Mathematics Education Journal
ISSN: 2549-1040 (e) | 1978-0044 (p)
https://doi.org/10.22342/mej.v19i3.pp567-584

Mathematical Insights into *Aboge* Calendar: Ethnomathematics Study of Javanese Cultural Heritage in Cikakak Village

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Abstract

Traditional calendars often embody complex knowledge systems that remain underexplored in formal academic discourse, especially in the context of indigenous mathematics. This study investigates the mathematical structures embedded in the Javanese *Aboge* calendar, a traditional timekeeping system preserved by the *Aboge* community in Cikakak Village, Banyumas Regency, Indonesia. While the official Javanese calendar has transitioned to the *Asapon kurup*, the *Aboge* community continues to follow the older *Aboge kurup*, rooted in local belief and tradition. Using a qualitative descriptive approach and ethnographic methods—comprising field observations, interviews, and collaborative computations—this study examines the underlying mathematical logic of the calendar. The findings reveal that the *Aboge* calendar applies modular arithmetic, particularly congruences modulo 7 (days) and modulo 5 (*pasaran*), to determine the first day of each month. These values follow recursive patterns modeled using mathematical formulas. Additionally, the Chinese Remainder Theorem is employed to calculate intervals between specific day-*pasaran* pairs, validating traditional practices through formal mathematical reasoning. The results demonstrate that the *Aboge* calendar encapsulates sophisticated mathematical concepts traditionally transmitted through memorization. This study highlights the value of cultural diversity and affirms the role of indigenous knowledge in sustainable development, reinforcing the importance of integrating local traditions into educational and heritage preservation efforts.

Keywords: *Aboge* Calendar, Cultural Diversity, Ethnomathematics, Javanese Cultural Heritage, Modular Arithmetic

How to Cite: Kusuma, A. B., Hanum, F., & Abadi, A. M. (2025). Mathematical insights into *Aboge* calendar: Ethnomathematics study of javanese cultural heritage in Cikakak village. *Mathematics Education Journal, 19*(3), 567-584. https://doi.org/10.22342/mej.v19i3.pp567-584

INTRODUCTION

Ethnomathematics is a field of study that investigates the interplay between mathematics and culture (Albanese & Perales, 2015). This discipline reveals how indigenous communities develop counting systems that are often linked to local environmental features, while artisans apply geometric principles in their crafts without formal mathematical training. Ethnographers conduct studies across various cultural environments, documenting mathematical ideas alongside other cultural practices (D'Ambrosio, 2007). The scope of culture encompasses not only the native population but also includes the workforce, artisan groups, urban communities, agricultural sectors, and professional organizations (D'Ambrosio, 2007). Ethnographers gather knowledge from both quantitative and qualitative practices, which include counting, weighing, measuring, comparing, sorting, and classifying (D'Ambrosio, 1999). The inquiry into ethnomathematics begins with questions about how ad hoc practices evolve into methods, how these methods develop into theories, and how theories ultimately lead to scientific discoveries (D'Ambrosio, 2007). By connecting mathematical practices to real-life cultural contexts, this research underscores the significance of ethnomathematics in understanding the mathematical competencies present within various communities.

Received February 7, 2025; Revised May 16, 2025; Accepted July 1, 2025

The learning framework of ethnomathematics aspires to promote ethical behavior and foster peace across multiple dimensions, including social and environmental aspects (D'Ambrosio, 1999). It acknowledges that all individuals possess equal rights and should benefit from progress (D'Ambrosio, 2017). To fulfill this objective, ethnomathematics situates mathematics within social, cultural, and historical contexts (Lerman, 2000). Situated within social, historical, and ecological contexts, this approach aligns with the Sustainable Development Goals (SDGs), particularly in its emphasis on promoting inclusive, context-based education and on safeguarding cultural heritage (UNESCO, 2017). This approach generates distinct forms of knowledge that arise from efforts to navigate diverse environments (D'Ambrosio, 2016). Ethnomathematics plays a vital role in restoring cultural dignity, enhancing creativity, bolstering cultural self-esteem, and broadening perspectives on humanity (D'Ambrosio, 1999, 2007). Furthermore, it fosters harmonious relationships between human behavior and nature (D'Ambrosio, 2007).

