Students' Obstacles in Learning Sequence and Series Using Onto-Semiotic Approach

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Abstract

Sequences and series is one of the mathematical topics that are related to everyday life. The topic is also taught at several levels of education in Indonesia. However, many students still experrienced difficulties in learning this topic. This study uses an interpretive paradigm that is part of the Didactical Design Research (DDR). This research aims to analyze students' learning obstacles on the topic of sequence and series using the onto-semiotic approach. To do so, written test consists of five questions related to the conceptual understanding of an arithmetic sequences and series was administered to 23 students from one of the senior high schools in Kota Tangerang Selatan followed by interviews with 4 students. The results show that learning obstacles are classified into epistemological, ontogenic, and didactical obstacles. Based on the onto-semiotics approach, the students had difficulties in defining a mathematical idea on sequences and series topics. They could convert a problem into mathematical model but were confused to use a proper procedure. It can be concluded that students still experience obstacles in learning sequences and series topic. The results of this study can be used by teachers as considerations in designing learning situation on the topic of sequence and series.

Keywords: Learning Obstacles, Sequences and Series, Onto-Semiotics Approach

Abstrak

Barisan dan deret merupakan salah satu topik matematika yang sangat erat dengan kehidupan sehari-hari. Selain itu, topik ini juga diajarkan di setiap tingkat sekolah di Indonesia. Meski demikian, siswa masih mengalami kesulitan dalam mempelajari materi ini. Penelitian ini menggunakan paradigma interpretif sebagai bagian dari penelitian desain didaktis (DDR). Tujuan penelitian ini untuk menganalisis hambatan belajar siswa pada topik barisan dan deret menggunakan pendekatan onto-semiotik. Untuk itu, peneliti mengujikan 5 soal terkait konsep dan aplikasi barisan dan deret aritmatika kepada 23 siswa di salah satu SMA Negeri di kota Tangerang Selatan dilanjutkan dengan wawancara terhadap 4 siswa. Hasil menunjukkan bahwa hambatan belajar siswa pada materi ini dapat digolongkan menjadi hambatan epistimologis, hambatan ontogenik, dan hambatan didaktis. Berdasarkan pendekatan onto-semiotik, siswa kesulitan dalam memberikan definisi beberapa simbol pada materi barisan dan deret aritmatika. Selain itu, siswa mampu mengubah suatu permasalahan ke dalam bentuk model matematika tetapi masih keliru dalam prosedur penyelesaian masalah. Dapat disimpulkan bahwa siswa masih mengalami hambatan dalam mempelajari topik barisan dan deret. Selanjutnya, hasil penelitian ini dapat digunakan oleh guru sebagai pertimbangan dalam mendesain pembelajaran barisan dan deret aritmatika di kemudian hari.

Kata kunci: Hambatan Belajar, Barisan dan Deret, Pendekatan Onto-Semiotik

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INTRODUCTION

Mathematics learning is related to three things: teachers, students, and learning materials (Suryadi, 2010). The fact is that the education system in Indonesia still focuses on assessment. Thus, most of the learning processes in the schools, including mathematics learning, still use the lecture method, which is based on memorizing formulas and using simple procedures (Firdaus, Kailani,

Bakar, & Bakry, 2015). As a result of using this method, the students participate passively because the teacher has become the only source of information. Students do not try to understand independently but they are used to being consumptive of the teacher's explanation. In this case, the students' thinking potential is not optimal, and their understanding is only partial. The impact is the accumulation of students' learning obstacles (Dewi, Suryadi, & Sumiaty, 2016).

Brousseau defined *obstacles* as a piece of knowledge obtained from the interaction of students with a didactical situation when acquiring knowledge, but this interaction leads to the formation of the wrong concept (Brousseau, 2002). Thus, learning obstacles are something that students experience due to external factors, not from their internal factors such as inaccuracy, carelessness, etc. Students may experience three types of learning obstacles, namely the ontogenic obstacles, epistemological obstacles, and didactical obstacles. The ontogenic obstacles arise because of the students' limitations at the time of their development. It is related to their mental readiness and cognitive maturity. The epistemological obstacle is an obstacle related to the limitations of the learning context when learning the concept for the first time. The didactical obstacle is an obstacle related to the unsuitable learning process (a didactical situation) (Brousseau, 2002; Suryadi, 2019b).

