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Collective Argumentation through Scaffolding: Homogeneous and Heterogeneous Groups in Solving Mathematics Tasks

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Abstract

Collective argumentation plays a crucial role in enhancing students' mathematical understanding through discussion. While previous studies have explored collective argumentation and group composition, only a limited number of research has examined the impact of ability-based grouping—both homogeneous and heterogeneous—on collective argumentation in mathematics learning. Based on this, the current research aims to explore collective argumentation in homogeneous and heterogeneous groups of students, supported by scaffolding, in solving mathematically and non-mathematically rich tasks. Using a qualitative approach with a case study design, the present study involved two groups of eighth-grade students, each consisting of six eighth-grade students with high, medium, and low abilities. Data were collected through recorded group discussions, observations, and interviews. After that, the collected data were analyzed using the Toulmin argumentation model. The findings reveal that homogeneous groups of high-ability students engaged more actively in idea exploration and generated dynamic arguments, incorporating key argumentation elements such as claims, data, warrants, rebuttals, and qualifications. In contrast, in heterogeneous groups, high-ability students dominated discussions, while lower-ability students were more passive and relied on scaffolding from teachers or peers. Furthermore, mathematically rich tasks were more effective in fostering in-depth discussions than non-mathematically rich tasks. These findings highlight the importance of strategic student grouping and scaffolding in promoting engagement and meaningful collective argumentation in mathematics learning.

Keywords: Collective Argumentation, Heterogeneous Group, Homogeneous Group, Mathematics Tasks, Scaffolding

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INTRODUCTION

Argumentation is a reasoning process based on facts and evidence to support or refute a particular statement, as well as a verbal, social, and rational activity that aims to build or reject claims using structured evidence (Aaidati et al., 2022; Ayalon & Nama, 2024; Hinton, 2021; Wang, 2020). In addition to defending a conclusion, argumentation also plays a role in building and validating knowledge, enabling critical reflection on a concept through social interaction and strengthening evidence (Chowning, 2022). Furthermore, argumentation fosters critical thinking by encouraging students to evaluate various points of view systematically and build reasoned conclusions. Therefore, argumentation is not only a tool to prove a conclusion, but also an important means of developing systematic, critical, and evidence-based thinking.

In mathematics learning, argumentation is a crucial aspect in helping students develop mathematical argumentation and concepts through interaction, making claims, and supporting them with valid evidence (Dogruer & Akyuz, 2020; Swidan, 2022). Its process allows students to evaluate their ideas, explore concepts in greater depth, and enhance their understanding through interactions with

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peers and teachers. Furthermore, argumentation promotes conceptual understanding (Romero Ariza et al., 2024), makes learning more meaningful (Gomez Marchant et al., 2021), and enhances students' mathematical problem-solving abilities (Indrawatiningsih et al., 2020; Iwuanyanwu, 2022). As a strategic component, it has also become a significant focus of mathematics education research because of its potential to deepen students' understanding of given materials and encourage critical engagement in the learning process (Mariotti et al., 2018).

However, although argumentation is often viewed as an individual activity, in mathematics learning, argumentation occurs more often in social settings, such as class discussions or group work. This situation is often referred to as collective argumentation (Krummheuer, 1995). In this context, individuals collaborate to construct a claim and support it with relevant evidence (Estrella et al., 2024). Furthermore, Krummheuer said that the concept of collective argumentation can be more clearly understood through a social framework that considers the structure within the model of social interaction (Bayindir et al., 2024; Krummheuer, 1995). This structure is closely related to Toulmin's argumentation model, which illustrates how teachers and students engage in discussions to collaboratively construct mathematical understanding. Cervantes-Barraza et al. (2020) found that fifth-grade students who applied the Toulmin extension model to construct mathematical proofs demonstrated complex interaction dynamics, in which peer discussions played a significant role in improving their reasoning. This collective argumentation process encouraged students to critique peers' ideas, clarify mathematical concepts, and negotiate meanings in greater depth, thereby enhancing their understanding. Within this context, the teacher acted as a facilitator by asking exploratory questions, emphasizing conceptual relationships, and guiding students in developing stronger claims. In addition, the teacher utilized the Toulmin extension model to help students validate their claims with relevant data, thereby encouraging deeper reflection and building more systematic mathematical argumentation.

Through constructing arguments, evaluating claims, and refining their thinking, students recognize that solving mathematical problems is not only about finding the correct answer but also about understanding the reasoning behind the solution. The effectiveness of collective argumentation is highly dependent on the design of the given task, with complex and challenging mathematical tasks shown to be more effective in stimulating critical discussion than procedural tasks that require only the application of specific steps (Foster, 2018; Yeo, 2017). Therefore, argumentation is not simply an individual attempt to persuade others but also can be regarded as a collaborative process in which students construct shared understanding through interactive discussion.

Having stated that, mathematical tasks can be categorized into two types: non-mathematically rich tasks, which emphasize applying learned methods and following established procedures, and mathematically rich tasks (Yeo, 2017). In contrast, mathematically rich tasks promote deeper reasoning, multiple solution strategies (Sevinc & Lesh, 2022) and active engagement in problem-solving discussions (Yeo, 2017). These tasks challenge students to investigate, collaborate, and justify their reasoning, fostering argumentation and conceptual understanding in the classroom (Ayalon et al., 2021; Fitriati et al., 2021).

However, in addition to the type of task assigned, group composition also plays a crucial role in shaping social interactions and the quality of discussions (Lloyd & Murphy, 2023). Heterogeneous groups of students with varying abilities can give way to students with higher abilities to support their less able peers, improving the quality of discussion and shared understanding among the members (Matthewes, 2020). In contrast, homogeneous groups consisting of students with similar levels of ability, although more focused on a common goal, can strengthen a more uniform understanding and increase the effectiveness of argumentation (Shuowen & Zhang, 2024). Several studies have shown that students in low-ability groups often have difficulty setting and critiquing tasks and rarely discuss data in depth. In contrast, high- and medium-ability groups are more active in discussing and analyzing data, contributing to their argumentation's productivity and quality (Ryu & Sandoval, 2015). Thus, the effectiveness of collective argumentation is highly dependent on the composition of the student group.

While research on collective argumentation and group composition exists, little attention has been given to the process of selecting group members. This has presented an opportunity to explore how variables such as mathematical ability, gender, and learning style influence argumentation dynamics (Ekawati et al., 2025). Although some studies have examined the effects of grouping, insights into how homogeneous and heterogeneous group compositions affect the quality of collective argumentation in mathematics learning still remain uncharted. Therefore, further research is needed to dive deep into understanding how grouping students based on ability can support the success of collective argumentation.