In the national context, ethnomathematics studies focused on culture in Indonesia have rapidly advanced (Kusuma et al., 2024), encompassing a wide range of Indonesian cultures. Research has examined various ethnic groups, including the Baduy (Arisetyawan & Yuda, 2019), Sasak (Subarinah et al., 2022), Arfak (Haryanto et al., 2017), Bugis (Pathuddin et al., 2023), Dayak (Mairing et al., 2024), Bali (Diputra et al., 2022), Sundanese (Muhtadi et al., 2017), and Javanese (Wiryanto et al., 2022). These studies draw upon various forms of representation, which can be categorized into sociofacts, *mantefak*, and artifacts (D'Ambrosio & Rosa, 2017). Notably, research pertaining to artifacts is more readily available than that concerning *sociofacts* and *mantefak.* Examples of artifact-related studies include investigations into temples (Parajuli, 2023), books (Utami et al., 2019), mosques (Kusno et al., 2022), batik (Hortelano & Lapinid, 2024), traditional food (Sakinah et al., 2023), and traditional houses (Mariamah et al., 2021; Setyaningrum & Untarti, 2024; Suharta et al., 2017). In contrast, an example of research focusing on *sociofacts* and mathematics is the study of the Baduy tribe (Arisetyawan et al., 2014). The diversity of these studies highlights the relevance of ethnomathematics in enhancing our understanding of Indonesian cultural practices. This multifaceted approach emphasizes the potential benefits of ethnomathematics in enriching the cultural landscape of Indonesia.

Focusing on a specific tradition, the Javanese calendar exemplifies how traditional practices adapt to modern life amid technological advancements. Despite the rise of new technologies that simplify human activities, traditional Javanese calculations continue to hold significance. For instance, the Javanese community employs traditional methods for estimating time, particularly in planning celebrations (Kamayanti & Ahmar, 2019). The Javanese calendar, established on July 8, 1633 AD, by Sultan Agung of the Mataram Kingdom, is a synthesis of the Hindu Saka and Islamic calendars (Fitrotun Nisa', 2021; Karjanto & Beauducel, 2021; Ricklefs, 1994). This calendar incorporates various cycles, including 5 days (*Weton*), 7 days (*Dinapitu*), 35 days (*Weton*), lunar months (*Wulan*), 210 days (*Pawukon*), and longer cycles such as 8 years (*Windu*) and 120 years (*Kurup*) (Karjanto & Beauducel, 2021).

The *Aboge* community, which actively preserves the Javanese calendar, exemplifies the integration of traditional practices with contemporary life (Fitrotun Nisa’, 2021). Although the calculations of the *Aboge* community generally align with traditional Javanese practices, notable distinctions exist, such as their unique interpretation of the *Kurup* cycle (Najiyah et al., 2022). This preservation of the *Aboge* calculation system reflects how traditional methods adapt to modern contexts, ensuring that the timing of religious holidays remains distinctive to the *Aboge* community. By maintaining these practices, the *Aboge* community illustrates the ongoing relevance of ethnomathematics in a rapidly changing technological landscape, highlighting how cultural heritage can coexist with modern advancements.

The *Aboge* community, known as "Islam *Aboge*" represents a unique cultural synthesis of local traditions and Islamic teachings (Sakirman, 2016). This community adheres to the teachings of Sheikh Siti Jenar and Suluk Syatariyyah (Prawiro, 2014) and continues to fulfill its religious obligations as Muslims, including prayer, fasting, and pilgrimage (Sakirman, 2016). However, debates surrounding the community's alignment with Islamic principles persist (Ramadhan et al., 2023). Cikakak Village in Banyumas Regency, Indonesia, is recognized as a traditional village where the *Aboge* community resides, and the government has designated it for cultural preservation (Government Regulation of Indonesia Number 10, 1993).

Several studies have explored the calculations of the *Aboge* community, including research on determining the start of Ramadan (Mahfudi et al., 2024), examining the timing of Ramadan and its mathematical elements (Prabowo et al., 2018), and comparing *Aboge* and *Asapon* calculations for religious holidays (Prabowo et al., 2022). Despite these contributions, a significant gap remains in the literature regarding a comprehensive analysis of the mathematical elements underlying the *Aboge* calendar. This study aims to address this gap by providing a detailed examination of the mathematical structures and principles that govern the *Aboge* calendar, thereby enhancing our understanding of its mathematical underpinnings.

By focusing on the mathematical aspects of the *Aboge* calendar, this research complements existing studies while offering new insights into the pedagogical value of ethnomathematics in preserving cultural heritage and enhancing mathematical literacy. The integration of traditional mathematical practices into modern educational frameworks is increasingly important, as research has shown that culturally contextualized mathematics education can enhance student engagement and understanding (Kurniawan et al., 2023). However, the specific challenge of integrating ethnomathematical practices, such as those of the *Aboge* calendar, into contemporary educational settings remains underexplored, underscoring the necessity of this study to bridge the gap.