Sequences and series is a mathematical topic that is closely related to everyday life. This topic is introduced to elementary students as pattern of numbers pattern, and then continued to the sequences and series topic at the secondary and high school level (MoEC, 2018). Students need to understand this topic well because it is taught continuously at several levels of school. Students, in fact, still experience learning difficulties and learning obstacles related to the conceptual understanding and the application of sequences and series. Some of the learning difficulties experienced by students include difficulty in determining the first and nth terms of a sequence (Hardiyanti, 2016; Harijani, Muhsetyo, & Susanto, 2016; Oktopiani, 2017; Wibowo, 2018); difficulty in applying rules and procedures in solving problems related to arithmetic sequences and series (Hardiyanti, 2016; Oktopiani, 2017); and the difficulty in identifying what is known from the word problem related to sequences and series then converting it into a mathematical model (Hardiyanti, 2016; Oktopiani, 2017; Septiahani, Melisari, & Zanthy, 2020; Wibowo, 2018). Some of the learning obstacles experienced by students include obstacles related to concept images of arithmetic sequences and series, students' proficiency in applying the rules in the concept of arithmetic sequences and series, the application of the concept of arithmetic sequences and series in life everyday life, and in connecting the concept of arithmetic sequences and series with other mathematical concepts (Fauzia, Juandi, & Purniati, 2017). Several previous studies have suggested some factors that caused students to experience difficulties and obstacles when learning arithmetic sequences and series, and they are: students do not understand the problems given; they lack precision in planning conclusions in problem-solving; they have low concentration and accuracy in mathematical calculations, and doubts when they find unusual results (Sumargiyani & Hibatallah, 2018; Zebua, 2020). They use many formulas or procedures as a concept of sequences and series topic. Learning obstacles arise when

students only memorize the formulas and rules of this concept without an understanding (Fauzia et al., 2017).

Understanding sequences and series requires students to possess knowledge of combining numbers and symbols in mathematical sentences in the form of equations, expressions, and functions (Mutodi & Mosimege, 2016). An onto-semiotic approach is an approach to understanding the meaning or nature (ontology) of mathematical objects covering three mathematical aspects: problem-solving activity, symbolic language, and organized logical and conceptual system (Godino, Batanero, & Roa, 2005; Montiel, Wilhelmi, Vidakovic, & Elstak, 2009). In general, semiotics is defined as a philosophical theory that deals with signs and symbols used to communicate specific information (Amin, Juniati, & Sulaiman, 2018). The mathematical object referred to in this research is anything that can be used, suggested, or pointed to when doing, communicating, or learning mathematics (Font, Godino, & D 'amore, 2007; Godino et al., 2005; Montiel et al., 2009). The onto-semiotic approach considers six primary entities, which are as follows: language (terms, expressions, notations, graphics); situations (problems, extra or intra-mathematical applications, exercises, etc.); definitions or descriptions of mathematical notions (number, point, straight line, mean, function, etc.); propositions, properties or attributes, which usually are given as statements; procedures or subjects' actions when solving mathematical tasks (operations, algorithms, techniques, procedures); and arguments used to validate and explain the propositions or to contrast (justify or refute) subjects' actions (Amin et al., 2018; Font et al., 2007; Rudi, Suryadi, & Rosjanuardi, 2020).

The onto-semiotic approach has been applied in several previous studies related to mathematics learning. It is applied to analyze the mathematical concepts of different coordinate systems in college students (Montiel et al., 2009), algebraic abilities based on the students' mathematics ability level (Amin et al., 2018), and also the difficulties of students in understanding and applying the Pythagorean theorem (Rudi et al., 2020). Besides being applied to students, the onto-semiotic approach can also be used to analyze the learning flow applied by the teacher. The didactical learning trajectories based on the constructive and objective models can be analyzed using the onto-semiotic approach (Godino, Rivas, Burgos, & Wilhelmi, 2018).

