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Spatial Thinking in Conic Sections: A Study of Undergraduate Mathematics Students by Sex and Spatial Category

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Abstract

Spatial thinking plays an important role in solving conic section problems. When students encounter problems involving shapes like parabolas, ellipses, and hyperbolas, they need not only grasp the algebraic representations but also visualize how a plane intersects a cone in three-dimensional space. Unlike previous studies that focused on psychometric tasks disconnected from classroom content, this study integrates spatial thinking directly into mathematical problem-solving, specifically in conic sections. The study examines undergraduate students' spatial thinking skills on conic sections, based on spatial thinking categories and sex differences. Twenty-five undergraduate students (4 males and 21 females) from the Mathematics Education Department at Universitas Muhammadiyah Malang, enrolled in an analytical geometry course and participated in a spatial thinking test. Additionally, four students (2 males and 2 females), representing both high and low spatial thinking abilities, were interviewed to provide deeper insights into students' spatial thinking. The results show that students with high spatial thinking abilities demonstrated strong mental visualization skills but had minor difficulties and errors in representing detailed components. Moreover, students with lower spatial thinking abilities have difficulties in visualizing complex objects and often misinterpret spatial representations. Furthermore, male students provided limited written explanations of spatial object characteristics, whereas female students faced difficulties in accurately drawing 3D spatial objects but compensated with effective 2D representations and verbal explanations. In conclusion, the findings show that the difficulties of spatial thinking are influenced by spatial thinking categories and gender differences, offering rich information to design more effective mathematics learning.

Keywords: Conic Section, Problem Solving, Sex Difference, Spatial Thinking, Spatial Thinking Category

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INTRODUCTION

Spatial ability becomes the research topic of many people since the past few decades. These abilities are needed to finished everyday task, such as reading map, navigating traffic, arranging room interior, and packing (Campos et al., 2019). Spatial ability is the ability to represent, transform, build and call symbols and non-linguistic information (Linn & Petersen, 1985), ability to receive visual image accurately, build a mental representation and imagine visual information, also ability to understand and manipulate spatial relation between objects (Nagy-Kondor, 2017). Furthermore, spatial ability is also ability to manipulate, organize, reason, interpret spatial relation in real life and imaginary (Atit et al., 2022), also define as mental ability to ease reasoning about space (Gagnier, 2020). Spatial abilities consist of some factors based on psychometric perspective, namely spatial perception, mental rotation, spatial visualization, and spatial orientation (Linn & Petersen, 1985; Lowrie et al., 2020).

Beside term 'spatial ability', also known term 'spatial thinking'. Some literature state that spatial thinking needed higher spatial ability, which is marked by analysis of specific cognitive demand of task

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given (The National Research Council, 2006). Spatial thinking involved thinking about shape and arrangements of object in space and about spatial process, such as objects deformation, and object movements or other entities through spaces. Spatial thinking also involved thinking about spatial representation of nonspatial entities (Hegarty, 2010). Spatial thinking include individual process and ability to perform specific tasks which need visualization and mental imagination of geometrical object in space, understand the spatial relation on object, also completing specific tasks or do geometrical transformation on these objects (Albarracín et al., 2021).

In this research, spatial thinking defined as an ability to build mental image, explore, manipulate, and use spatial objects to complete a particular task (Albarracín et al., 2021; Linn & Petersen, 1985; Nagy-Kondor, 2017; The National Research Council, 2006). The indicators of spatial thinking from this definition are as follows.

1) Building a mental image of a spatial object using the given information (Albarracín et al., 2021; Linn & Petersen, 1985),
2) Exploring spatial objects by identifying and defining their properties (Nagy-Kondor, 2017), define spatial relations (Atit et al., 2020; Bintoro, 2021; Nagy-Kondor, 2017), find simple spatial objects in complex spatial objects (Newcombe & Shipley, 2015), or observing spatial objects from various perspectives (with or without a distractor) (Bintoro, 2021),
3) Manipulate spatial objects, including rotation, translation, reflection, resizing (Albarracín et al., 2021), and
4) Utilizing spatial objects to respond to questions, particularly in this study, to construct new knowledge (Albarracín et al., 2021; Gummer & Mandinach, 2015).