This research builds upon existing investigations of the *Aboge* calendar by systematically exploring the mathematical calculations and patterns inherent within the calendar system. While previous studies have primarily focused on converting the *Aboge* calendar to the Gregorian calendar, the present study aims to (1) identify and analyze the mathematical structures embedded in the *Aboge* calendar, and (2) examine their relevance to modern mathematical principles such as cyclicity and modular arithmetic. Employing ethnographic research methods, this study captures the cultural context and lived experiences of the *Aboge* community (D'Ambrosio, 2007).

METHODS

This study employs ethnographic research methods specifically tailored to investigate the *Aboge* calendar. Ethnography, as a qualitative research approach, allows for an in-depth exploration of cultural practices and beliefs, thereby facilitating a nuanced understanding of the distinct experiences and perspectives of communities from diverse cultural backgrounds. In this context, the ethnographic method is particularly relevant as it enables the identification and analysis of mathematical patterns inherent in the *Aboge* calendar system. By examining the spoken and written language, signs, gestures, and numerical representations associated with this cultural calendar, this research aligns with the principles of ethnomathematics, which seeks to uncover the mathematical knowledge embedded in specific cultural contexts (D'Ambrosio, 2007; Spradley, 1979).

This study focuses on the *Aboge* calendar tradition in Cikakak. Data were collected through field observations, documentation, and interviews. A literature review was not conducted, as no written records of the *Aboge* calendar were found within the community. The knowledge of calendrical calculations and date determination is transmitted orally from generation to generation, relying on memorization rather than written documentation. The study's respondents included the public relations representative of the *Aboge* community in Cikakak, the head of the local hamlet, the *juru kunci* (traditional custodian), and the local historian of Cikakak. These individuals were selected not only for their expertise in *Aboge* calculations but also for their esteemed status within the community. When residents seek calendrical guidance, they typically consult either the public relations officer or the *juru kunci*. Following preliminary discussions, data were gathered through semi-structured interviews, designed to elicit insights into the cultural values and calendrical practices associated with the *Aboge* tradition in Cikakak.

During this study, the exploration of ethnomathematics was guided by four overarching questions: "Where should one begin the search?", "What methodologies should be employed in the search?", "How can one recognize significant findings?", and "What strategies are effective for understanding these findings?" (Alangui, 2010). Data collection was conducted through a combination of images, videos, and field notes, with field notes serving as the primary source of data. Given the absence of a definitive guidebook for determining the *Aboge* calendar, observations and interviews provided essential insights into the subject matter.

The informant directly taught the calendar calculation process, illustrating mathematical concepts and their applications through practical examples. This instructional process was meticulously documented in the field notes and supplemented with video recordings. The video documentation served as an auxiliary resource, allowing for cross-verification of field notes in cases where additional context or clarification was needed. While image data was utilized to capture the authentic conditions observed in the field, it was not regarded as the primary data source due to the lack of literature or guides on calculating the *Aboge* calendar.

Table 1. Design of ethnography study

General Questions	Initial Answers	Starting Point	Specific Activity
Where to start looking?	In *Aboge* calendar in Cikakak, there were mathematical practices	Culture	Conduct interviews with people or figures who know the *Aboge* calendar in Cikakak.
How to look?	Examining aspects of *Aboge* calendar in Cikakak associated with mathematical practices.	Regularity of calendar calculation patterns	Determine ideas related to formal mathematics in the *Aboge* calendar in Cikakak
What is it?	Look for evidence of formal mathematical findings in the *Aboge* calendar	Matching calculation results between manual calculations and formal mathematics	Identify formal mathematical results found and then clarify calculations manually
What does it mean?	The truth of the findings is assessed	*Aboge* calculation expert	Describes the relationship between the formal mathematical knowledge system and traditional mathematics that is carried out from generation to generation

Furthermore, source triangulation was employed to analyze the data and explore the relationship between mathematical knowledge and the *Aboge* calendar in Cikakak. This methodological approach facilitated cross-verification of information, thereby enhancing the reliability and depth of the findings (Sari et al., 2024). The triangulation process involved comparing interview results among different informants. Discrepancies were addressed by consulting additional informants to ensure a comprehensive understanding. Following the triangulation, data validation was conducted by consulting with the public relations representative of the *Aboge* community, who serves as a key resource for community engagement and expertise regarding the *Aboge* calendar.

Subsequently, the data was analyzed individually to ascertain the presence of formal mathematical elements. The *Aboge* community conveyed that only a limited number of components involved regular calculations, suggesting the potential inclusion of formal mathematical principles. However, they candidly acknowledged their lack of formal mathematical knowledge, indicating that their calculations are based on traditional teachings passed down through generations. The results were described in the results of this study, as shown in Table 1 based on 4 general questions.