Finally, in the collective argumentation process, the teacher's role as a facilitator is crucial. Teachers can directly contribute to the argumentation component, ask questions, and take other actions to support the process (Zhuang & Conner, 2024).Teachers' support in collective argumentation aligns with the concept of scaffolding in groups, which serves to support the content and process of argumentation (Sevinc & Lesh, 2022), both by providing guiding questions that sharpen arguments and helping students clarify their ideas (Sevinc & Lesh, 2022; Zhuang & Conner, 2022). Teacher intervention through scaffolding aims to expand students' proximal development zone to understand better argumentation activities (Peng & Tao, 2022; Xi & Lantolf, 2021). Thus, scaffolding helps in better arguments and encourages students to be actively involved in discussions and solving mathematical problems independently.

The current research aims to describe collective argumentation with scaffolding in groups of students with homogeneous and heterogeneous ability compositions in completing mathematics tasks. The findings of this study are expected to provide new insights into the description of group composition in the collective argumentation process with scaffolding so that it can guide educators in designing teaching strategies to be more responsive to differences in student abilities.

METHODS

This study uses a qualitative approach with a case study design to explore in depth the phenomenon of collective argumentation that occurs in groups of students with homogeneous and heterogeneous ability compositions when solving mathematical tasks (Alam, 2021). A qualitative approach was applied because it allowed the researchers to understand the dynamics of discussion, social interaction, and argumentation processes that develop in natural settings (Tümen-Akyıldız & Ahmed, 2021). Furthermore, the researchers played an active role in data collection and analysis, providing deeper insight into the studied phenomena.

Research Subjects

The focus of this study was on homogeneous and heterogeneous groups, each consisting of three students. The first group was a homogeneous group with students possessing high mathematical ability, while the second group was a heterogeneous group consisting of one high-ability student, one medium-ability student, and one low-ability student. In total, this study recruited six students selected from 68 eight-grade students of a secondary school in Banjarmasin who had studied linear equation material. The selection of subjects was based on the results of a mathematical ability test that included questions about integers, linear equations of one variable, number patterns, quadrilaterals, and comparisons. This test measured the students' cognitive levels at levels C3 (applying) and C4 (analyzing). The mathematical ability test consisted of 5 descriptive questions validated by three mathematics education lecturers holding doctoral degrees and more than 15 years of teaching experience. The researchers assigned scores based on established guidelines. These scores were then categorized according to the criteria outlined in Table 1. The students' mathematic abilities based on a previous study (Henra et al., 2024) are displayed in Table 1.

Table 1. Student mathematic ability categories

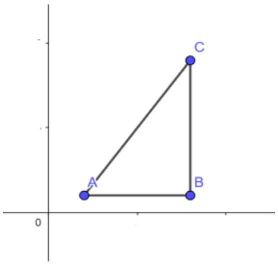
Category	Student Grades
High	$80 \leq \text{score} \leq 100$
Medium	$60 \leq \text{score} < 80$
Low	$0 \leq \text{score} < 60$

In this study, there were two different groups that were analyzed because homogeneous groups with high ability tend to be more effective in collaborating, while heterogeneous groups (high, medium, and low) provide a more complete picture of the dynamics of the discussion, including the challenges faced by lower-ability students (Shuowen & Zhang, 2024). Only the students who were willing to be respondents were selected for this study.

Mathematics Tasks

The mathematics tasks used had been validated by the same validator as the mathematics ability test. Tasks were divided into two types: mathematically-rich tasks and non-mathematically-rich tasks. Mathematically-rich tasks are tasks designed to encourage students to think critically and apply various mathematical concepts in more open and complex contexts, while non-mathematically-rich tasks are tasks that are more focused on applying mathematical procedures or rules that have been taught. Table 2 illustrates the details of the mathematics tasks that the research subjects will complete.

Table 2. Mathematics tasks for explore argumentation collective with scaffolding

Mathematics Tasks	Types of Mathematics Tasks
Amat, Budi, and Cici form a standing formation on the field as shown in the following diagram. Point A represents Amat's position, point B represents Budi's position, and point C represents Cici's position.  Next, Dian comes to join the formation. The positions of Dian, Amat, and Cici are on the same line. Amat is at position (2,1). The distance between Amat and Budi is 6 dm. The distance between Amat and Cici is 10 dm. The distance between Cici and Dian is the same as the distance between Dian and Amat. Determine Dian's standing position. Explain how you determined Dian's position.	Mathematically-rich tasks
Given point A (1,1), point B (4,4), point C $(2,-2)$, point D (7,3), point E (2,4), and point F (6,0). Line u passes through point A and point B. Line v passes through point C, and point D. Line w is a line that passes through point E and point F. Explain how the relationship is (1) between line u and line v (2) between line w and line u	Non-mathematically rich tasks

Research Stage

After the research subjects, namely homogeneous and heterogeneous groups, had been determined, the mathematical tasks were given to each group. The assignment was carried out separately on the same day, where each group did assignments in a separate room to avoid interference from other students. Homogeneous groups did the tasks first, then followed by heterogeneous groups. The time lag between the two groups was 10 minutes to prevent the possibility of leakage of information between groups. All interactions in group discussions and between groups and researchers were recorded in video and audio. The researchers observed the course of group discussions. In this case, the researchers acted as facilitators who guided discussions, reminded students about prerequisite knowledge, and supported the development of solutions, conclusions, and arguments. In addition, researchers also ensured that each group reached a mutual agreement on the conclusions they made. After the group had completed the assignment, the researcher interviewed them to explore the students' thinking and arguments more about their completed tasks. The interview was carried out the next day after the task was completed. The whole process was carried out in Indonesian, and then the results were translated into English for further analysis.

The data collected was analyzed using a collective argument reconstruction developed by Cervantes-Barraza et al. (2020). This analysis was carried out on the transcript of the discussion record, student answer sheet, and interview to identify the contribution of each student to the argument process. In analyzing the data, the researchers utilized an interactive model (Miles et al., 2014), which consisted of three common activity flows, namely data reduction, display, and drawing conclusions. The reduction data was carried out by filtering and selecting relevant data from the discussion transcript, sheet answers, as well as student interviews. Every form of interaction in the argument was categorized into six components of argumentation, namely data, claims, warrant, support, qualifications, and refutations, which are presented on Table 3. The scaffolding provided in the form of questions, as referenced by Bikmaz, included verification and clarification, invited students to provide clues, and encouraged student participation (Agoestanto et al., 2020; Bikmaz et al., 2010; Mahharrini et al., 2020). These components helped in understanding how students compile, support, and evaluate their claims in the discussion. Finally, conclusions were drawn based on the findings that have been presented.