Spatial thinking plays a crucial role in mathematical problem-solving (Buckley et al., 2019). In several previous studies, researchers measured students' spatial thinking skills in the form of solving geometrical problems (Albarracín et al., 2021; Buckley et al., 2019; İbili et al., 2019; Pavlovičová et al., 2022), especially on the topic of conic sections (Dintarini et al., 2024; Salinas & Pulido, 2017). When solving conic section problems such as parabolas, ellipses, and hyperbolas, students require not only algebraic representations but also visual representations, which involve constructing mental images of spatial objects. For example, students must visualize how a plane intersects a cone in a three-dimensional space. Spatial thinking skills enable students to comprehend the orientation and position of the plane relative to the cone, as different plane orientations produce different conic sections. For instance, to understand an ellipse, students need to visualize a plane intersecting the cone at an angle smaller than the cone's apex, resulting an ellipse.

Previous research also demonstrated differences in spatial thinking between males and females (Harris et al., 2020; Reilly et al., 2017; Wei et al., 2016). However, various studies have reported mixed results regarding these differences. Wei et al. (2016) found that males outperformed females on approximate arithmetic problems that require visuo-spatial processing. In contrast, Harris et al. (2020) conducted two studies: the first, with 43 girls and 41 boys (average age = 11.19 years), suggested that females outperformed males in spatial orientation skills. In their second study, involving 498 girls and 405 boys (average age = 13.83 years), they reported no significant differences between boys and girls in spatial thinking and mathematics ability. Furthermore, the analysis examined these differences based on key spatial skills, including spatial visualization, spatial orientation, and mental rotation. The findings revealed that girls excelled in spatial visualization, while boys performed better in spatial orientation. However, no significant differences were observed between the two groups in mental rotation abilities. However, these findings can be influenced by various factors, including genetic factor, contribution of sex-hormones, experience and social environments (Reilly et al., 2017).

A deeper investigation into the impact of sex differences on spatial thinking abilities in conic section problems could contribute valuable insights to the existing literature while informing the development of more inclusive teaching strategies. This study aims to explore undergraduate students' spatial thinking skills in conic sections, considering both spatial thinking categories and sex differences. By analyzing how students from different categories and genders approach these problems, the research seeks to enhance understanding of the role of spatial abilities and sex differences in higher education.

METHODS

This paper is part of a larger research project, with earlier findings on abstraction published separately (Dintarini, et al., 2024). While previous research emphasized abstraction, this study explores students' spatial thinking abilities in solving mathematical problems. Adopting a qualitative case study approach, it examines real-life examples to gain a deeper understanding of how students interact with spatial problems. The study provides insights into variations in spatial thinking across different categories and between genders, highlighting specific challenges in visualization and the application of abstract concepts in real-world classroom settings (Cohen et al., 2018).

Instruments

This study utilized a spatial thinking test consisting of three conic section problems. The test scores were used to assess students' spatial thinking abilities, categorizing them into three levels: high (score > 80), medium (70 ≤ score ≤ 80), and low (score < 70). These categories were adapted from the course assessment scale of Universitas Muhammadiyah Malang. The instrument was developed through a collaborative effort involving a research team consisting of a professor specializing in Mathematics Education, a Mathematics lecturer with over 20 years of teaching experience, and a doctoral student in Mathematics Education. The instrument then underwent a rigorous validation process, both logically and empirically. The analyses yielded positive results in terms of content validation, pilot testing, and reliability (Dintarini, et al., 2024; Dintarini, et al., 2024). The test is shown in Figure 1 below.

Problems

Draw double vertical right circular cone, of which the Vertices are coincident. Then,

1. Draw a plane that pass through either the top or bottom of the cone that yield a conic section resulting an ellipse, then explain your answer.
2. Draw a plane that pass through either the top or bottom of the cone that yield a conic section resulting a parabola, then explain your answer.
3. Draw a plane that pass through both of cones that yield a conic section resulting a hyperbola, then explain your answer.