RESULTS AND DISCUSSION

Aboge, also referred to as Alip Rebo Wage, denotes a specific year within the *Kurup* cycle of the Javanese calendar. Currently, the Javanese calendar has transitioned into the *Asapon Kurup*; however, the *Aboge* community, including residents of Cikakak Village, has not adopted this change. Instead, they continue to utilize the preceding *Kurup*, known as the *Aboge Kurup* (Prabowo et al., 2020). This adherence to the *Aboge Kurup* is rooted in the teachings of Mbah Tholih, a revered figure within the community. The *Aboge* community firmly believes that their calendar is fixed and will remain applicable indefinitely.

Aboge Calendar

Aboge calendar had currently entered the 4th year cycle or *Ja* year. The first day in *Ja* year coincided with Tuesday *Pahing*, and this year was often called *Jasapaing* year.

Table 2. Benchmark for the day and *pasaran* in Jasapaing year

Day	Code	*Pasaran*	Code
Selasa (Tuesday)	*Ji* (*Siji*/One)	*Pahing*	Ji (*Siji*/One)
Rebo (Wednesday)	*Ro* (*Loro*/Two)	*Pon*	Ro (*Loro*/Two)
Kemis (Thursday)	*Lu* (*Telu*/Three)	*Wage*	Lu (*Telu*/Three)
Jemuah (Friday)	*Pat* (*Papat*/Four)	*Kliwon*	Pat (*Papat*/Four)
Setu (Saturday)	*Mo* (*Limo*/Five)	*Manis*	Mo (*Limo*/Five)
Ahad (Sunday)	*Nem* (*Enem*/Six)		
Senen (Monday)	*Tu* (*Pitu*/Seven)		

The *Aboge* calendar consists of days, *pasaran*, and months. There were 7 days in *Aboge* calendar, namely Monday (*Senen*), Tuesday (*Selasa*), Wednesday (*Rebo*), Thursday (*Kemis*), Friday (*Jemuah*), Saturday (*Setu*), and Sunday (*Ahad*). Following this discussion, 5 *pasaran* were included in *Aboge* calendar, namely *Pon, Wage, Kliwon, Legi,* and *Pahing*. The calendar also consisted of 12 months, namely *Muharam, Sapar, Mulud, Robingul Akhir, Jumadil awal, Jumadil akhir, Rajab, Sadran/Sya'ban, Pasa, Sawal, Apit/Dzulqangidah*, and *Hajj/Dzulhijjah* (Muhtada, 2023).

The first day of each year was known to be the starting point for *Aboge* calendar. Based on that time, benchmark was created for day, *pasaran,* and month. Following the discussion, the benchmark was used to determine *Aboge* calendar. The standard for day and *pasaran* in *Jasapaing* year were shown in Table 2.

Aboge calendar had abbreviations for easy remembering of first day of each month. These abbreviations were *Ramjiji (Muharam siji-siji), Parluji (Sapar telu-siji), Ludpatma (Mulud Papat-Lima), Ngukirnemma (Robingul Akhir Enem-Lima), Waltupat (Jumadil Awal Pitu-Papat), and Dakhirrapat (Jumadil Akhir Lara-Papat)*. Other acronyms included *Jablulu (Rajab Telu-Telu), Dranmalu (Sadran Lima-Telu), Sanemra (Pasa Enem-Lara), Waljira (Sawal Siji-Lara), Pitroji (Apit Lara-Siji)*, and *Jiapatji (Haji Papat-Siji)* (Prabowo et al., 2019), which were all applied to each year. The meaning of these acronyms, for example, *Ramjiji (Muharam siji-siji)* was in the month of *Muharam Siji* (1) - *Siji* (1), or written as *Muharam* 1-1. The first number showed day code, and the second signified *pasaran* code. Table 2 showed the meaning of day and *pasaran* number code. Since the first number was 1, first day of month of *Muharram* was on Tuesday. In addition, since the second number was 1, first *pasaran* of month of *Muharram* was on *Pahing*. The benchmark for month in *Jasapaing* year was shown in Table 3.

Table 3. Benchmark for month in *Jasapaing* year

Abbreviation	Month	Code	Day	*Pasaran*
Ramjiji	*Muharam*	1-1	*Selasa* (Tuesday)	*Paing*
Parluji	*Sapar*	3-1	*Kemis* (Thursday)	*Paing*
Ludpatma	*Mulud*	4-5	*Jemuah* (Friday)	*Manis*
Ngukirnemma	*Robingul Akhir*	6-5	*Ahad* (Sunday)	*Manis*
Waltupat	*Jumadil awal*	7-4	*Senen* (Monday)	*Kliwon*
Dakhirrapat	*Jumadil akhir*	2-4	*Rebo* (Wednesday)	*Kliwon*
Jablulu	*Rajab*	3-3	*Kemis* (Thursday)	*Wage*
Dranmalu	*Sadran/Sya'ban*	5-3	*Setu* (Saturday)	*Wage*
Sanemra	*Pasa*	6-2	*Ahad* (Sunday)	*Pon*
Waljira	*Sawal*	1-2	*Selasa* (Tuesday)	*Pon*
Pitroji	*Apit/Dzulqangidah*	2-1	*Rebo* (Wednesday)	*Paing*
Jiapatji	*Haji/Dzulhijjah*	4-1	*Jemuah* (Friday)	*Paing*