Table 3. Description of component argumentation

Component Argumentation	Description
Data	Statement underlying the conclusion made
Claim	Statement of conclusion based on data supported by warrant
Warrant	Statement connecting data with a claim
Backing	Statement supporting the warrant
Qualifier	A statement stating the condition or requirement under which the claim applies or a statement of the degree of confidence in the claim
Rebuttal	Statement refuting claim applies or warrant

Cervantes-Barraza et al. (2020)

RESULTS AND DISCUSSION

The distribution of test results by ability category is presented in Table 4. Six of 68 students were purposively selected and assigned to two groups. The first group was homogeneous, consisting of three high-ability students from class 8A. The second group was heterogeneous, comprising three students from class 8B with varying abilities: high, medium, and low. According to the mathematics teacher, all six students have strong communication skills. During the task completion, one of the researchers acted as a facilitator, guiding group discussions, reminding students of prerequisite knowledge, and supporting the development of solutions, conclusions, and arguments. The researcher ensured consensus on the conclusions reached by all group members. The distribution of students' test results based on ability and gender is displayed in Table 4.

Table 4. Distribution of students based on mathematic ability test results and gender

Category	Sum of Students				
	8A		8B		Total
	Male	Female	Male	Female	
High	4	0	1	5	10
Medium	6	5	20	18	49
Low	2	4	1	2	9
Sum	12	9	22	25	68

Collective Argumentation with Homogeneous Group Scaffolding

The results of the collective argumentation activities related to mathematically rich tasks are summarized as follows. Each group member shared relevant information to complete the task collaboratively. For instance, student 1 (S1) identified Amat's position at (2,1), and student 3 (S3) provided details about the formation of Amat, Budi, and Cici, including the distances between Amat and Budi (6 dm) and Amat and Cici (10 dm). Student 2 (S2) contributed the positions of Dian, Amat, and Cici on the same line and posed the task question regarding Dian's position. As a result, all members agreed on the information presented. The group then focused on the diagram and calculated the distance between B and C using the Pythagorean theorem. The researcher asked the group to clarify their understanding about formula. The conversation is illustrated in the following transcript.

Researcher : *"How can we find the distance between B and C?"*
S1 : *"Use Pythagoras, ma'am"*
Researcher : *"Could you explain how the Pythagoras can help us?"*
S1 : *"The formula is $c^2 = a^2 + b^2$. In this triangle, I think a is AB, b is BC, and c is AC."*
Researcher : *"Ok, S1 has suggested that a is AB, b is BC, and c is AC. How about the others?"*
S2 : *"I think that is wrong, I think a should be BC, b should be AC, and c should be AB."*
Researcher : *"We have two different positions of a, b, and c. S3, what do you think?"*
S3 : *"I used a different approach. I didn't make label like that. I did it like this* (show his work (see "Claim" at Figure 1))*"*
Researcher : *"So, what is the Pythagoras theorem?"*

S1	:	*"Pythagoras formula is the square of the hypotenuse is equal to the sum of the squares of the other two sides."*
Researcher	:	*"Which side is the hypotenuse in this triangle?"*
S2	:	*"The hypotenuse is AC because it's opposite the right angle and is the longest side."*
Researcher	:	*"So, if AC is the hypotenuse, which side should we label as c?"*
S1	:	*"a should be BC, b should be AC, and c should be AB"*
Researcher	:	*"S2, S3, do you agree with this?"*
S2	:	*"Yes, ma'am, I agree with S1. So, the position of c is AC or the hypotenuse, while a is AB and b is BC. They are at the two other sides."*
S3	:	*"Yes, I agree."*

In the conversation above, the researcher attempted to clarify the positions of points a, b, and c, as mentioned by S3. The Pythagoras formula is used as a warrant to determine the distance between B and C. Figure 1 shows the collective argumentation built by the student group and the researcher in finding the distance between B and C.