Figure 1. The conic sections problems

Participants and Data Collection

This study involved a class of undergraduate students from the Mathematics Education Department at Universitas Muhammadiyah Malang, enrolled in the Analytic Geometry course. The class consisted of 25 students (4 male and 21 female). A spatial thinking test was administered with a 60-minute time limit, conducted simultaneously during class hours in a structured setting to ensure individual and orderly work. Students' responses were then evaluated and categorized into three levels, as outlined in Table 1.

Table 1. The category of student's spatial thinking

Number	Score Interval	Number of students		Category
		M	F	
1	$Score > 80$	1	6	High
2	$70 \leq score \leq 80$	2	2	Medium
3	$Score < 70$	1	13	Low
The Total of Students		4	21	

To gain a deeper understanding of students' spatial thinking abilities, a careful selection process was conducted. Two students were chosen from each of the high and low categories, allowing the study to focus on the most pronounced differences in abilities and characteristics. By selecting these two categories, the researcher could better analyze the contrast between students with strong spatial thinking skills (high) and those requiring improvement (low). This approach also helped minimize homogeneity in the medium category, which may not have provided as clear insights as the groups with more distinct ability differences.

To explore the qualitative aspects of students' spatial thinking, a structured selection process was implemented for the interview sessions. The criteria for selection included: 1) ensuring representation from both high and low spatial thinking categories, 2) selecting both male and female students from each category, and 3) choosing students whose responses closely reflected those of their peers within the same category. Based on these criteria, four students were selected: S1 (male) and S2 (female) represented the high spatial thinking category, while S3 (male) and S4 (female) exemplified the low spatial thinking category.

Data Analysis

The data analysis in this study adhered to the interactive data analysis framework outlined by Miles, Huberman, and Saldaña (2014). The interactive data analysis approach is highly suitable for this research because of its interactive, iterative nature, which allows researchers to flexibly interpret complex data, both quantitative and qualitative. This study examines the spatial thinking processes of undergraduate students in the context of conic sections, requiring a detailed analysis of both test results and interviews. The research follows three key phases: data condensation, data presentation, and conclusion formulation. During data condensation, student work was classified into three categories—high, medium, and low—leading to the careful selection of four students. These results were then thoroughly reviewed and systematically coded based on descriptors aligned with spatial thinking indicators (see Table 2).

Table 2. The indicators of spatial thinking

Number	Indicator	Descriptor	Codes
1.	Building a mental image of a spatial object based on the information provided	1. Able to draw double vertical right circular cones of which the vertices are coincident.	B_1
		2. Able to draw planes with various slopes that pass through either the top or bottom of the cone.	B_2
		3. Able to draw the result of intersection between plane and cone, that formed parabola, ellipse, and hyperbola.	B_3
2.	Exploring spatial objects	1. Describe the characteristics of the plane that cuts the cone to form a parabola.	E_1
		2. Describe the characteristics of the plane that cuts the cone to form an ellipse.	E_2
		3. Describe the characteristics of the plane that cuts the cone to form a hyperbola.	E_3
3.	Manipulate spatial objects (rotation, translation, reflection, etc.)	1. Able to draw a rotated/reflected cone.	M_1
		2. Able to draw a rotated/reflected plane.	M_2
		3. Able to do the translation to the cone or the plane.	M_3
4.	Utilizing spatial objects to construct new knowledge	1. Using cones and planes with various slopes to discover the concept of a parabola.	U_1
		2. Using cones and planes of varying slopes to discover the concept of an ellipse.	U_2
		3. Using cones and planes with various slopes to discover the concept of a hyperbola.	U_3

