanding of algebra depends not only on their cognitive abilities but also on the influence of their surrounding environment. This was evident in class VIII.8 students at Junior High School 24 Palembang. Effective mathematical communication with the teacher positively influences students' success in understanding algebra. Additionally, group discussions, presentations and question-and-answer sessions, and peer teaching—where students who grasp the concepts help those who do not—serve as forms of oral mathematical communication that facilitate the construction of students' understanding. These findings correspond to the fact that the correlation coefficient value obtained, which indicates a strong positive relationship between students' algebraic thinking abilities and their oral mathematical communication abilities. In other words, an improvement in students' algebraic thinking abilities is accompanied by a corresponding enhancement in their oral mathematical communication skills.

Commognitive approach also has an important role in encouraging a relationship between students' algebraic thinking abilities and oral mathematical communication abilities. The learning process, provided by researchers through instruments designed to elicit indicators of these two abilities, supports this connection. Through group worksheets activities, students not only generate algebraic ideas to solve problems but are also required to collaborate, discuss, and work together with peers. This process enables students to combine algebraic thinking skills with good oral communication in solving algebra-related problems. Therefore, the observed relationship between students' algebraic thinking and oral mathematical communication abilities demonstrates a strong correlation.

This strong positive relationship between algebraic thinking abilities and students' oral mathematical communication abilities is also closely linked to the teacher's role in designing learning activities that integrated oral mathematical communication as a tool to support students' understanding of algebraic concepts and their real-life applications. In practice, researchers realize that students from diverse backgrounds have different approaches to understanding concepts. Data indicate that a small number of students with relatively good algebraic thinking skills still exhibit weak oral mathematical communication skills, and vice versa—some students with strong oral communication skills lack sufficient algebraic thinking abilities (Blanton et al., 2015). However, the role of oral mathematical communication here is not merely as a learning method, but rather as a means of translating mathematical language, which has its own uniqueness. This mathematical language can sometimes act as a barrier for students in understanding abstract mathematical concepts, particularly algebra. Strong oral mathematical communication skills are certainly beneficial for students in understanding algebra, and conversely, students with strong algebraic thinking skills are likely to demonstrate good oral mathematical communication skills.

This is also one of the factors that explains why the level of relationship between students' algebraic thinking abilities and their oral mathematical communication skills through commognitive learning falls into the category of a strong positive relationship. The relationship between these two abilities cannot be separated from the extent to which one contributes to the development of the other. The researchers calculated the coefficient of determination using a regression equation based on the collected data, resulting in a coefficient of determination value of 0.55. This value shows that the formation of oral mathematical communication skills is significantly influenced by algebraic thinking abilities. Among the various factors that influence the formation of oral mathematical communication skills, more than half of the process is attributed to algebraic thinking skills (Byers, 2016). This relatively high value aligns with the notion that the application of algebraic thinking fosters students' skill to understand mathematical language in a more formal and accurate manner. Students' skill to communicate is also developed through abstract symbols, which students must be able to represent both in written and visual form. However, this value does not suggest that algebraic thinking is the sole factor in developing students' mathematical communication abilities; other contributing factors also play a role (Suwardi, 2022; Zayyadi et al., 2019). This aligns with the data showing that some students still exhibit an imbalance between these two abilities.

The learning design embedded in the commognitive approach played a pivotal role in fostering these skills. Through the integration of worksheets, group problem-solving tasks, and whole-class presentations, students were given opportunities to externalize their algebraic reasoning in both written and spoken forms. This is consistent with Sfard's (2008) theory, which posits that thinking and communicating in mathematics are inherently intertwined. As students moved between discursive practices—talking, writing, modelling—they engaged in a process of mathematizing real-world contexts, which further reinforced their understanding. Moreover, the learning involving engaging communication allowed for meaningful contextualization, helping students transition from procedural operations to conceptual comprehension (Byers, 2016; Kontorovich, 2021).

The study also identified individual variances in the strength of this correlation. While most students who demonstrated strong algebraic thinking also exhibited proficient oral communication skills, a few exceptions were noted. This suggests that while algebraic competence significantly contributes to students' ability to communicate mathematically, it is not the sole determinant. Factors such as linguistic fluency, confidence in public speaking, and familiarity with mathematical discourse conventions may also influence performance (Zayyadi et al., 2019; Suwardi, 2022). As such, instructional strategies that explicitly support discourse norms in mathematics—such as structured talk moves, sentence starters, and peer-feedback protocols—should be incorporated to scaffold students' verbal expression alongside their symbolic reasoning.