Based on the facts above, research related to students' learning obstacles on the topic of sequences and series and research related to the application of the onto-semiotic approach has been done. However, research related to students' learning obstacles using the onto-semiotic approach, especially on the topic of arithmetic sequences and series has never been carried out. Therefore, this study aims to analyze students' learning obstacles on the topic of sequence and series using the onto-semiotic approach.

METHODS

This study uses an interpretive paradigm that is part of the Didactical Design Research (DDR). The interpretive paradigm relates to the perspective of a person or group of people and the process of forming the meaning of knowledge (Creswell, 2013; Suryadi, 2019a). In this case, it is the result of a didactical situation. The interpretive paradigm identifies students' learning obstacles in the arithmetic sequence and series topic in this research. This research is the first step in didactical design research. This research can be continued using a critical paradigm to form a new didactical design based on the learning obstacles found.

Research subjects were selected using a purposive sampling technique, considering that the subjects had studied the arithmetic sequence and series material. The subjects are class XII students (17-18 years old) of a high school in Kota Tangerang Selatan. Research data were collected using a test and interviews. The test consists of 5 questions, validated by experts, and was related to the topic of arithmetic sequences and series. 23 students did the test in 40 minutes. After being tested, the students' answers were analyzed using the onto-semiotic approach by focusing on three components, namely descriptions, situations, and procedures. Based on the test answers, 4 students who were indicated of having learning obstacles were selected to conduct the interviews. Semi-structured interviews were conducted to clarify students' answers and identify factors that caused students to experience learning obstacles. The results of the interview were transcribed. Data in the form test answers and interviews were analyzed to conclude the types of learning obstacles experienced by students.

RESULTS AND DISSCUSSION

Based on the students' test answers and interview results, when learning arithmetic sequence and series, the students experienced three types of learning obstacles: epistemological obstacles, ontogenic obstacles, and didactic obstacles. The obstacles were analyzed using an onto-semiotic approach that focuses on three components: situations, definitions or descriptions of mathematical ideas, and procedures for solving mathematical problems. The notions of arithmetic sequences and series that are used in the description below are the first term (a), the common difference (b), the nth term (U_n), and the nth partial sum (S_n).

Epistemological Obstacles

There are many ways to identify epistemological obstacles. It can be analyzed through students' conceptual understanding and how students relate one to another mathematical concept. One factor indicating epistemological obstacles was when students know which concepts to use, but there was a slight misunderstanding of using the concepts. Figure 1 shows the questions about the concept of the

first term and the formula for the *n*th term of arithmetic sequences Figure 2 shows the students' answers to this question.

2. Jumlah 10 suku pertama suatu barisan aritmatika adalah 145. Diketahui jumlah suku keempat dan kesembilan dari barisan ini sama dengan lima kali suku ketiganya. Tentukan:
   a. suku pertama barisan tersebut,
   b. rumus suku ke-n barisan tersebut.

Translation:
The sum of the first ten terms of an arithmetic sequence is 145. Given that the sum of the fourth and the ninth term is equal to five times the third term. Determine:
a. the first term of the sequence
b. the formula for the *n*th term of the sequence

Figure 1. Question no.2

Figure 2. The answer of student A

The following is a translated transcript of the researcher (R) interviewing Student A (A):

Interview 1

R   : *Mention the information given by this question?*

A   : *Given the sum of the first ten terms of an arithmetic sequence is 145, so $S_{10} = 145$. Then the total of the fourth and ninth terms of this sequence equals five times the third term, which mean $S_4 + S_9 = 5U_3$. The question is to find the first term (a) and U_n.*

R   : *Now, explain the procedure that you took to solve this problem.*

A   : *First, because the question asked for first term (a), we write the equation $(a + 3b) + (a + 8b) = 5(a + 2b)$*

R   : *What equation is that? What does $(a + 3b) + (a + 8b) = 5(a + 2b)$ mean?*

A   : *This equation uses the formula U_n, Ma'am, so $U_4 = (a + 3b)$, plus $U_9 = (a + 8b)$, equals $5U_3 = 5(a + 2b)$.*

R   : *Previously, you wrote it as $S_4 + S_9 = 5U_5$. Why are did suddenly use U_4 and U_9?*

A   : *Because, to find the first term (a), you have to use the formula U_n. Whereas the question said the total of the fourth and ninth terms, so I write it as S_n.*

