

Arithmetic Sequences and Series Learning Using Realistic Mathematics Education Assisted by Augmented Reality

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Abstract

Effective mathematics learning is a significant challenge in improving students' understanding and skills in mathematical concepts. One exciting approach to enhancing mathematics education is Realistic Mathematics Education (RME), which emphasizes understanding mathematical concepts through real-world contexts. This research combines the RME approach with Augmented Reality (AR) technology to facilitate the learning of sequences and arithmetic series. This research aims to create a learning trajectory that can help students understand the concept of arithmetic sequences and series using the context of the Borobudur Temple in grade 8th of SMP Negeri 4 Semarang. This research uses a design research method which consists of three stages, namely preliminary design, design experiment (pilot experiment and teaching experiment), and retrospective analysis. In this research, a series of learning activities were designed and developed based on the RME approach. This research involved 6 grade 8th students. The result of this research is a learning trajectory that includes a series of learning processes in three activities, namely: 1) Observing Borobudur Temple videos to understand the definition and characteristics of sequences and series; 2) finding the formulas and results of sequences and series; 3) and solving contextual problems related to sequences and series. The activities carried out can help improve students' understanding of the sequences and arithmetic series material. The research results show that the context of the Borobudur Temple can help students understand the concept of sequences and arithmetic series. Apart from that, the results of this research add options for local wisdom that can be used as a context for mathematics learning, especially sequences, and arithmetic series.

Keywords: Augmented Reality, Mathematics Learning, Realistic Mathematics Education

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INTRODUCTION

The importance of understanding mathematical concepts lies in the fact that to comprehend new concepts, an understanding of previous concepts is required (Nasir, 2017; Simon, 2017). Students will be more capable of remembering and applying a mathematical concept they have learned to solve various types of mathematical problems if they possess a strong understanding of mathematical concepts.

A strong understanding of mathematical concepts, such as arithmetic sequences and series, is essential not only in academic contexts but also in everyday life. However, students often need help comprehending and applying these mathematical concepts due to a lack of connection between what is learned in the classroom and the real world. For example, one common issue is the scarcity of questions about sequences and series that can be connected to everyday life situations. Hardiyanti (2016) revealed that some difficulties students face in solving sequence and series problems include challenges in specifying known information, formulating questions, and determining the solution steps in story problems. Other difficulties involve determining the formula for the *nth* term of an arithmetic or

geometric sequence and understanding the concept of the first term in a sequence.

The use of context in mathematics education allows mathematical concepts to become relevant (Müller & Kohlhase, 2009; Beswick, 2011; Nursyahidah et al., 2023). Practical and relevant mathematics learning is a continuous challenge faced by educators. For example, when students learn the concept of arithmetic sequences without understanding how the concept can be applied in real-world contexts such as financial problems, scheduling, or project planning. Furthermore, a lack of connection also occurs when teaching focuses solely on theoretical aspects without providing relevant practical examples or applications. Students may need help to identify the relevance and usefulness of arithmetic sequence and series concepts in everyday life if there is no apparent connection between what they learn in class and real-world situations.

To address this challenge, the Realistic Mathematics Education (RME) approach has emerged as an intriguing solution. This approach emphasizes understanding mathematical concepts through realworld contexts. Mathematics at RME will not be taught as ready-made formulas students remember (Nursyahidah & Albab, 2021; Van Zanten & Van den Heuvel-Panhuizen, 2021). In the context of RME, students are encouraged to explore and understand mathematical concepts by seeing how they can be applied in everyday situations (Anwar et al., 2012). This can help students recognize the relevance and utility of mathematics in their lives (Nusaibah & Murdiyani, 2017). In addition to the RME approach, the development of AR technology has opened new opportunities in mathematics learning (Gargrish et al., 2021). AR combines the physical world with virtual elements, creating a more interactive and engaging learning experience (Mustaqim, 2016). In the context of mathematics learning, AR allows students to interact with mathematical concepts in an environment that closely resembles the real world.