The implication of this study extends to both curriculum design and pedagogical practice. A deliberate integration of algebraic thinking and oral communication within mathematical instruction not only promotes a deeper conceptual grasp of algebra but also prepares students for collaborative problem-solving in real-life contexts. This reinforces the assertion that communication is not merely a byproduct of mathematical understanding but a vital component of it (Venegas-Thayer, 2019; Amalia et al., 2023). Hence, adopting a commognitive lens in mathematics education offers a powerful framework for bridging the gap between thought and expression, thereby enhancing both cognitive and communicative dimensions of learning.

CONCLUSION

Based on the analysis of data collected from eighth-grade students at Junior High School 24 Palembang, a strong positive correlation (r = 0.75) was found between students' oral mathematical communication skills and their algebraic thinking skills when taught through a commognitive-based learning approach. This statistically supports the alternative hypothesis (Ha), indicating that students who communicate their reasoning effectively are also more capable of engaging in algebraic thinking. In addition to the statistical analysis, a detailed examination of students' written responses revealed varied levels of mathematical communication skills. Among the 29 students assessed, four students were categorized as excellent, nine students as good, 11 students as average, three students as poor, and two students as very poor. These assessments were based on three indicators: written test responses, mathematical expressions, and diagrammatic representations. Most students successfully identified known information, interpreted the question, and formulated algebraic models, showing evidence of algebraic reasoning. However, fewer students could accurately express solutions graphically or symbolically, especially in terms of constructing coordinate graphs and performing algebraic computations. These findings suggest that the commognitive framework supports not only oral interaction but also fosters the development of written mathematical communication and algebraic thinking. Students' written work reflected varying levels of integration between conceptual understanding and symbolic representation, pointing to the importance of continued support in helping students externalize their thinking both verbally and in written form. Another long-term implication of this research also contributes significantly to the advancement of basic education by generating evidence-based knowledge that informs educational policies, enhances instructional practices, and promotes the improvement of student learning outcomes. However, this study is limited by its small sample size and single-school setting, which may affect the generalizability of the findings. The future studies of the relationship between students' mathematical communication skills and algebraic thinking through a commognitive-based learning approach can be enriched by addressing several critical dimensions. First, expanding the sample size beyond the confines of a single institution would be necessary to ensure that the findings are widely applicable across different educational contexts. A larger and more diverse sample could provide deeper insights into the variations of students' communication and algebraic abilities, enhancing the robustness of the results. Such an approach would facilitate a more comprehensive understanding of how commognitive frameworks operate across varying demographic and socio-economic backgrounds, which have been shown to impact educational outcomes.

ACKNOWLEDGMENTS

We would like to sincerely thank the knowledgeable validators for their insightful comments. We also appreciated the time and cooperation of the teachers and students who took part in the study. We also like to express our gratitude to our supervisors and colleagues for their help and direction during the study process. We also thank Universitas Sriwijaya for its financial assistance from Sriwijaya University Research Grant year 2023, contract number SP DIPA-023.17.2.677515/2023.

DECLARATIONS

Author Contribution	:	WDP: Conceptualization, Writing - original draft, Investigation, and Visualization; Z: Writing - review & editing, Supervision, Validation; RIIP: Writing - review & editing, Supervision, Methodology; CH: Writing - review & editing, Supervision, Investigation
Funding Statement	:	The authors extend their sincere appreciation to Universitas Sriwijaya for funding this study, Sriwijaya University Research Grant year 2023, contract number SP DIPA-023.17.2.677515/2023.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

REFERENCES

Amalia, Y. R., Pratiwi, W. D., Aisyah, N., Kurniadi, E., Araiku, J., & Astuti, P. (2023). Quantitative thinking level of high school students for geometry learning based on Rigorous Mathematical Thinking (RMT) approach. *AIP Conference Proceedings*, *2811*(1). https://doi.org/10.1063/5.0142259

Blanton, M., Brizuela, B. M., Gardiner, A. M., Sawrey, K., & Newman-Owens, A. (2015). A learning trajectory in 6-year-olds' thinking about generalizing functional relationships. *Journal for Research in Mathematics Education*, *46*(5), 511–558. https://doi.org/10.5951/jresematheduc.46.5.0511

Byers, P. (2016). Knowledge claims in cognitive development research: Problems and alternatives. *New Ideas in Psychology*, *43*, 16–27. https://doi.org/10.1016/J.NEWIDEAPSYCH.2016.03.004

Chan, M. C. E., Moate, J., Clarke, D., Cunnington, R., Díez-Palomar, J., Friesen, M., Haataja, E., Hošpesová, A., Kuntze, S., Nieminen, J. H., Novotná, J., Ochôa, X., Sherwell, C., Trần, D., & Tuohilamp., L. (2022). Learning research in a laboratory classroom: A reflection on complementarity and commensurability among multiple analytical accounts. *ZDM*. https://doi.org/10.1007/s11858-022-01330-0