R   : *Please continue the explanation regarding the procedure.*

A   : *After getting the equation, I moved the segment, ma'am, then got the result b = 3a. Next, I suppose that b = 3 then a = 1.*

R   : *Is it ok to determine the first term by considering a number for the common difference (b) like that?*

A   : *I do not know, Ma'am. But for this question, after I consider a number, I tried to arrange up to the 10th term of the sequence, and it turns out that the sum is 145.*

R   : *All right. Then how did you answer the question in point b?*

A   : *At first, I was confused about the question, Ma'am. I only remembered the general formula of the arithmetic sequence. So, I just substituted a and b to this formula.*

R   : *Is it necessary to answer the question like that or do we need to solve it?*

A   : *Yes, Ma'am. In my class, this is a correct answer.*

Based on the Student A's answers in Figure 2 and the translated transcript of Interview 1, it was found that Student A made a mistake in capturing information from the questions. Student A thought that the sum of the 4th and the 9th term of the sequence was S_4+S_9. However, Student A used the correct definition in finding the first term. Another mistake was made when Student A determined the common difference (b) and the first term (a) of this sequence. Student A took a common difference (b) and the first term (a) with a number. Even though the students' answers are correct for this problem, they will find it difficult if they work on similar problems that have different numbers. They also had difficulty answering questions related to the nth term formula. Student A only replaced *a* and *b* in the general formula for arithmetic sequences without solving them and constructed the specific *n*th term formula for the asked sequence. This showed that students' conceptual understanding of arithmetic sequences and series was not enough. This mistake might be due to the limited context when the students learned arithmetic sequences and series. As a result, students might experience learning obstacles called epistemological obstacles. Based on the onto-semiotic approach, the students had difficulty defining a mathematical idea. In this case, the students incorrectly defined the common difference (b), the nth term (U_n), and the nth partial sum (S_n). In addition, the students did not perform correct mathematical problem-solving procedures.

Epistemological obstacles were also found when the students tried to solve problems related to the middle term of arithmetic sequences. Figure 3 shows the problem regarding the concept of the *n*th term, the middle term, and the sum of the *n*th partial term of the arithmetic sequence and series. Figure 4 shows the answer of Student B.

> 3. Suku tengah suatu barisan aritmatika adalah 19. Jika suku pertama barisan tersebut adalah 4 dan suku keempatnya adalah 13, tentukan jumlah semua suku barisan tersebut.

Translation:
The middle term of an arithmetic sequence is 19. If the first term is 4 and the fourth term is 13, find the sum of this sequence's terms.

Figure 3. Question no.3

Figure 4. The answer of student B

The following is a translated transcript of the researcher (R) interviewing Student B (B):

Interview 2
R   : *Mention the information given by this question?*
B   : *Given that $a = 4$, $U_4 = 13$, and $U_t = 19$. The question asked for S_n.*
R   : *Yes, great. Now explain your ways to answer this question*
B   : *Yes, Ma'am. We have to find n first. I used the 4th term to find out the common difference (b) of this sequence. After that, since the first term was already known, we only needed to arrange the arithmetic sequence. We found that the middle term is the 5th term, so we have ten terms in this sequence. It means the question asked for S_{10}.*
R   : *What is the middle term of a sequence?*
B   : *The middle term is the term that divides all the terms in the sequence into two parts.*
R   : *So, if the middle term is the 5th term, why are all the terms 10?*
B   : *Yes, because $5 \times 2 = 10$, Ma'am.*
R   : *Why is it multiplied by 2?*
B   : *It is because the middle term is known.*
R   : *As you know, is there a specific formula for the middle term?*
B   : *Actually yes, Ma'am, but I forgot it. I just manually arranged the term.*

Based on the answers of Student B in Figure 4 and the translated transcript of Interview 2, it was found that Student B experienced a misunderstanding in defining the middle term of a sequence. Student B assumed that the "n" used in the middle term is doubled to determine all terms in a sequence. The students had already known that the middle term is the term that divides all the terms in the sequence into two parts. However, they had not understood the concept of the middle term that the number of terms in the right and left sides of the middle term must be the same. This mistake might be due to the limited context of the middle term when the students learned it. As a result, students might experience learning obstacles called epistemological obstacles. Based on the onto-semiotic approach, it showed that students had difficulty defining the idea of the arithmetic middle term. In addition, the students did not perform correct mathematical procedures in solving this problem.