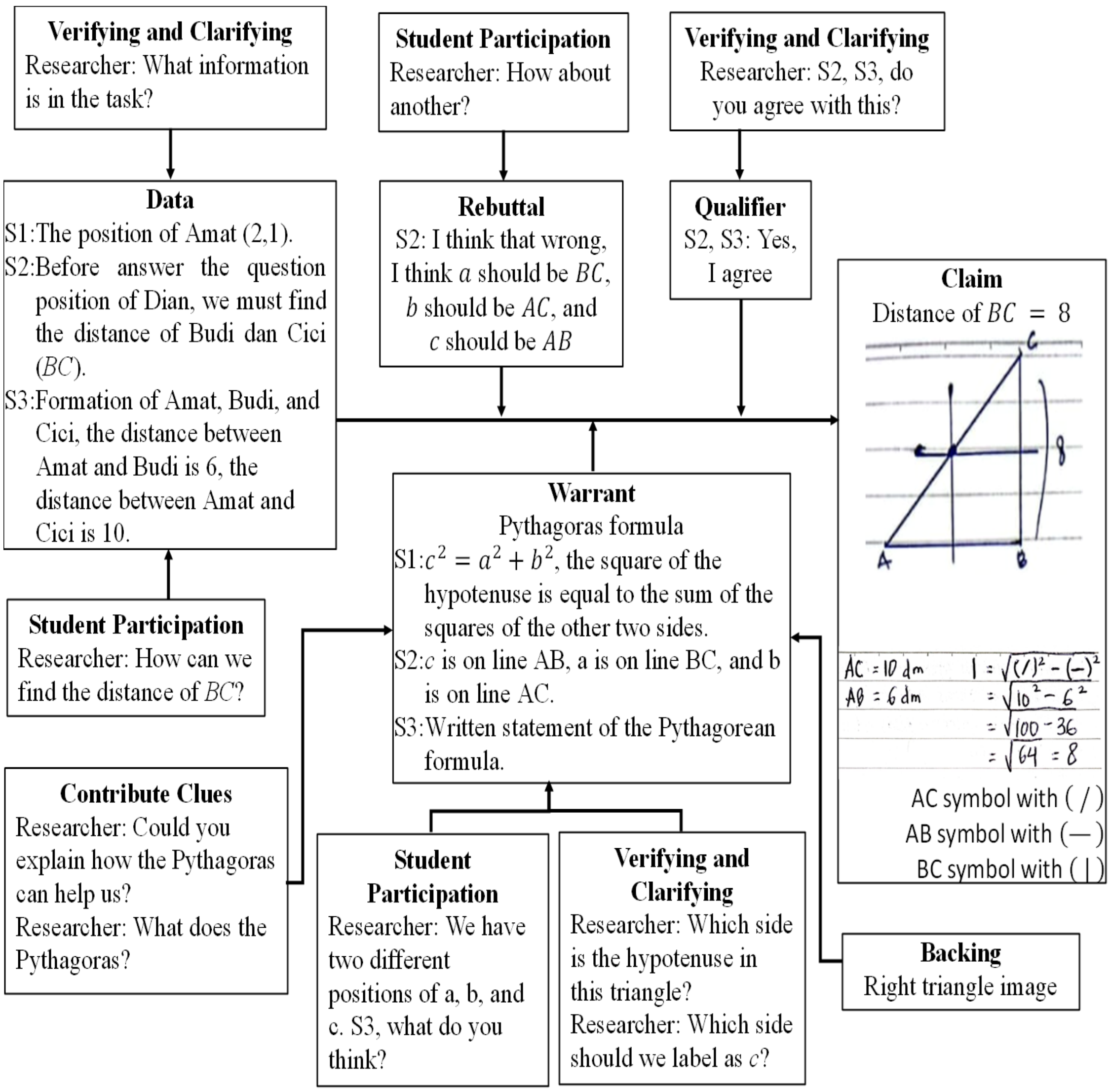


Figure 1. Diagram collective argumentation finding the distance between B and C

After the group of students had obtained the distance BC, they counted eight units upwards from Budi's position to determine Cici's position, obtaining Cici's position (8,9). When determining Dian's position, S2 proposed a solution by dividing the distances between A and B, B and C into two equal parts, arguing that Dian's position was right in the middle. From this solution, Dian's position was obtained at the point $(5,5)$. However, the other group members still looked confused and did not agree with this way of determining Dian's position. The researcher then asked questions to continue the discussion. The following is a snippet of the conversation that took place.

S2 : *"I tried to find Dian's position by dividing the distance of line AB and line BC into two equal parts. Since Dian is supposed to be in the middle, dividing each segment in half should place her at the center point. So, dividing AB gives 3, and counting 3 from Amat brings us to 5. Similarly, dividing BC, which is 8, gives us 4, and counting 4 from Budi also brings us to 5. Therefore, Dian's position should be* $(5,5)$*."*

S1 : *"I'm not sure if that's true."*

S3 : *"I'm confused too. Why does dividing into two put Dian in the middle?"*

Researcher : *"Good questions, S1 and S3. S2, could you explain why you think dividing each line in half will give Dian's exact position at* $(5,5)$*?"*

S2 : *"Dian is in the middle, so dividing AB and BC in half will put Dian at the midpoint between Amat and Budi."*

Researcher : *"Okay, everyone else, any other ideas for determining Dian's position?"*

S3 : *"I have a different way. I make a right triangle and a rectangle."*

Researcher : *"Can you explain it?"*

S3 : *"The area of two triangles has the same area as the rectangle. We divide it into 3 equal areas."*

S2 : *"We need to find a point so that the areas are equal."*

S1 : *"I think it makes sense to me."*

S2 : *"Yes, I think that's a good idea."*

S3 : *"I did like this"*
(S3 proceeds to sketch out the triangles and rectangle, showing that the areas on both sides of the midpoint align if Dian's position is at $(5,5)$ (see "Claim" at Figure 2))

S1 : *"Yes, both methods—dividing the distance and using the area—give the same result."*

S2 : *"I agree with that. It means we can use two ways."*

In this conversation excerpt, a conclusion emerges based on mutual agreement among group members. S3 put forward a qualification, which stated agreement with S2's answer, although using a different method, followed by S1, who agreed with S3's answer. Although S3 and S2 utilized different solution methods, both obeyed the same basic comparison rules. Figure 2 shows the collective argumentation built by the student group and the researcher in finding Dian's position.

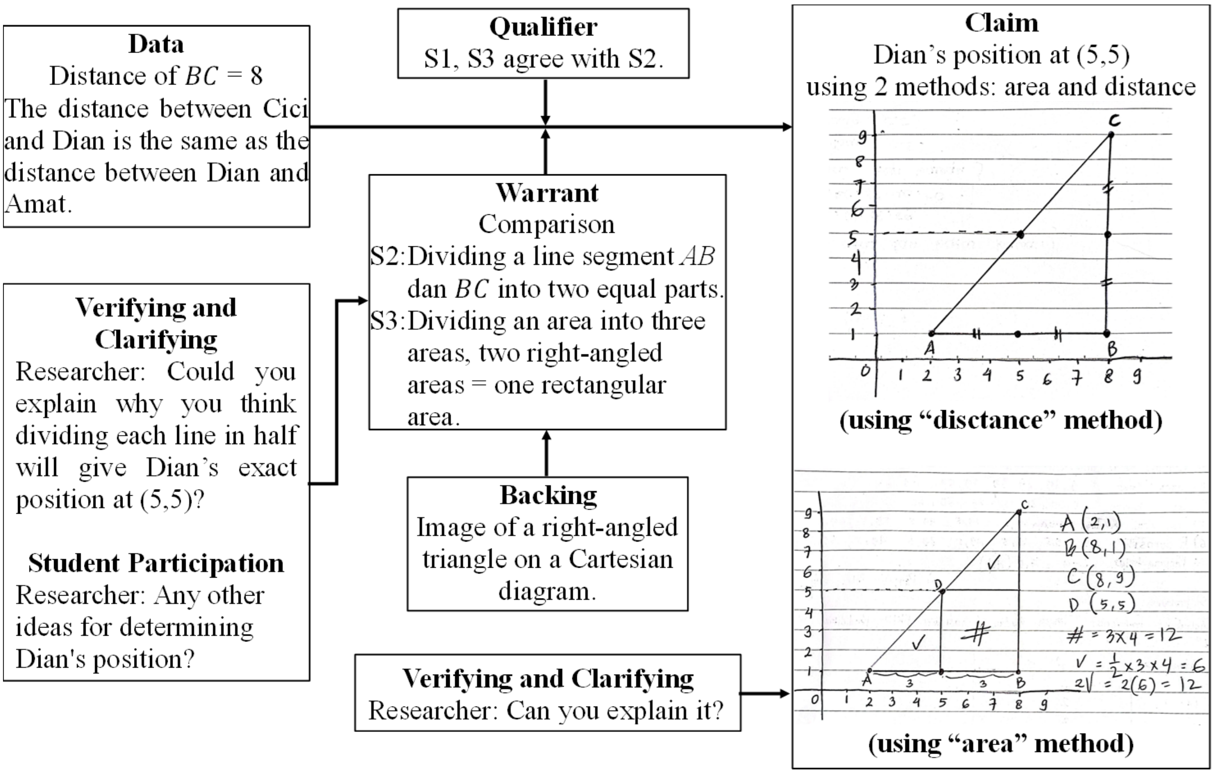


Figure 2. Diagram collective argumentation finding Dian's position

Student groups completed non-mathematically rich tasks after finishing mathematically rich tasks. Based on the task information, the group decided to plot a point on the Cartesian plane. From this point, S2 asserted that lines u and v were parallel, while lines w and u intersected. However, S2's statement was refuted by S3, stating that lines w and u were perpendicular to each other. S2 replied by explaining that perpendicular lines consist of one vertical line and one horizontal line, which formed a 90°. The researcher then asked the groups how they could determine whether two lines intersect, are perpendicular, or are parallel. The following is an excerpt from their discussion.