There were various special features of month reference found in *Aboge* calendar. The first exceptional quality included order of months and code for each month was often the same. Order of months started from month of Muharram and ended with Hajj. The process was special because it allowed ethnographers to analyze more deeply about repetition as well as how there was a relationship between months and repetition. The similarity of code in each month was also extraordinary, even though year changed, code was always the same. Following the discussion, the second distinctive characteristic was that number of days in each month had the same pattern. In sequence, number of months of *Muharram, Sapar, Mulud, Robingul Akhir*, ..., *Apit, Hajj* was 30, 29 30, 29, ..., 30, 29 (Ja'far Sodiq, 2022). The third exceptional feature was that code used two numbers, including the first number signifying day, and the second number showing *pasaran*.

Mathematical Modeling in Determining Month Code in Aboge Calendar

The origin of month code, according to Mr Suyitno, as public relations of *Aboge* Community was hereditary memorization. When the pattern of the number of days each month was viewed, code formula was present. The source of month code was passed down from generation to generation. *Muharram* was the first month at beginning of year, therefore, code 1-1 as an initial reference was very reasonable.

Sapar occurs in the month of *Muharram*, which is the second month of the Javanese calendar, consisting of 30 days. Consequently, the first day of *Sapar* falls on the 31st day of the year. To determine the day of the week for the 31st day, we calculate $31 \div 7 = 4$ with a remainder of 3, which can be expressed mathematically as $31 \equiv 3 \; mod \; 7$. In addition, to ascertain the 31st *pasaran*, we perform the calculation $31 \div 5 = 6$ with a remainder of 1, represented mathematically as $31 \equiv 1 \; mod \; 5$. The remainder from these divisions serves as a code for identifying the second month. Thus, this study concludes that the first day of the second month corresponds to the designation 3-1. This calculation is corroborated by the *Aboge* memorization table (Table 3), which confirms that the *Sapar* month is represented as 3-1.

The selection of 7 and 5 as divisors is not arbitrary; the number 7 is employed to reflect the total number of days in a week, while the number 5 corresponds to the total number of *pasaran* within the community. Consequently, the numbers 7 and 5 are frequently employed in calculations related to the calendar.

In the third month, *Mulud* is derived from the total number of days in the months of *Muharram* and *Sapar*, which amounts to 30 + 29 = 59 days. Consequently, the first day of the month of *Sapar* falls on the 60th day of the year. Utilizing a method similar to the previous calculations, we determine the day of the week for the 60th day by calculating $60 \div 7 = 8$ with a remainder of 4, which can be mathematically expressed as $60 \equiv 4 \; mod \; 7$. To ascertain the 60th *pasaran*, we perform the calculation $60 \div 5 = 6$ with a remainder of 0. Since there is no designation for the number 0 in the *pasaran* series, this remainder is interpreted as representing the number 5; mathematically, this is expressed as $60 \equiv 5 \; mod \; 5$.

Based on this analysis, the remainder from these divisions serves as a code for identifying the third month. Therefore, this study concludes that the first day of the third month corresponds to the designation 4-5. This calculation further illustrates the applicability of the modulo pattern. As shown in Table 3, the code for the month of *Mulud* is indeed 4-5, which aligns precisely with the results of the calculations conducted.

If the calculations are extended, the initial month codes for subsequent months can be determined through modulo 5 and 7. Thus, it can be concluded that mathematical modeling using modulo 5 and 7 is effective for establishing the initial month codes in the *Aboge* calendar.

Mathematical Modeling in the Month Code Pattern in Aboge Calendar

The initial month code for day or first number in sequence were 1, 3, 4, 6, 7, 2, 3, 5, 6, 1, 2, 4 (as shown in Table 3). From this sequence, the difference between numbers was shown in Figure 1.