Another epistemological obstacle was seen when the students tried to solve problems related to the application of arithmetic sequences and series. The incorrect ways that the students applied in their previous mathematical concepts were the potential factors causing students to experience epistemological obstacles. Mathematics is a science built by interconnected concepts, so students must be able to apply them correctly. Figure 5 show the question number 4 about the application of arithmetic sequences and series. Epistemological obstacles can be seen in the answers of students who answered correctly (Figure 6) and incorrectly (Figure 7).

4. Firza dan Zacky mulai menabung di bank pada saat yang sama. Pada awal menabung, Firza menabung Rp.800.000 dan tiap bulan ia menabung Rp.15.000 lebih banyak dari uang yang ditabungkan pada bulan sebelumnya. Pada awalnya, Zacky menabung Rp.1.000.000 dan tiap bulan berikutnya ia menabung lebih sedikit Rp.10.000 dari uang yang ditabungkan pada bulan sebelumnya. Mungkinkah jumlah uang yang ditabung Firza dan Zacky bernilai sama? Jika mungkin, pada bulan keberapakah hal itu terjadi ?

Translation:

Firza and Zacky started saving money in the bank at the same time. In the beginning, Firza saved Rp.800.000, and every month he saved Rp.15.000 more than the last month. At first, Zacky saved Rp.1.000.000, and each following month he saved Rp.10.000 less than his previous month. Is it possible for Firza and Zacky to save the same amount? If possible, in what month did this occur?

Figure 5. Question number 4
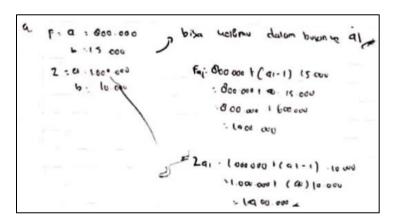

Figure 6. The answer of student B (Correct)

Figure 7. The answer of student C (Incorrect)

The following is a translated transcript of the researcher (R) interviewing Student B (B):

Interview 3

R   : *What do you understand from this problem?*

B   : *There are 2 cases, Ma'am. I mean, there are two sequences, the sequence of Firza and Zacky. For Firza, $a = 800.000$, then because he saved more, so $b = 15.000$. For Zacky, $a = 1.000.000$, then because he saved less, $b = -10.000$.*

R   : *Can a common difference (b) be a negative number?*

B   : *Yes, Ma'am. If the common difference of the sequence is negative, then it is decreasing sequence.*

R   : *All right, now try to explain how you did this problem.*

B   : *I used the manual method, Ma'am. I simplify the numbers in the thousands. For example, Firza, 800 plus 15 becomes 815, plus 15 for the following few terms. I did the*

same for Zacky. The first term 1000 minus 10 becomes 990, minus 10 again until we find a common number with the term in Firza sequence. It was in the ninth term.

R   : *Why did you using this method?*

B   : *Because I did not think of any other way, Ma'am. I just tried to count it manually.*

R   : *If you had to use a formula to solve this problem, what formula do you think you should use?*

B   : *I will use the U_n formula, Ma'am. It is impossible to use the S_n formula because the question is not related to the sum of the nth partial term.*

The following is a translated transcript of the researcher (R) interviewing Student C (C):

Interview 4

R   : *What information did you get from this question?*

C   : *From the question, we know that the first term (a) and the common difference (b) for each of Firza's and Zacky's sequences. For Firza $a = 800.000$ and $b = 15,000$, while for Zacky $a = 1.000.000$ and $b = 10.000$. We were asked for the same possible value and in what month.*

R   : *Are you sure about that? Try to pay attention to the common differences that you write.*

C   : *I am sure, Ma'am. For Firza, because every month he saves 15.000 more, then $b = 15.000$. For Zacky, because every following month he saves 10.000 less, then b = 10,000.*

R   : *Does the phrase "more" and "less" not affect the difference?*

C   : *I only looked at the numbers, Ma'am. The important thing is that it has a fixed common difference every month.*