S2	:	*"Based on the points I've drawn, line* u *and line* v *are parallel, and line* w *intersects line* u*."*
S3	:	*"Or maybe they're perpendicular?"*
S2	:	*"Perpendicular means one line is vertical and the other is horizontal, forming a* 90°*."*
Researcher	:	*"How can we be certain whether the lines are intersecting, perpendicular, or parallel?"*
S1	:	*"Hmm, we could calculate the gradient to check."*
Researcher	:	*"Good idea! How would we do that?"*
S2	:	*"I think we can use the formula y = mx + c or maybe this* $\left(\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\right)$*"*
S3	:	*"That's the equation of a line, not the gradient"*
S2	:	*"But in the equation of a line, there is a gradient. The slope, m, represents the gradient."*
S1	:	*"Yes, exactly. We have two points, A and B, we can find the gradient of line* u *between them."*

S2 : *"I got the equation is $y = x$ and the gradient of line u as 1."*
S3 : *"And for line w, I got -1"*
Researcher : *"Now, based on the gradients you found, and the image S3 created, what can you conclude?"*
S2 : *"I remember if perpendicular if $m_u \times m_w = -1$, gradient u $= 1$ and gradient w $= -1$, so u and w are perpendicular"*
S1 and S3 : *"Yes, it's perpendicular."*

Figure 3 shows the diagram of collective argumentation of the non-mathematically rich task of a homogeneous group.

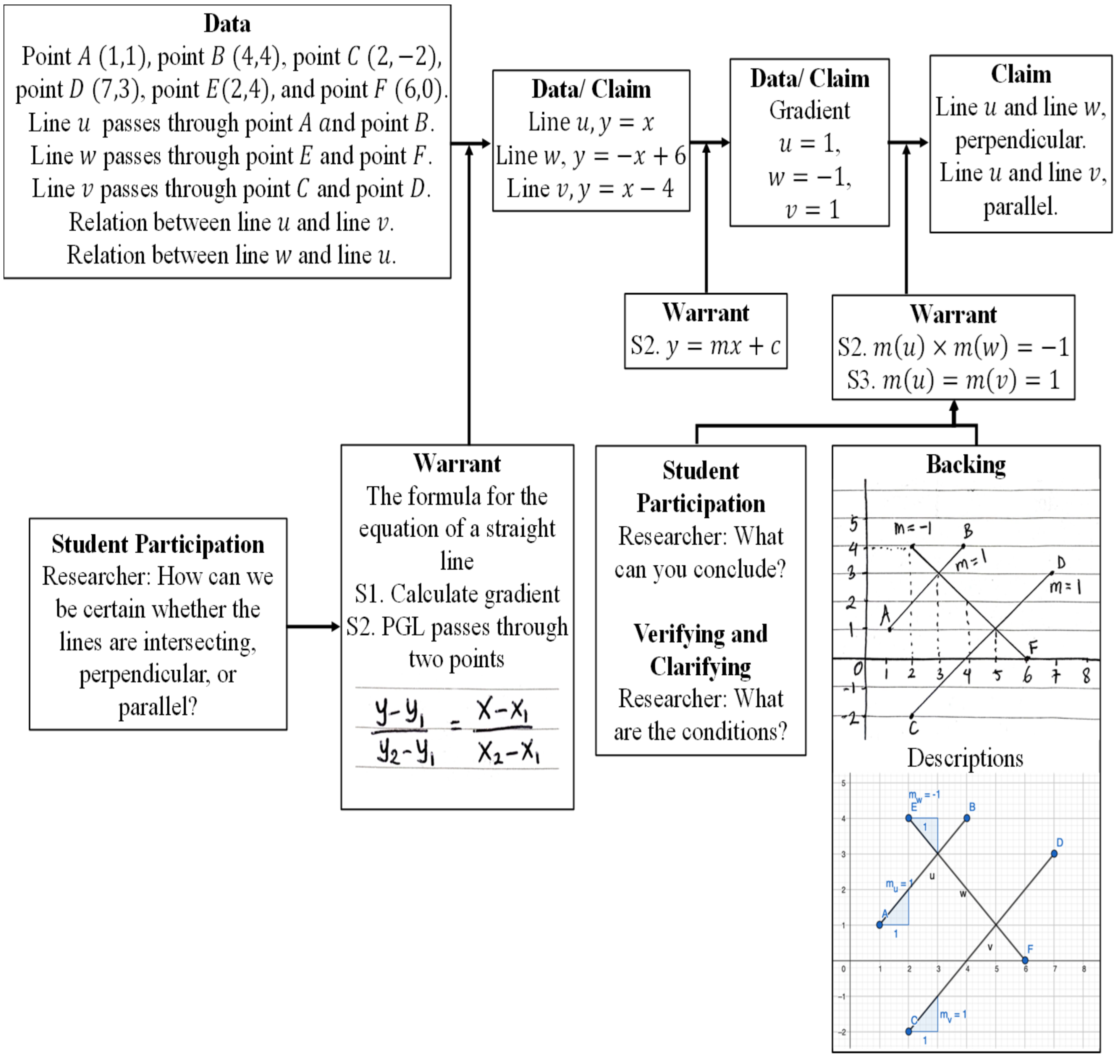


Figure 3. Collective argumentation of non-mathematically rich task of a homogeneous group

When completing mathematical tasks rich in concepts, they showed argumentation skills that allowed the exploration of several approaches and the achievement of consensus. All members actively participated in producing ideas, demonstrating a deep understanding of the concept of proportions and their relation to several mathematical strategies. This enriching discussion, with more diverse and in-depth solutions, aligns with the characteristics of rich mathematical tasks that tend to produce non-single solutions (Fitriati et al., 2021; Foster, 2018). This kind of task encourages students to integrate several concepts and apply critical thinking, increasing their exploration and understanding (Ayalon et al., 2021).

Through collective arguments, it has been stated that students propose solutions and strengthen their understanding through interaction (Cervantes-Barraza et al., 2020; Cetina-Vázquez et al., 2019; Swidan, 2022). In this study's case, two completion methods arose when students completed rich mathematical tasks. Although all students had high mathematical abilities, collective arguments still provided benefits in increasing their understanding. S1, for example, could gain a deeper understanding of proportions in the right triangle through an explanation from S2 and S3. S2 proposed how to determine the position of Dian using distance, while S3 explored alternative methods using the area (see Figure 2). This exchange of ideas further enriched their mathematical understanding. As for the mathematical task, this group utilized a straight-line equation to determine the line gradient.

The argument structure produced in homogeneous groups when completing rich mathematical tasks includes all components of arguments, including data, claims, warrants, support, rebuttals, and qualifications. The qualifications are strengthened by comprehensive evidence, including presenting alternative solutions during completion. Likewise, the rebuttals proposed by individuals in the group improved and clarified students' understanding of applying the Pythagorean theorem (as illustrated in Figure 1). As for the mathematical task that was not rich, the argument components produced include data, claims, warrants, and support.