Ching, M., Chan, E., & Sfard, A. (2020). On learning that could have happened : The same tale in two cities. *Journal of Mathematical Behavior*, *60*(October), 100815. https://doi.org/10.1016/j.jmathb.2020.100815

Daher, W., & Swidan, O. (2019). Low achieving students' realization of the notion of mathematical equality with an interactive technological artifacts. *Eurasia Journal of Mathematics Science and Technology Education*, *15*(4). https://doi.org/10.29333/ejmste/103073

Denscombe, M. (2014). *Open UP Study Skills: The Good Research Guide for Small-Scale Sosial Research Project* 4th Eds. London: Open University Press. https://www.librarysearch.manchester.ac.uk/discovery/fulldisplay?vid=44MAN_INST:MU_NUI&search_scope=MyInst_and_CI&docid=alma992975780418601631&lang=en&context=L&adaptor=Local%20Search%20Engine&query=creator,exact,Knoke,%20David.,AND

Lu, J., Tuo, P., Feng, R., Stephens, M., Zhang, M., & Shen, Z. (2022). Visualizing commognitive responsibility shift in collaborative problem-solving during computer-supported one-to-one math tutoring. *Frontiers in Psychology*. https://doi.org/10.3389/fpsyg.2022.815625

Gallego-Sánchez, I., Martín-Molina, V., Caro-Torró, I., & Izquierdo, J. M. G. (2023). Spanish primary school students' engagement with a non-traditional method when adding and subtracting: Ritualised versus exploratory participation. *Education 3-13*. https://doi.org/10.1080/03004279.2023.2171739

Handayani, R., & Wandini, R. R. (2020). The effect of using interactive multimedia on conceptual understanding in mathematics learning [In Bahasa]. *Intersections*, *5*(2), 30. https://doi.org/10.47200/intersections.v5i2.553

Handayani, U. F. (2021). Students' mathematical communication in solving tug-of-war algebra problems [in Bahasa]. *ARITMATIKA: Jurnal Riset Pendidikan Matematika*, *2*(2), 67–78. https://doi.org/10.35719/aritmatika.v2i2.58

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51–52*, 128–145. https://doi.org/10.1016/J.IJER.2011.12.015

Khaidir, C., Fitriza, R., Yumariza, N., & Wahid, R. (2024). Communication skills and mathematical disposition in implementing geogebra-assisted problem-based learning strategies. *Alifmatika Jurnal Pendidikan Dan Pembelajaran Matematika*, *6*(1), 51–62. https://doi.org/10.35316/alifmatika.2024.v6i1.51-62

Kontorovich, I. (2021). Pre-university students square-root from squared things: A commognitive account of apparent conflicts within learners' mathematical discourses. *The Journal of Mathematical Behavior*, *64*, 100910. https://doi.org/10.1016/J.JMATHB.2021.100910

Kristiani, Y. D., Juandi, D., & Putri, R. I. I. (2024). Students' mathematical communication skills on straight-line equation using PMRI and collaborative learning. *Mathematics Education Journal*, *18*(2), 163–180. https://doi.org/10.22342/jpm.v18i2.pp163-180

Lu, J., Zhang, Y., & Li, Y. (2023). Visual analysis of commognitive conflict in collaborative problem solving in classrooms. *Frontiers in Psychology, 14*. https://doi.org/10.3389/fpsyg.2023.1216652

Makgakga, T. P. (2023). Twinned Teachers' Mathematical Discourse Using Problem-Solving. *Journal of Culture and Values in Education*, *6*(2), 138–157. https://doi.org/10.46303/jcve.2023.13

Marsitin, R., & Sesanti, N. R. (2018). Limit learning with apos theory and maple to develop mathematical communication and critical thinking. *Proceedings of the University of Muhammadiyah Malang's 1st International Conference of Mathematics Education*. Malang: Universitas Muhammadiyah Malang. https://doi.org/10.2991/incomed-17.2018.12

Matthews, P. G. (2024). Reflecting from the border between mathematics education research and cognitive psychology. *Journal for Research in Mathematics Education*, *55*(2), 68-75. https://doi.org/10.5951/jresematheduc-2023-0229

Nauli, N., Harisman, Y., Armiati, & Yerizon. (2024). Junior high school students' habits of mind in solving mathematical problems. *Mathematics Education Journal*, *18*(1), 39–58. https://doi.org/10.22342/jpm.v18i1.pp39-58

Olive, J., Makar, K., Hoyos, V., Kor, L.K., Kosheleva, O., Sträßer, R. (2009). Mathematical Knowledge and Practices Resulting from Access to Digital Technologies. In: Hoyles, C., Lagrange, JB. (eds) *Mathematics Education and Technology-Rethinking the Terrain*. New ICMI Study Series, vol 13. Springer, Boston, MA. https://doi.org/10.1007/978-1-4419-0146-0_8