R   : *All right, now try to explain how to solve this problem.*

C   : *I looked for the possible value of n, Ma'am. For example, I chose $n = 10$, then substitute it to the "anib formula" with the first term (a) and common difference (b) in each sequence. It turned out that the results were still far different. Next, I selected $n = 20$, and did the same step as before and so on until I found the same value when n = 41.*

R   : *Why did you use this method?*

C   : *This is the easiest way, Ma'am. We only need to substitute what is known to the "anib formula".*

R   : *What do you mean by the "anib formula"?*

C   : *It is the nth term formula for arithmetics sequence, Ma'am. $U_n = a + (n - 1)b$.*

R   : *Oh, that formula. Why do you call this formula the "anib formula"?*

C   : *It was from the teacher Ma'am, to make it easier to remember.*

Other students answered the question using a similar way to the student in Figure 6. Based on this answer and the transcript of interview 3, it was concluded that the students understood the concept of the first term and the common difference in the arithmetic sequence application problem. However, students have difficulty in finding the correct procedure to solve this problem. Students catch the word "common" in the question with a general understanding, so the student only counted manually, searching for the common term from the two known sequences. This student did not use previous knowledge regarding the similarities of the two mathematical equations. These mistakes may be due to the limited use of sequence and series contexts in learning. So that students are not getting

used to applying previous knowledge to the context of learning they have just encountered. As a result, students might experience learning obstacles called epistemological obstacles. Based on the onto-semiotic approach, students had not followed the correct procedure in solving problems.

Based on the students' answers in Figure 7 and interview 4, it can be seen that the student could change the arithmetic sequence application questions to a mathematical model but were still mistaken in determining the common difference, which was a negative common difference. This mistake shows that students do not understand the whole concept of common difference. Then the procedure used was also inappropriate. These mistakes may be due to the limited context of common differences when it was taught in the first learning. As a result, the students might experience learning obstacles called epistemological obstacles. Based on the onto-semiotic approach, it means that the students have difficulty defining a mathematic idea, and a common difference of arithmetic sequences. In addition, students had not performed proper mathematical procedures in solving this problem. The word *"anib formula"* used by students for memorizing the *n*th term formula is discussed in the didactical obstacles section.

Epistemological obstacles related to the application of arithmetic sequences and series were also seen in students' answers to question number 5. Similar to the previous case, the students' mistakes when applying previous concepts that they had understood to new problems became a factor that cause students to experience obstacles. The following Figures show the questions and a student's answers.

5. Sisi-sisi suatu segitiga siku-siku membentuk barisan aritmatika. Jika panjang sisi terpendeknya adalah 24 cm, tentukan:
   a. panjang ketiga sisi segitiga tersebut,
   b. keliling segitiga siku-siku tersebut.

Translation:
The sides of a right triangle form an arithmetic sequence. If the length of the shortest side is 24cm, determine:
   a.   the length of the three sides of the triangle,
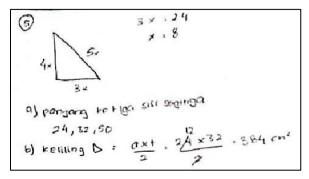   b.   the perimeter of the right triangle.

Figure 8. Question number 5

Figure 9. The answer of student A

The following is a translated transcript of the researcher (R) interviewing Student A (A):

> *Interview 5*
> R   : *Describe the procedure to solve this problem.*
> A   : *First, because it is a right triangle, it means that the sides are $3x$. $4x$, and $5x$. Since the shortest side is $24\ cm$, then $3x = 24$, and $x = 8$. Then I multiplied the other sides by $x = 8$, which were $4x = 4(8) = 32$, and $5x = 5(8) = 40$.*
> R   : *What do you mean by x?*
> A   : *x is the multiplication of the right triangle's side, Ma'am. In arithmetic sequences, it is called common difference, Ma'am.*
> R   : *Then why did you conclude that the lengths of the sides of the right triangle are $3x, 4x$, dan $5x$?*
> A   : *The easiest way to remember the sides of a right triangle is 3, 4, 5—these numbers of Pythagorean triple appears most frequently in questions.*
> R   : *Does the Pythagorean triple have to be $3, 4, 5$?*
> A   : *No, Ma'am, it could be $6, 8, 10$ or $9, 12, 15$. Eh, that is a multiple of $3, 4, 5$. In this problem, because the shortest side a multiple of 3, which is 24cm, then 3, 4, 5 Pythagorean triples can be used, Ma'am.*
> R   : *What if the shortest side is not a multiple of 3? For example, the shortest side is 14cm.*
> A   : *It means that the shortest side is the first term, which is $a = 14$, then the other side is $14 + b$, and the hypotenuse is $14 + 2b$.*
> R   : *After that, what procedure did you do then?*
> A   : *I am confused, Ma'am. I have not thought of what to do after this.*