Moreover, it was found that interactions in homogeneous groups tend to be more egalitarian, with each member actively contributing by expressing ideas, asking questions, and challenging opinions. This finding aligns with the research of Shuowen & Zhang (2024), which notes that students in homogeneous groups with high abilities are more often involved in in-depth discussions and focus on complex problem-solving strategies. Meanwhile, scaffolding provided by researchers in homogeneous groups focuses on clarifying concepts and stimulating in-depth thought, allowing students to explore alternative approaches such as comparing distance and area. Researchers should provide minimal scaffolding and gradually reduce student reliance to encourage independence and deeper understanding (Van de Pol et al., 2019).

Collective Argumentation with Heterogeneous Group Scaffolding

The heterogeneous groups consisted of three students: a high-ability student (S4), a medium-ability student (S5), and a low-ability student (S6). Students were encouraged to collaborate and discuss within their groups to improve their understanding of the assigned task. S4 began by reading the main details about Amat, Budi, and Cici's positions and noting the distances between Amat, Budi, Amat, and Cici. This information became the basis for group discussion and diagrams on the coordinate plane. S5 added that the distance between Amat and Budi was six dm, while the distance between Amat and Cici was ten dm. S6 mentioned that the distance between Cici and Dian was the same as between Dian and

Amat. This is following conversation determines Cici's position.

Researcher	:	*"So, how do we determine Cici's position?"*
S6	:	*"Cici's position* (8,11). *Amat's position is* (2,1)*, then I count 6 units to the right to find Budi's position. I measured along this line; I got 10 units (pointing to line AC for 10 units)."*
S5	:	*"The distance from Amat to Cici is 10 units, so we measured along this line (pointing to line AC for 10 units). We count 10 units up, which gives us 11. So, Cici's position is (8,11)."*
Researcher	:	*"Is the distance the same for each side of this triangle?"*
S4	:	*"It doesn't seem so, ma'am. This side* (pointing to the diagonal line) *is longer than this one* (pointing to the vertical line)*."*
Researcher	:	*"What kind of line is formed by Amat and Budi, Amat and Cici, or Budi and Cici? Are the lines the same?"*
S5	:	*"The line between Amat and Budi is horizontal, Amat to Cici is diagonal, and Budi to Cici is vertical."*

The conversation that occurred above shows that S6 made an inappropriate claim regarding Cici's position, which was later agreed by S5. However, the researcher tried to direct the group to better grasp the concept of the length of the side of a triangle. S4 also tried to convey his views on this matter. The collective argumentation diagram that emerged from this conversation can be described as follows in Figure 4.

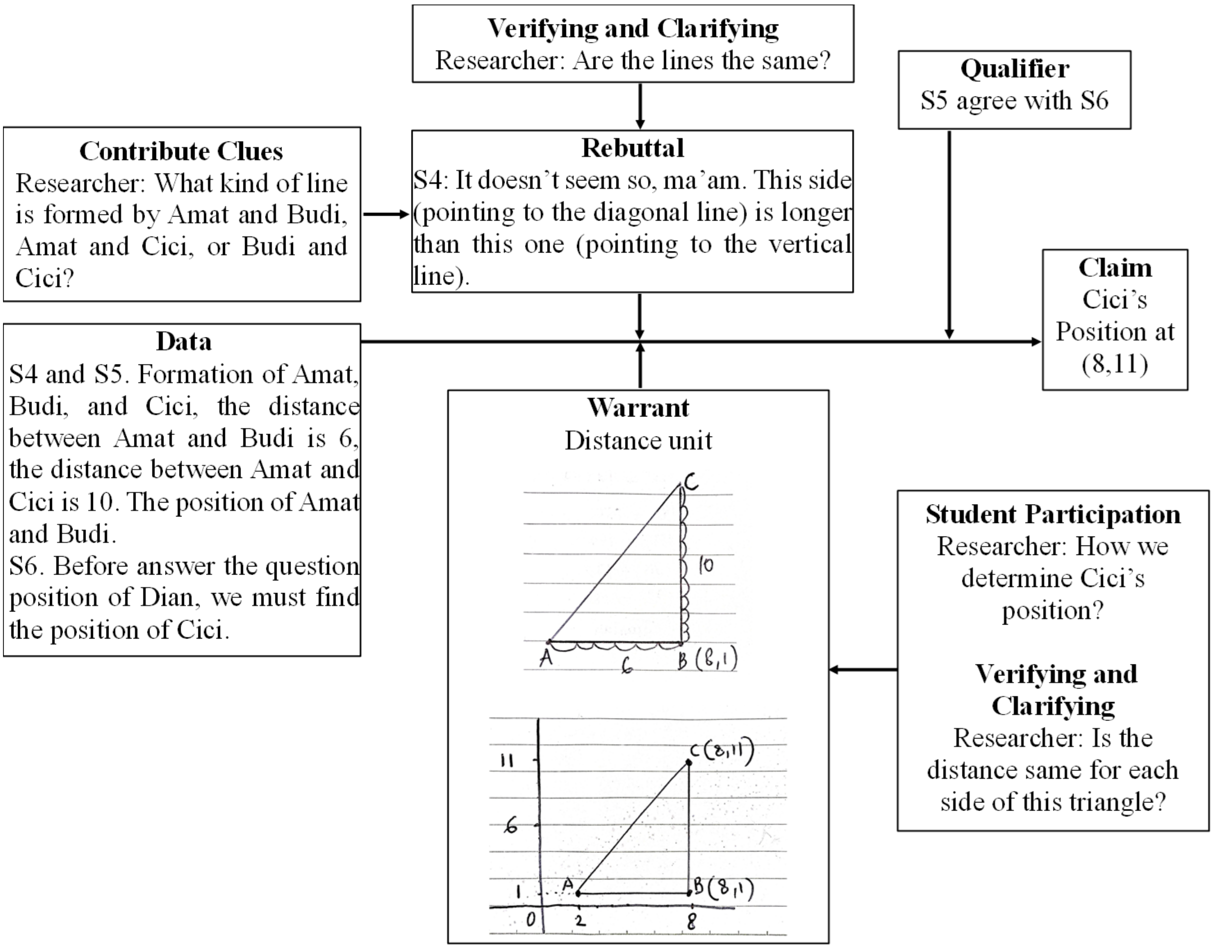


Figure 4. Collective argumentation about Cici's position in heterogenous groups

Furthermore, after the group members had understood that the unit distance on the horizontal, vertical, and oblique lines was not the same, the next step was to determine Cici's position, which began by identifying the distance between Budi and Cici. The group utilized Pythagoras for find the correct position of Cici. After that, the group looked for Dian's position. In this group, the way to get Dian's position was only one solution offered, using a distance comparison. Here is an excerpt from the conversation that took place.