This research focuses on the implementation of the RME approach assisted by AR in teaching arithmetic sequences and series. This research uses the design research method developed by Gravemeijer and Cobb (2006). Design research is a methodology that aims to increase the impact and transformation of educational research into practice (Nursyahidah et al., 2021). The researcher will explore how the combination of the contextual RME approach with AR technology can facilitate a better understanding and application of these mathematical concepts by students. The novelty in this research lies in the use of Borobudur Temple in AR to design learning about arithmetic sequences and series, which has not been done before. With the RME approach and the assistance of AR technology, this learning design is expected to help students understand the concepts of arithmetic sequences and series. Previous research has demonstrated that reflective learning using the RME approach can enhance students' understanding of the concept of artihmetic sequence (Gravemeijer, 2012; Nursyahidah et al., 2021).

Based on the background described above, the author is conducting research on transformative learning design for 8th-grade students using the context of Borobudur Temple. The learning will be designed in a Hypothetical Learning Trajectory (HLT) with the support of AR technology. By integrating RME and AR in mathematics education, the researcher hopes to make a positive contribution

to efforts aimed at improving the quality of mathematics education and helping students understand how relevant mathematics is in their everyday lives.

METHODS

The research methodology employed in this study is known as design research. Design research is a systematic approach aimed at generating novel theories, artifacts, and practical models that both explain and have an impact on learning in authentic educational settings (Akker et al., 2006). It typically comprises three distinct stages, as outlined by Gravemeijer and Cobb (2006).

- 1. Preliminary Design: During this initial phase, activities center around the creation of a Hypothetical Learning Trajectory (HLT). Hypothetical Learning Trajectory (HLT) in the context of learning sequences and series aims to outline the ideal learning design and expected thought processes of students as they comprehend the concepts of arithmetic sequences and series. In the initial stage of this research, the researcher designed a learning design associated with the context of Borobudur Temple.
- 2. Design Experiment: In the subsequent stage, the designed interventions are tested with students in an experimental setting. This testing phase is crucial for assessing whether the expectations and hypotheses formulated during the preliminary design align with the actual outcomes observed. The designed and tested intervention was learning with the context of Borobudur Temple incorporated into it.
- 3. Retrospective Analysis: At this stage, the collected data will be analyzed to assess the outcomes of the experiment or intervention, aiming to understand the extent to which learning objectives are achieved. Subsequently, the findings will be compared with the expectations and hypotheses formulated during the initial design phase. In this article, we will primarily focus on presenting the experimental results, offering a detailed account of the testing of the HLT within small groups.

The participants in this research consisted of six 8th-grade students from one state junior high school in Semarang. These six students were categorized into three ability levels: two with high abilities, two with moderate abilities, and two with low abilities. This selection represents the whole and can strengthen the generalizability of research results to the entire population, as it includes variations in levels of ability or specific characteristics. The determination of students' ability levels was based on teacher recommendations and their performance in previous learning materials. By selecting students from three different proficiency levels, the research can represent the entire spectrum of abilities in the class. This helps in understanding how this learning approach can impact students with diverse proficiency levels. Furthermore, data collection techniques during the research encompassed the use of video recordings to capture the learning process, conducting interviews, and collecting completed student worksheets.

RESULTS AND DISCUSSION

The results of this research are a learning pathway in the context of ethnomathematics at Borobudur Temple, integrated with AR to understand the material of sequences and series in grade 8. This learning pathway includes three activities adapted to the characteristics of Realistic Mathematics Education (RME) to facilitate students' understanding of sequences and series. The three activities in this learning pathway comprise: 1) observing videos to understand the definition and characteristics of sequences and series; 2) finding the formulas and results of sequences and series; 3) and solving contextual problems related to sequences and series. The understanding and results of students' work in this learning pathway can be observed from the results of assignments and interviews. The following is a detailed explanation of the results of this research.