Based on the answers of Student A in Figure 9 and the transcript of Interview 5, it was found that Student A understood this question quite well. She also tried to apply her previous knowledge related to right triangles to this problem. However, her previous knowledge was not sufficient. She only guessed the length of every side of a right triangle without knowing a concept of its comparison. She did not use the triple Pythagoras for finding the length of every side of the right triangle. Even though the student understood the problem and could translate this question into an arithmetic sequence model, Student A could not solve this problem correctly. This obstacle might be due to the limitations of the arithmetic sequence and a right triangle context when they were first taught. As a result, students might experience learning obstacles called epistemological obstacles. Based on the onto-semiotic approach, the definition of a common difference (b) is well understood, but there were still obstacles in the problem-solving procedure for this question.

Ontogenic Obstacles

Based on their nature, ontogenic obstacles are divided into three types: psychological, instrumental, and conceptual ontogenic obstacles (Suryadi, 2019b). Psychological ontogenic obstacles are related to psychological aspects, such as motivation and low interest in learning that caused students to feel unprepared to learn. Instrumental ontogenic obstacles are technical obstacles so that students do not to follow the learning process because they do not understand the core concept of the

topic learned. Conceptual ontogenic obstacles are related to the incompatible conceptual level of learning with students' learning experience. The students' answers show ontogenic obstacles when there was a discrepancy between their ways of thinking and the material(Wahyuningrum, Suryadi, & Turmudi, 2019). Figure 10 shows the answers of Student D, followed by the transcript of interview 6.

Figure 10. The answer of student D

The following is a translated transcript of the researcher (R) interviewing Student D (D):

Interview 6

R   : *Explain the procedure that you took to solve this problem.*

D   : *From the question, we know that Firza's first term (a Firza) is 800.000, and Zacky's first term (a Zacky) is 1.000.000. I do not know the initial common difference (b) for these sequences. Because Firza's savings increase over time, I wrote bFirza= initial b + 15.000, and because Zacky's savings decrease over time, I wrote bZacky= initial b-10. 000. Next, I supposed that initial b equals zero, then I multiplied the common difference by a trial and error number. The result showed the exact amount when I multiplied the common difference with 8 then adding the first term of each sequence. This means that the amount of their savings will be the same in the 8th month.*

P   : *What do you mean by "initial difference"?*

D   : *It was the first difference of the sequence, Ma'am.*

P   : *How do you know a common difference (b) between a sequence?*

D   : *It is $U_2 - U_1$ Ma'am.*

P   : *How about $U_4 - U_3$? Is it okay?*

D   : *Yes, ma'am, the formula for finding a common difference $U_n - U_{n-1}$.*

P   : *What do you mean by $(8 \cdot 15.000) + 800.000$?*

D   : *That is 8 from trial and error.*

P   : *Why did you use this method? Is there a formula like this?*

D   : *I don't know Ma'am, I forgot the formula. I just used logic.*

P   : *Do you know the general formula for arithmetic sequences?*

D   : *I also forgot, Ma'am. I only remembered that the arithmetic sequence is a sequence that has a common difference.*

Based on the answer of Student D in Figure 10 and the transcript of interview 6, it can be seen that Student D's understanding of the arithmetic sequence concept is not comprehensive. In the interview, Student D said that he forgot the general *n*th formula for arithmetic sequences, but from the answer, it seemed that the student used this formula, even though it was not quite correct. Student D knew the concept of common difference (*b*) in arithmetic sequences, but was incorrect in interpreting it when it was applied to the question. The student's mistake showed that lack of understanding the core concept of arithmetic sequences, the *n*th term formula. Sari et al. (in Octriana, Putri, &

Nurjannah, 2019) stated that the nth term formula (the general formula for a sequence) is the initial spear when learning number patterns. This obstacle is called the instrumental ontogenic obstacle. Based on the onto-semiotic approach, students could change the problem into mathematical models but were incorrect in the problem-solving procedures. Students were still confused about defining mathematical ideas.