Otten, M., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (2019). The balance model for teaching linear equations: A systematic literature review. *International Journal of STEM Education*, *6*(1). https://doi.org/10.1186/s40594-019-0183-2

Pinto, A., Cooper, J. (2023). Boundary Crossing in Curriculum Reform. In: Shimizu, Y., Vithal, R. (eds) *Mathematics Curriculum Reforms Around the World*. New ICMI Study Series. Springer, Cham. https://doi.org/10.1007/978-3-031-13548-4_28

Rabin, J. M., Fuller, E., & Harel, G. (2013). Double negative: The necessity principle, commognitive conflict, and negative number operations. *The Journal of Mathematical Behavior*, *32*(3), 649–659. https://doi.org/10.1016/J.JMATHB.2013.08.001

Rahmatina, D., Nusantara, T., Parta, I. N., & Susanto, H. (2022). Statistical reasoning process of students in decision making using commognitive framework. *Acta Scientiae*, *24*(3), 63–88. https://doi.org/10.17648/acta.scientiae.6603

Rahmawati, A., Cholily, Y. M., & Zukhrufurrohmah, Z. (2023). Analyzing students' mathematical communication ability in solving numerical literacy problems. *Mosharafa: Jurnal Pendidikan Matematika*, *12*(1), 59–70. https://doi.org/10.31980/mosharafa.v12i1.1938

Rahmawati, M. S., & Soekarta, R. (2021). Social Media-based e-learning and online assignments on algebraic materials. *Jurnal Pendidikan Matematika*, *15*(2), 175–190. https://doi.org/10.22342/jpm.15.2.13714.175-190

Setyowati, S., Purwanto, P., & Sudirman, S. (2022). Cognitive analysis of students in solving circle problems reviewed from the learning style [in Bahasa]. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *6*(2), 2336–2351. https://j-cup.org/index.php/cendekia/article/view/1625

Sfard, A. (2012). Introduction: Developing mathematical discourse-Some insights from communicational research. *International Journal of Educational Research*, *51–52*(June), 1–9.

https://doi.org/10.1016/j.ijer.2011.12.013

Shinno, Y., & Fujita, T. (2021). Characterizing how and when a way of proving develops in a primary mathematics classroom: A commognitive approach. *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739x.2021.1941365

Sukasno, Zulkardi, Putri, R. I. I., & Somakim. (2024). Students' cognitive processes in understanding fractions through the tourist context. *Mathematics Education Journal*, *18*(1), 27–38. https://doi.org/10.22342/jpm.v18i1.pp27-38

Taherdoost, H. (2022). What are different research approaches? Comprehensive review of qualitative, quantitative, and mixed method research, their applications, types, and limitations. *Journal of Management Science & Engineering Research*, *5*(1), 53–63. https://doi.org/10.30564/jmser.v5i1.4538

Venegas-Thayer, M.A. (2019). Integration from a commognitive perspective: an experience with mathematics and music students. In: Doig, B., Williams, J., Swanson, D., Borromeo Ferri, R., Drake, P. (eds) *Interdisciplinary Mathematics Education*. ICME-13 Monographs. Springer, Cham. https://doi.org/10.1007/978-3-030-11066-6_4

Viirman, O. (2014). The function concept and university mathematics teaching. *Published Dissertation*. Karlstad University Studies: Sweden. https://www.diva-portal.org/smash/get/diva2:693890/fulltext01.pdf

Weingarden, M., & Buchbinder, O. (2023). Teacher Learning to Teach Mathematics via Reasoning and Proving: A Discursive Analysis of Lesson Plans Modifications. *Frontiers in Education, 8*. https://doi.org/10.3389/feduc.2023.1154531

Zayyadi, M., Nusantara, T., Subanji, Hidayanto, E., & Sulandra, I. M. (2019). A commognitive framework: The process of solving mathematical problems of middle school students. *International Journal of Learning, Teaching and Educational Research*, *18*(2), 89–102. https://doi.org/10.26803/ijlter.18.2.7

Zayyadi, M., Nusantara, T., Hidayanto, E., Sulandra, I. M., & Sa'dijah, C. (2020). Content and pedagogical knowledge of prospective teachers in mathematics learning: Commognitive framework. *Journal for the Education of Gifted Young Scientists*, *8*(1), 515–532. https://doi.org/10.17478/jegys.642131

Zulu, M. W., & Mudaly, V. (2023). Unveiling problem-solving strategies of pre-service mathematics teachers: a visual and discursive exploration. *Eurasia Journal of Mathematics Science and Technology Education, 19*(7), em2299. https://doi.org/10.29333/ejmste/13344