Activity 1. Watch the Video to Understand the Definition and Properties of Sequence and Series



Figure 1. Video of the Candi Borobudur

In Activity 1, students were asked to explore the video context related to the facts about Borobudur Temple (see Figure 1) and its relevance to the material presented to understand sequences and series. Activity 1 begins by dividing students into three groups, each consisting of 2 students with high abilities, 2 students with medium abilities, and 2 students with low abilities. Next, the teacher provides students with a worksheet containing instructions and a series of activities that students will carry out to understand the material on sequences and series. After that, students are tasked with calculating the sequence of sequences and the sum of series within the stupas at Borobudur Temple. Student answers can be seen in Figure 2.

jumlah stupa pada masing-masing teras Candi Borobudur

Teras	Jumlah Stupa
Teras 1	32
Teras 2	24
Teras 3	16

Translated to English:

Number of stupas on each terrace of Borobudur Temple? (see figure)

Figure 2. Students' answers regarding the number of stupas at each level of Borobudur Temple

Based on Figure 2, students successfully identifying and finding arithmetic sequences and series may be easy for students because students know it. The role of the teacher is necessary to provide stimulation to students so that the findings and information obtained can help them recognize the concept of arithmetic sequences and series and even define them.

Activity 2a. Determine the Formula for an Arithmetic Sequence



Translated to English:

If the first term of an arithmetic sequence (U1) is denoted by a and the difference denoted by b then the formula for the nth term of the sequence can be derived a following. (see figure)

Figure 3. Arithmetic series formula

Activity 2 is to determine the initial formula for an arithmetic sequence. In this activity, students and their groups look for the origin of the arithmetic sequence formula through the project they are working on. In this case, students are expected to be able to know the formula for arithmetic sequences. Apart from using Student Activity Sheets, students can also take advantage of supporting applications that have been designed according to the material being studied (see Figure 3). Here, the teacher directs students to work on group projects according to Student Activity Sheets guidelines. After students work on a group project, they are asked to answer the questions. At the end of the activity, students conclude the arithmetic sequence formula and do the exercises given. Students' answers regarding the arithmetic sequence formula can be seen in Figure 4.



Translated to English:

If the first term of an arithmetic sequence (U1) is denoted by a and the difference denoted by b then the formula for the nth term of the sequence can be derived a following. (see formula on figure)

Conclusion

So, the formula for the nth term of an arithmetic sequence is. (see Figure)

Figure 4. Student answers regarding the arithmetic series formula

Based on Figure 4, students can determine the formula for an arithmetic sequence from the project being implemented and can solve examples of arithmetic sequence problems.

Activity 2b. Determine the Arithmetic Series Formula from the Project

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Penjumlahan n suku deret aritmatika disimbolkan dengan S<sub>n</sub>, dan S<sub>n</sub> ditentukan
oleh :
                                                                    S_n = U_1 + U_2 + U_3 + \dots + U_{n-1} + U_n
Substitusikan U_1 = a, U_2 = (a + b), U_3 = (a + 2b), U_{n-2} = (U_n - 2b),
                                               U_{n-1} = (U_n - b) diperoleh :
                        S_n = a + \dots + u_n \dots + \dots + \dots + \dots + u_n \dots + U_n \dots (*)
Jiko urutan suku-suku penjumkahan pada persamaan (*) itu dibalik, diperoleh:
                      S_n = U_n + \dots + a \dots + a \dots + a
          Jika kita jumlahkan masing masing ruas pada persomaan (*) dengan persamaan (**).
          maka akan diperaleh :
       S_n = a = (a + (\dots + \dots) + (\dots + \dots) + \dots + (\dots - \dots) + \dots + (\dots - \dots) + U_n)
       S_n = -U_n + (U_{-1} - \dots) + (U_{-1} - \dots) + \dots + (\dots + \dots) + (\dots + \dots) + a
     a_{1}S_{1} = (a_{1}a_{1} + U_{1}) + (a_{2}a_{2} + U_{1}) + (a_{2}a_{2} + U_{1}) + (a_{3}a_{2} + U_{1}) + (a_{3}a
        \dots S_n = \dots \dots (\dots + U_n)
          S<sub>n</sub> = ( ..... + ..... + ( n - ..... ) ...... )
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Translated to English:
The sum of n terms of an arithmetic series is symbolized by Sn, and Sn is determined by:
(see formula on figure)
Substitute (see formula on figure)
If the order of the addition terms in equation (*) is reversed, we obtain:
(see formula on figure)
If we add each side of equation (*) with equation (**), we get:
(see formula on figure)