Didactical Obstacles

Didactical obstacles can be identified through students' answers which emphasize a mathematical notion or formula without understanding the concept (Wahyuningrum et al., 2019). The didactical obstacle was identified by looking at the similarity of students' answers about the nth term formula in question number 2. When solving this problem, most students only replaced the first term (a) and the common difference (b) from the general nth term formula without solving them and constructing a specific nth term formula for the sequence in question. Based on students' answers in Figure 2 and transcript of interview 1, it was concluded that most students answered this question only by replacing a and b in the general formula because they were used to doing it in class. The teacher considered the answer as correct. As a result, the students did not understand that the specific nth term formula for each sequence can be constructed.

There was another problem with the nth term formula of arithmetic sequences. Some students did not write plus sign in the nth term formula in their answers. Student A was one of the students who made this mistake. Figure 2 shows the answers of Student A. In interview 1, it was found that this student forgot to write a plus sign in the answers. However, other facts were found when doing interview 4 with Student C. This student mentioned "anib formula" for the general nth term formula of arithmetic sequence. This nickname is probably the reason why most of the students did not write a plus sign in the nth term formula. This mistake happened due to memorizing the word given by the teacher.

The word "anib formula" for the general nth term formula of arithmetic sequences is basically one of the strategies used by the teacher to help students in memorizing the formulas. This strategy is also known as the mnemonic strategy. Brigham and Brigham (2001) defined mnemonics as a structured way to help people memorize and recall information. In line with this, Verdianingsih (2020) stated that the mnemonic strategy is used to help people remember information through coding, maintenance, and recall procedures, both in long-term and short-term memory. In learning mathematics, the mnemonic strategy is usually used for a topic that requires more memory, such as trigonometry (Ardika & Sardjana, 2016). To be effective, when using the mnemonic strategy, the teacher needs to allocate time to show each step in building the mnemonic. After that, the teacher demonstrates how the mnemonic works and allows students to practice independently and memorize the formed mnemonics (Miller & Mercer, 1993). There was a statement that mnemonics give a

narrow impression of learning mathematics. It tells students that their job is to memorize and follow procedures without thinking about why they are doing what they are doing and without understanding the mathematical concepts (Graybeal & Strickland, 2018).

The word "anib formula" was initially formed to help students memorize the nth term formula for arithmetic sequences. In fact, it has the potential to cause learning obstacles. This obstacle is due to the wrong choice of word by the teacher. This word tended to make students only memorize and use procedures without understanding the concept. As a result, there were learning obstacles which called the didactical obstacles. Based on the onto-semiotic approach, this obstacle is related to the definition and description of mathematical ideas related to the symbols used (Font et al., 2007; Godino et al., 2005; Rudi et al., 2020).

CONCLUSION

Based on the results of the analysis of student answers and interviews with students, we concluded that students experienced three types of learning obstacles: epistemological obstacles, ontogenic obstacles, and didactical obstacles. Students experienced epistemological obstacles due to their incomplete conceptual understanding and they were not accustomed to applying one mathematical concept into another. Students also experienced ontogenic obstacles when they could not understand the core concepts (key concepts) of arithmetic sequence and series topic. In addition, the learning process carried out by the teacher also became a factor in which students experience didactical obstacles. The onto-semiotic approach helped the researchers to focus on what needed to be analyzed. This research analyzed the students' understanding of definitions or descriptions of mathematical notions, students' ability to understand and change mathematical situations, and students' problem-solving procedures. This research provides an initial overview of students' conditions when studying the sequence and series topic. The results of this research provide an initial overview to the teacher regarding the conditions experienced by students. With these results, the future design of arithmetic sequences and series learning should avoid the identified obstacles.

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