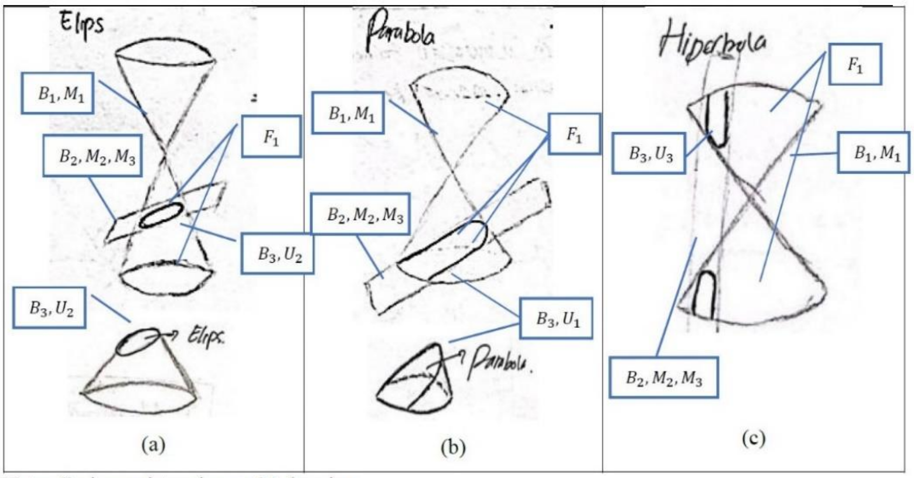
Following the review of test results, any areas requiring further clarification were explored through follow-up interviews. The interview data were then coded, and the combined findings were synthesized into the research conclusions. The conclusions of this study are grounded in the outcomes of data presentation and analysis, integrating key findings and insights gathered throughout the research process. To ensure the validity of the findings, method triangulation was employed, combining test and interview methods to validate, verify, and confirm the results while gaining a deeper understanding of students' spatial thinking processes (Flick, 2018).

RESULTS AND DISCUSSION

Building on the aim of study outlined in the introduction, this section presents the distinct narratives of four students' work, carefully selected to represent different levels of spatial thinking. These narratives provide valuable insights into each student's cognitive processes, offering diverse perspectives on spatial thinking.

The Spatial Thinking of High-Level Male Student (S1)

In the context of spatial thinking, S1 was a male student with a high-category spatial thinking score. This subsection explores the underlying factors contributing to S1's advanced spatial thinking abilities, offering insight into the complexities of high-level spatial thinking in males. S1's test results are presented below (see Figure 2).



Note: F_1: inconsistencies on S1 drawing

Figure 2. The S1' image of cone cut by a plane yielding: (a) an ellipse, (b) parabola and (c) hyperbola

Figure 2 illustrates that S1 successfully met the first indicator of spatial thinking by building a mental image of a double cone intersected by a plane, resulting in an ellipse (Figure 2(a)), a parabola (Figure 2(b)), and a hyperbola (Figure 2(c)). Since direct measurement of mental image is not feasible, researchers evaluated students based on their ability to draw and explain the corresponding spatial objects. S1 notably sketched two vertical right-angled circular cones, accurately depicting the double cone images mentioned in the problem (B_1), draw a plane at a certain inclinations and positions (B_2). Moreover, S1 drew an ellipse resulting from plane intersections (See Figure 2 (a), B_3, U_2), drew a parabola resulting from plane intersections (See Figure 2 (b), B_3, U_1) and drew hyperbola resulting from plane intersections (See Figure 2 (c), B_3, U_1). Additionally, S1 included a drawing of a truncated cone to further illustrate how the intersection between the cone and the plane forms an ellipse and a parabola. However, since S1 did not depict a truncated cone for the hyperbola, this aspect required further clarification during the interview.

Referring again to Figure 2, S1's illustration of two vertically aligned right circular cones with coincident vertices showcases their ability to represent a rotated or reflected cone (M_1). To create this drawing, S1 first sketched a vertical right circular cone. Then, by visualizing a second cone with a shared vertex, S1 completed the illustration. This process required spatial manipulation skills, including rotating or reflecting the cone through its apex. When drawing the plane, S1 likely applied spatial transformations, such as rotations or translations, to accurately position it for intersecting the cone and forming an ellipse, a parabola, and a hyperbola (M_2, M_3). S1 demonstrated strong spatial manipulation skills, effortlessly rotating and reflecting cones to represent a double cone and accurately illustrating planes intersecting cones to form geometric shapes.