Figure 5. Arithmetic series formula

Activity 3 aims to determine the formula for arithmetic series. After knowing the arithmetic series formula, students are expected to be able to determine it (see Figure 5). This activity begins with students working on a project in Student Activity Sheets based on the teacher's direction. This worksheet is designed to stimulate students in determining arithmetic series formulas. During the learning process, the teacher accompanies and helps students in working on activity sheets. The teacher conducts discussions to find out the strategies used by students in solving the problems given. Students' answers regarding the artihmetic series formula can be seen in Figure 6.

Penjumlahan
n suku deret aritmatika disimbolkan dengan $S_n, \, dan \, S_n \, ditentukan oleh :$

$$S_n = U_1 + U_2 + U_3 + \dots + U_{n-1} + U_n$$

Substitusikan $U_1 = a$, $U_2 = (a + b)$, $U_3 = (a + 2b)$, $U_{n-2} = (U_n - 2b)$,

$$U_{n-1} = (U_n - b) \quad \text{diperoleh} :$$

$$S_n = a + (a + b) + (a + b) + \dots ((4a - b) + ((4a - b)) + U_n \dots (c))$$

Jika urutan suku-suku penjumlahan pada persamaan (*) itu dibalik, diperoleh:

 $S_n = U_n + U_{n-1}b_0 + U_{n-1}b_1 + \dots + (a+2b) + (a+b) + u_{n-1} + u_{n-1}$

Jika kita jumlahkan masing masing musi pada personaan (*) dengan persamaan maka akan diperoleh: $S_n = a + (...R_1 + |k_{...}|) + (...R_{...} + 2|k_{...}|) + \dots + (!k_{N_1} - 2!k_{N_2}) + (!k_{N_1} - !k_{...}|) + U_n$ $S_k = U_n + (U_{N_1} - ...k_{...}) + (U_{N_1} - ...k_{2}) + \dots + (...R_{.} + 2!k_{...}) + (...R_{.} + ...k_{...}) + a$ $\frac{1}{2} S_n = (...R_{...} + U_{N_1}) + (...R_{...} + U_{N_1}) + \dots + (...R_{.} + 2!k_{...}) + (...R_{.} + U_{N_2}) + (...R_{.} + U_{N_2}) + (...R_{.} + U_{N_2}) + (...R_{.} + U_{N_2}) + (...R_{.} + U_{N_1}) + \dots + (...R_{.} + U_{N_2}) + (...R_{.} + U_{N_2})$ Translated to English:

The sum of n terms of an arithmetic series is symbolized by Sn, and Sn is determined by:

(see formula on figure)

Substitute (see formula on figure)

If the order of the addition terms in equation (*) is reversed, we obtain:

(see formula on figure)

If we add each side of equation (*) with equation (**), we get:

(see formula on figure)

Figure 6. Student answers regarding the arithmatic series formula

Students can determine the arithmetic series formula from the project they are working on and are able to solve examples of arithmetic series questions.