Researcher : *"How to determine Dian's position?"*

S4 : *"Here, ma'am, in the middle of line AC, because Dian, Amat, and Cici are on the same line."*

Researcher : *"How do you determine the exact point?"*

S6 : *"In the middle, right? So, it should be divided into two equal parts."*

Researcher : *"Divided into two–what does that look like exactly?"*

S5 : *"The distance from Cici to Dian should be the same as the distance from Dian to Amat. So, Dian is in the middle."*

S4 : *"We'll split the lines BC and AB in half. If we divide the distance between A and B AB by 2, we get 3. Dividing BC by 2, we get 4. Then, we count 3 steps from Amat's position and 4 steps from Budi's position. So, Dian's position is* (5,5)*."*

Researcher : *"How about S6, do you think there is another way to determine Dian's position?"*
S6 : *"No, ma'am. I agree with it."*

Figure 5 is the diagram of collective argumentation from a heterogeneous group in finding the Position of Dian.

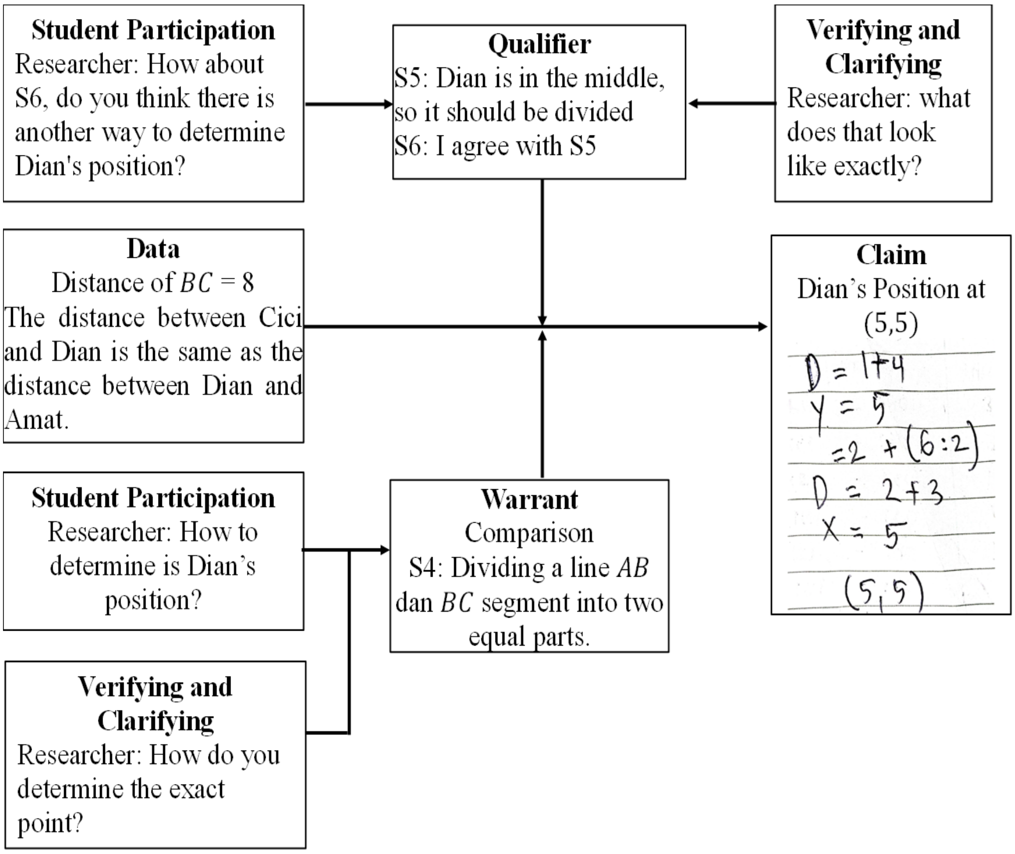


Figure 5. Diagram collective argumentation from a heterogeneous group in finding position of Dian's

Similar to the previous group, in this group they also worked on non-mathematically rich tasks after solving the mathematically rich task. The researcher began to engage in discussion with students by asking for information in non-mathematically rich tasks. Then, the group members began to put forward claims that they believed. Here is an excerpt from the conversation of the non-mathematically rich task completion activity in the group regarding the relationship between perpendicular lines. In general, the diagram of collective argumentation with scaffolding that occurred in the group's non mathematically rich task is as follows.

Researcher : *"How do we know they're parallel, perpendicular, or intersect?"*
S4 : *"From this picture, ma'am. See, the line u dan v, they don't intersect."*
Researcher : *"If two lines don't intersect, are they necessarily parallel?"*
S4 : *"Well, ma'am, these lines are parallel because they're positioned like this."*
Researcher : *"Does anyone else have an idea about what ensures two lines are parallel?"*
S5 : *"We can use the gradient, ma'am."*
S4 : *"Lines are parallel if they have the same gradient."*

Researcher : *"How do we find the gradient?"*
S4 : *"Use this formula,* $m = \frac{y_2 - y_1}{x_2 - x_1}$*"*
Researcher : *"Anyone else, any suggestions?"*
S6 : *"No, I think that is the formula."*
S4 : *"I've already calculated it: the gradient of line* u*, through points* A *and* B*, is one."*
S5 : *"The gradient of line* v *is also one"*
S6 : *"It means they're parallel."*
Researcher : *"How about the line* u *and* w*?"*
S5 : *"The gradient of line* w *is -1."*
S4 : *"Line* u *and* w *are perpendicular, because* $m_u \times m_w = -1$*."*

Figure 6 is a diagram of the collective argumentation of the heterogeneous non-mathematically rich tasks.