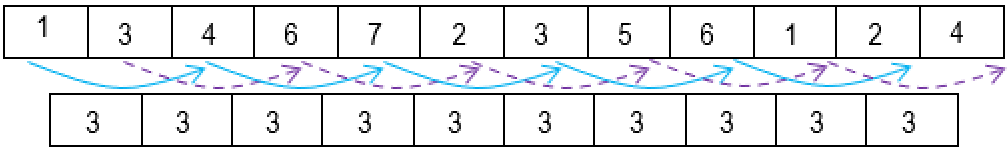

Figure 1. Day code pattern for each month in *Aboge* calendar

The Figure 1 showed the number patterns formed, which were divided into two groups, namely number pattern for odd and even months. Each number pattern was in modulo 7 forms, and after reaching 7, the process was repeated to 1. Moreover, the first number pattern was for odd months (months 1, 3, 5, 7, 9, 11), which had number pattern of 1, 4, 7, 3, 6, and 2. To determine kth month of odd month, it was formulated as $k = 1 + (l - 1)2 = 2l - 1$, with l being the sequence pattern. To evaluate the value of day (m) in lth sequence in kth month, $m = 1 + (l - 1)3 = 3l - 2$ was equated, with m being member of modulo 7 set. Following this process, Equation 1 was obtained in odd months during the finding.

$$\begin{cases} k = 2l - 1, \; k \text{ was the } l\text{th odd month, and } l \text{ was the sequence pattern} \\ \quad m = 3l - 2, \; m \text{ was the value of the day in modulo 7} \end{cases} \tag{1}$$

The second number pattern was for even months (months 2, 4, 6, 8, 10, 12), which had number pattern of 3, 6, 2, 5, 1, 4. To determine pth month of even month, the formula was $p = 2 + (q - 1)2 = 2q$, where q was the sequence pattern. To evaluate the value of day (r) in qth sequence in pth month, $r = 3 + (q - 1)3 = 3q$ was equated, with r being member of modulo 7 set. Relating to the process, Equation 2 was obtained in even months in this study.

$$\begin{cases} p = 2q; p \text{ was the } q\text{th even month, and } q \text{ was the sequence pattern} \\ \quad r = 3q; \; r \text{ was the value of the day in modulo 7} \end{cases} \tag{2}$$

When finding first day of 11th month, Equation 1 was used. The first step was to determine the order of pattern by substituting $k = 11$ into formula $k = 2l - 1$, and the result was $11 = 2l - 1 \leftrightarrow l = 6$. Value of l was then substituted into $m = 3l - 2$, and value of m was $m = 3(6) - 2 = 16$. Since m was in modulo 7, the result was $m = 2$. Following the discussion, the first day was on 2nd day or Wednesday for 11th month (Table 2). When finding first day of 8th month, Equation 2 was used. The first step in the Equation was to determine order of pattern by substituting $p = 8$ into formula $p = 2q$, to obtain $8 = 2q \leftrightarrow q = 4$. Value of q was then substituted into $r = 3q$, to obtain value of r, which was $r = 3(4) = 12$. Since r was in modulo 7, $r = 5$ was obtained during the study. Therefore, first day was on 5th day or Saturday for 8th month (Table 2).

Pasaran sequence in initial *pasaran* code was written as 1, 1, 5, 5, 4, 4, 3, 3, 2, 2, 1, 1 (Table 3). From this sequence, the difference between numbers was shown in Figure 2.

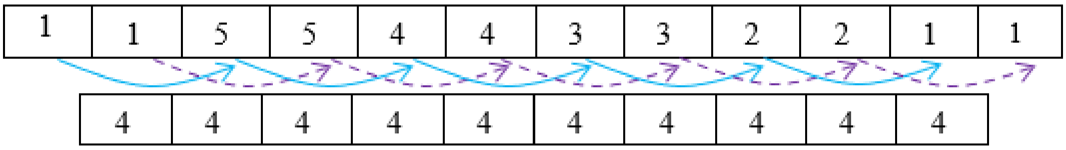

Figure 2. *Pasaran* code pattern for each month in *Aboge* calendar

The Figure 2 showed the number patterns formed, which were divided into two groups including odd-month and even-month number patterns. Each number pattern was modulo 5, and after reaching number 5, the process was repeated to 1. Moreover, the first number pattern for odd months (months 1, 3, 5, 7, 9, 11), which had number pattern of 1, 5, 4, 3, 2, 1. To determine kth month of odd month, formula $k = 1 + (l-1)2 = 2l - 1$ was used, where l was the sequence pattern. $m = 1 + (l-1)4 = 4l - 3$ was used to determine *pasaran* value (m) in lth sequence in kth month, with m being member of modulo 5 set. Relating to this process, market determination in odd months was obtained using Equation 3.

$$\begin{cases} k = 2l - 1; k \text{ was the } l\text{th odd month}, and\ l \text{ was the sequence pattern} \\ m = 4l - 3; m \text{ was the } pasaran \text{ value in modulo 5} \end{cases} \quad (3)$$