Activity 3. Solve Contextual Problems Related to Arithmetic Sequences and Series

- Jumlah mobil dalam parkiran wisata candi borobudur pada tanggal 1 Agustus 2023 sebanyak 80, kemudian pada tanggal 6 dan 7 berjumlah 105 dan 110. Ternyata jumlah mobil tersebut membentuk barisan aritmatika. Maka:
 - a. Tentukan beda pada barisan aritmatika tersebut?
 - b. Berapa banyak jumlah mobil pada tanggal 10 dengan ketentuan tidak ada hari libur?

Translated to English:

The number of cars in the Borobudur Temple tourist parking lot on August 1, 2023, was 80, then on the 6th and 7th it was 105 and 110. It turned out that the number of cars formed an arithmetic sequence. So: a. Determine the difference in the arithmetic series?

b. How many cars will there be on the 10th with the condition that there are no holidays?

Figure 7. Contextual problems related to arithmetic sequences and series

Activity 3 is to solve contextual problems related to arithmetic sequences and series. Students are given a number of questions related to arithmetic sequences and series that have been studied in previous activities (see Figure 7). In working on Student Activity Sheets, the teacher constantly monitors and provides direction to all students. Students' answers regarding the arithmetic sequence formula can be seen in Figure 8.

Penyelesaian:	Translated to English:
Penyelesaian: a. un = a + (n-1) b. n = 80 + (w - 1) 5 = 80 + (9) 5 = 80 + 45 = 125	Completion: a. $U_n = a + (n - 1) b$ n = 80 + (10 - 1) 5 = 80 + (9) 5 = 80 + 45 = 125
.btanggal10. = 1.35Nabil	b. 10th = 125 cars

Figure 8. Student answers contextual problems related to arithmetic sequences and series

The initial stage of design research is the experimental preparation stage through several activities, namely conducting a literature review, reviewing competitions that students have and need, and designing HLT. Preparations need to be made at the experimental stage to develop an initial strategy for designing learning. In understanding the material of arithmetic sequences and series, a learning design that can make it easier for students to understand is needed. The context used as a learning resource helps students understand the material of arithmetic sequences and series.

Based on the results above, it is found that mathematics connected to real-world contexts makes it easier for students to understand the material. This is consistent with the research conducted by (Sullivan et al., 2003; Brown, 2017; Nursyahidah et al., 2023). The students' understanding proves this, as demonstrated in the discussion of the student activity sheet, where students can comprehend the arithmetic sequence formula through systematic steps.

Not only that, in this study, there is AR, which makes students more interested in the learning material—judging from the students' enthusiasm when the video showcasing the context of Borobudur temple connected with AR was presented. This is in line with the research conducted by (Lu & Liu, 2015; Mustaqim, 2016; Kurzaeva et al., 2020).

CONCLUSION

The research results involve the construction of a Hypothetical Learning Trajectory for building arithmetic sequences and series. This trajectory is designed within the context of Borobudur Temple. The primary aim is to facilitate students' understanding of the material. The intention is to create a more meaningful learning experience for the students. This research produces a learning trajectory for arithmetic sequence and series material to make it easier for students to understand the concept which consists of 3 activities, namely: 1) observing the video borobudur temple to find out the definition and characteristics of arithmetic sequence and series; 2) find the formula and arithmetic sequence and series results; 3) and solving contextual problems related to arithmetic sequence and series. The results of this research show that through a series of activities that have been designed it can help students understand concepts in arithmetic sequence and series material using the context of the borobudur temple. Apart

from that, the results of this research also add alternative local wisdom that can be used as a context for mathematics learning in arithmetic sequence and series material.

Furthermore, the observed positive impact on student engagement and understanding suggests broader applications of RME and AR in mathematics education. As technology continues to evolve, integrating innovative approaches like AR can significantly contribute to making mathematics more accessible, enjoyable, and applicable in real-world scenarios. A suggestion for future research is to expand the use of AR in mathematics education by associating it with contexts or traditions as a learning approach. This can add diversity and relevance to the teaching methods.

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