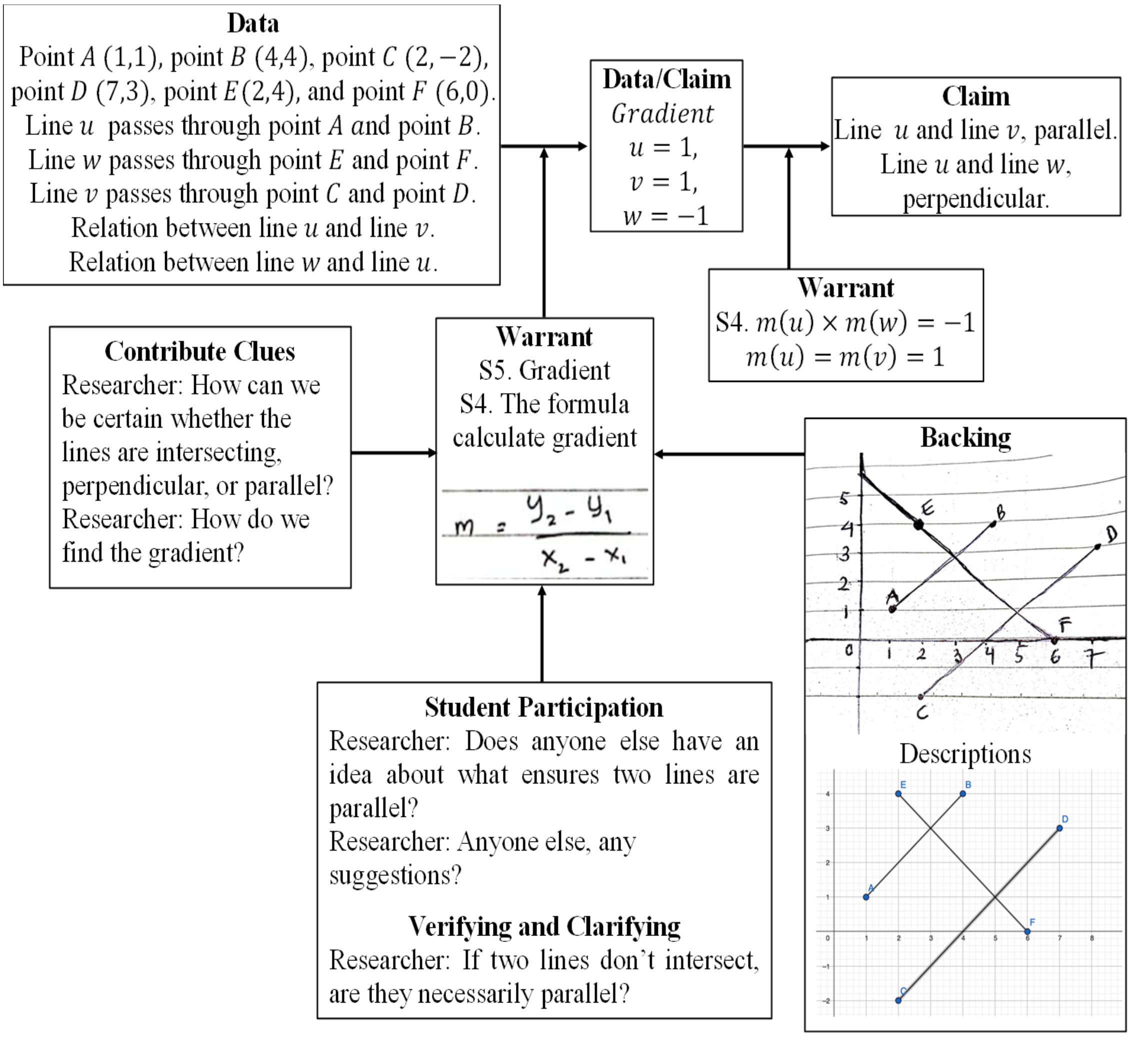


Figure 6. Collective argumentation of the heterogeneous non-mathematically rich tasks group

When completing the mathematics task, S4 acted as the facilitator of the group, supporting the other members' mathematical understanding by working with the researcher. The student with high ability proposed solutions, while other students with lower abilities contributed but relied on clarification and guidance when needed. In this group, the student with high ability played a dominant role in the discussion, clarifying concepts and leading the direction of the argumentation. Although the argumentation structure still included components such as data, claims, warrants, and support, the argumentation was often less dynamic because students with lower abilities tended to follow rather than actively engage in critical discussion. The argumentation components produced in the rich mathematics task include data, claims, warrants, support, rebuttals, and qualifications, while the non-rich mathematics task includes data, claims, warrants, and support.

When completing the non-rich mathematics task, the heterogeneous group tended to follow the procedures taught by the teacher, as proposed by S4, by determining the gradient of the line as the basis for the solution. All members accepted S4's proposal. This task brought up the argumentation components of data, claims, and warrants without significant contributions from students with lower abilities. This suggests that low-complexity tasks did not provide enough challenge to stimulate complex discussion or critical engagement from all group members.

Scaffolding in heterogeneous groups serves as a means of balancing ability, allowing lower-ability students to join discussions led by higher-ability peers. In the present study, the researcher acted as a facilitator, offering structured support through carefully designed questions to encourage contributions, clarify ideas or strategies, and verify student understanding (Conner, 2022; Conner et al., 2023; Demiray et al., 2022; Gomez Marchant et al., 2021). Questions such as "Is the distance the same for each side of this triangle?", "What kind of line is formed by Amat and Budi, Amat and Cici, or Budi and Cici? Are the lines the same?" and "How do you determine the exact point?" were used to clarify concepts and engage all students in the discussion. However, contributions from low-ability students were more passive, as they often relied on guidance from more advanced peers.

Interaction dynamics in heterogeneous groups show that although these groups increase interaction and engagement, high-ability students often dominate the conversation and contribute more. Medium and low-ability students tend to follow the direction given by more advanced peers, with relatively fewer contributions in choosing problem-solving methods. This can be seen in Figure 4 and Figure 5, which show that high-ability students provide more warrants for the group's claims.

These findings align with research by Lloyd & Murphy (2023) and Shuowen & Zhang (2024), which highlight that although heterogeneous groups provide opportunities for collaboration, the engagement of lower-ability students in in-depth mathematical reasoning is still limited. When completing the mathematically rich task, understanding how to calculate the length of horizontal, vertical, and oblique lines using unit segments remained a challenge. S6 and S5 assumed that the unit segments made for the horizontal, vertical, and oblique lines were the same length, causing them to make an incorrect claim. S4 tried to help their peers by explaining that the unit segments they made were not the same and offered an alternative solution using the Pythagorean formula. This was where the exchange of understanding occurred between students with high, medium, and low abilities.

Likewise, when determining the next claim, S4 remained more dominant in the discussion, but S5 still played a role in helping to complete the task by contributing ideas for solving the problem.

CONCLUSION

In conclusion, both groups exhibited different dynamics in completing tasks. In homogeneous groups, all members actively participated in discussions, explored multiple approaches, and gained a deeper understanding of mathematical concepts. Arguments in these groups were more dynamic, incorporating all essential elements, such as data, claims, warrants, support, rebuttals, and qualification, while interaction remained egalitarian, with minimal scaffolding to encourage independent reasoning. Conversely, in heterogeneous groups, high-ability students acted as facilitators, leading to less dynamic arguments as lower-ability students followed directions rather than actively contributing. Although scaffolding helped balance abilities, lower-ability students' involvement in solving complex problems remained limited. Homogeneous groups fostered deeper collective arguments, whereas heterogeneous groups increased interaction but struggled to ensure equal participation. To enhance discussions, teachers should implement mathematically rich tasks and encourage collaboration among students. Scaffolding should be more targeted to ensure that all members actively contribute. Further exploration of group composition based on factors such as learning styles and interests is needed to understand its impact on argumentation structures. Developing teacher competencies in effective scaffolding, including the use of technology, is also crucial. Lastly, further research should design conceptually rich mathematical tasks and examine collective argumentation across different educational levels to maximize student engagement.

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