The second number pattern for even months (months 2, 4, 6, 8, 10, 12) in determining the *pasaran* had number pattern of 1, 5, 4, 3, 2, 1. To calculate pth month in even month, formula $p = 2 + (q-1)2 = 2q$was used, where q was the sequence pattern. $r = 1 + (l-1)4 = 4q - 3$ was used to determine *pasaran* (r) in qth order in pth month, with r being member of set modulo 5. Therefore, Equation 4 was obtained in even months during the study.

$$\begin{cases} p = 2q; p \text{ was the } q\text{th even month, and } q \text{ was the sequence pattern} \\ r = 4q - 3; r \text{ was the } pasaran \text{ value in modulo 5} \end{cases} \quad (4)$$

When finding first day of 7th month, Equation 3 was used. The first step was determining pattern sequence by substituting $k = 7$ into formula $k = 2l - 1$, and the result was $7 = 2l - 1 \leftrightarrow l = 4$. value of l was then substituted into $m = 4l - 3$, and value of m was $m = 4\,(4) - 3 = 13$. Since m was in modulo 5, this study obtained $m = 3$. Following the discussion, first *pasaran* was on 3rd *pasaran* or Wage for 7th month (Table 2). When finding first market in 10th month, Equation 4 was used. The first step was to determine pattern sequence by substituting $p = 10$ into formula $p = 2q$ to obtain $10 = 2q \leftrightarrow q = 5$. Value of q was then substituted into $r = 4q - 3$, to obtain value of r, which was $r = 4(5) - 3 = 17$. Since r was in modulo 5, this study obtained $r = 2$. Relating to the process, first *pasaran* was on 2nd *pasaran* or Pon for 10th month (Table 2). In Month Code Pattern in *Aboge* Calendar, there was mathematical modeling in the form of number pattern $= 2l - 1, m = 3l - 2, p = 2q\ r = 3q, k = 2l - 1, m = 4l - 3, p = 2q$ and $r = 4q - 3$.

Mathematical Modeling in Determining the Distance between Days

Table 3 showed code for first day of each month, and the problem that often occurred in society was calculating how long it would take to reach a specific day. For example, the society was currently in the month of *Sapar*, and the question was how long would fasting be conducted. Solving this problem was often conducted manually or with multiples of 35. Since month of *Sapar* had code (3,1), and every time it reached (3,1), 35 days had passed. Due to the distance between *Sapar* and *Pasa* being 7 months, maximum distance was 210 days. Therefore, the finding showed that 5 times multiples of 35 was 175. The distance (3,1) was then calculated manually to month of *Pasa* (6,2) as shown in Table 4. The result showed that from (3,1) to (6,2) there were 31 days. Distance between months of *Sapar* (3,1) and *Pasa* (6,2) was 175 + 31 = 206.

Table 4. Manual calculation from (3-1) to (6-2)

No	Kode	No	Kode	No	Kode	No	Kode
1	3-1	9	4-4	17	5-2	25	6-5
2	4-2	10	5-5	18	6-3	26	7-1
3	5-3	11	6-1	19	7-4	27	1-2
4	6-4	12	7-2	20	1-5	28	2-3
5	7-5	13	1-3	21	2-1	29	3-4
6	1-1	14	2-4	22	3-2	30	4-5
7	2-2	15	3-5	23	4-3	31	5-1
8	3-3	16	4-1	24	5-4	32	6-2

The calculation of distance between the days was evaluated mathematically, and day code for each month was known, as shown in Table 3. From this case, distance between *Sapar* (3,1) and *Pasa* (6,2) was obtained. Following the process, the difference between the two months was (3,1). System of Equations 5 was obtained when modulo 5 and 7 were used.

$$x \equiv 3 \bmod 7$$
$$x \equiv 1 \bmod 5 \qquad (5)$$

Chinese remainder theorem was used to solve system of Equations 5. The solution to congruence system was

$$x \equiv 3 \bmod 7 \quad => \; a_1 = 3, m_1 = 7$$
$$x \equiv 1 \bmod 5 \quad => \; a_2 = 1, m_2 = 5 \qquad (6)$$

From Equation 6, the following Equation was viewed including:

$$(m_1, m_2) = 1$$
$$M = m_1 m_2 = 7\,.\,5 = 35$$
$$M_1 = \frac{M}{m_1} = 5$$

$M_2 = \frac{M}{m_2} = 7$

$\left(\frac{M}{m_1}\right) b_1 \equiv 1(mod\ m_1) \rightarrow 5b_1 \equiv 1(mod\ 7) \qquad \rightarrow b_1 = 3, 10, 17, 24, 31, \ldots$

$\left(\frac{M}{m_2}\right) b_2 \equiv 1(mod\ m_2) \rightarrow 7b_2 \equiv 1(mod\ 5) \rightarrow 2b_2 \equiv 1(mod\ 5) \quad \rightarrow b_2 = 3, 8, 13, 18, 23, \ldots$

For $b_1 = 3$ and $b_2 = 3$, using Chinese remainder theorem, the result was obtained.

$x = M_1.b_1.a_1 + M_2.b_2.a_2 = 5.3.3 + 7.3.1 = 45 + 21 = 66$

This process was not possible since distance between month of *Sapar* and *Pasa* was 7 months. Therefore, correct period was between 7 × 29 and 7 × 30 or 203 to 210. Due to incorrect process, other values of $b_1 = 10$ and $b_2 = 8$ was applied. For $b_1 = 10$ and $b_2 = 8$, using Chinese remainder theorem, the result was obtained.

$$x = M_1.b_1.a_1 + M_2.b_2.a_2 = 5.10.3 + 7.8.1 = 150 + 56 = 206$$

Since value of x was in the range of 203 to 210, the value of $x = 206$ was accurate. Consequently, distance between the commencement of month of *Sapar* and *Pasa* was 206 days. The number of days between months of *Sapar* and *Pasa* was 206 days when calculated manually as shown in Table 3. During this study, mathematical modeling was used to determine distance between days using Chinese remainder theorem.

The results of exploring moon code modeling in *Aboge* calendar added references and knowledge about the use of culture in mathematical learning. This study found one form of modeling, namely Modulo 5 and 7. The results of this modeling search were consistent with the outcomes of previous studies on the birth and death determination system in Javanese calendar (Prahmana et al., 2021), determination of time of *tedhak siten* (Wiryanto et al., 2022) and calculation of Javanese *primbon* (Utami et al., 2019). Moreover, modeling occurred similarly because Javanese calendar benchmark used 7 days and 5 *pasaran*. This process further added to treasury of mathematics and use of culture in learning.

Other search results related to mathematical modeling found in this study determined distance between days. In other studies related to calculating length of fasting, the process was also associated with using Chinese remainder theorem (Prabowo et al., 2018). These results further strengthened the idea that calculations in *Aboge* calendar could be approached applying Western mathematical calculations, used as material for packaging learning contextually based on culture. This research underscores the value of incorporating cultural knowledge into mathematics education, aligning with the Sustainable Development Goals (SDGs), particularly Goal 4 on inclusive and equitable quality education and Goal 11 on safeguarding intangible cultural heritage (UNESCO, 2017). By appreciating cultural diversity in learning contexts, this approach supports culturally responsive pedagogy and contributes to sustainable development through the preservation and revitalization of indigenous knowledge systems.

CONCLUSION

The *Aboge* calendar is exceptional due to its constancy and the unique codes assigned to each day, month, and year. This study successfully identified three mathematical models within the month codes of the *Aboge* calendar: the nature of modulus, number patterns, and the application of the remainder theorem. These findings provide valuable insights into how ancient calculation processes were influenced by Western mathematical principles. Community members actively engage in calculating the month codes, the intervals between days, and the mathematical patterns inherent in the fixed month codes. This research underscores the potential for further exploration of mathematical modeling within the *Aboge* calendar, particularly regarding the year codes. Such investigations could significantly enrich the integration of cultural heritage into mathematics education, demonstrating that traditional knowledge systems can offer profound mathematical insights. Ultimately, this study highlights the importance of preserving and understanding the *Aboge* calendar not only as a cultural artifact but also as a source of mathematical learning and innovation. Furthermore, the findings contribute directly to the objectives of the Sustainable Development Goals. Specifically, they support SDG 4 by promoting inclusive and equitable quality education that is locally relevant, and SDG 11 by safeguarding intangible cultural heritage as a resource for sustainable urban and community development. Preserving and revitalizing indigenous knowledge systems such as the Aboge calendar not only strengthens cultural diversity but also fosters a more inclusive, pluralistic approach to knowledge in the 21st century.

ACKNOWLEDGMENTS

The authors are grateful to Mr. Suyitno, Mr. Suladi, Mr. Akim, Mr. Sulam, and Mr Edi Catit from Cikakak village, Banyumas Regency, who helped with this study. The authors are also grateful to Universitas Muhammadiyah Purwokerto and Yogyakarta State University, who supported this study and publication.

DECLARATIONS

Author Contribution	:	ABK : Conceptualization, collecting data, Writing - Original Draft, Editing, Visualization, and Methodology. FH : Data analysis results, Writing - Review & Editing, and Formal Analysis. AMA : Validation and Supervision.
Funding Statement	:	This research is supported by Universitas Muhammadiyah Purwokerto, with a decree number A11-IV/740-

S.Re/UMP/IV/2020.

Conflict of Interest : The authors declare no conflict of interest.

Additional Information : Additional information is available for this paper.

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