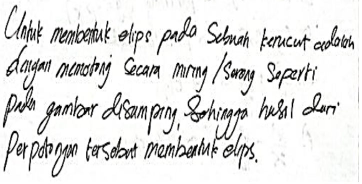
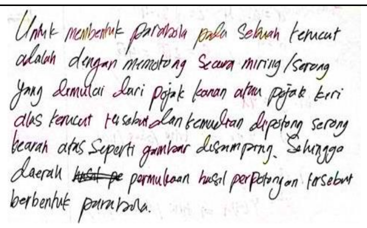
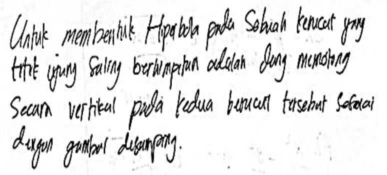
 Untuk membentuk elips pada Sebuah kerucut adalah dengan memotong Secara miring/Serong Seperti pada gambar disamping, Sehingga hasil dari Perpotongan tersebut membentuk elips. (a)	 Untuk membentuk parabola pada Sebuah kerucut adalah dengan memotong Secara miring/serong yang dimulai dari pojok kanan atau pojok kiri alas kerucut tersebut dan kemudian dipotong serong kearah atas Seperti gambar disamping. Sehingga daerah ~~hasil pe~~ permukaan hasil perpotongan tersebut berbentuk parabola. (b)	 Untuk membentuk Hiperbola pada Sebuah kerucut yang titik ujung Saling berhimpitan adalah deng memotong Secara vertikal pada kedua kerucut tersebut Sebagai dengan gambar disamping. (c)
English version: To form an ellipse on a cone, the plane must intersect it obliquely, as illustrated in the image. This intersection produces an elliptical shape (E_1).	**English version:** To form a parabola on a cone, the plane must cut obliquely at an angle, starting from either the right or left corner of the base, as shown in the image. This intersection results in a parabolic shape (E_2).	**English version:** To form a hyperbola on a double cone, the plane must cut through it vertically, as illustrated in the image. This intersection produces a hyperbolic shape (E_3).

Figure 3. S1' explanation of the image of a cone cut by a plane yielding: (a) an ellipse, (b) a parabola, and (c) hyperbola

S1 stated that an ellipse is formed when a plane intersects the cone at an oblique angle (see Figure 3(a)). However, while the drawing was accurate, S1 did not provide a detailed explanation or explicitly describe the plane's characteristics in their test response. For a parabola, S1 explained that the plane cuts obliquely starting from the cone's base (see Figure 3(b)). Although S1 attempted to clarify by mentioning the starting position of the cut, the explanation remained somewhat unclear. Interestingly, S1's approach differed from the conventional method. Instead of visualizing a plane cutting from the lateral surface to the cone's base, S1 imagined it starting from the bottom corner and moving upward through the lateral surface. Meanwhile, for obtaining a hyperbola, S1 demonstrated a clear understanding by accurately explaining that it is formed by vertically slicing a double cone (see Figure 3 (c), E_3). Although S1's verbal explanation lacks clarity, the researcher believes that this is due to difficulty in articulating ideas rather than a lack of understanding. S1's explanation, supported by the figure he drew, provides sufficient evidence of his mastery of spatial thinking. However, to address certain inconsistencies—particularly in the use of dotted and thick lines—the researcher conducted a follow-up interview for clarification. The following is an excerpt from the interview.

R	:	*"Describe the characteristics of the plane that cut the cone so that it yields an ellipse, and parabola section."*	(1)
S1	:	*"To yields an ellipse, the plane needs oblique and all the side of plane need cut the lateral surface of cone (E_2). To yields a parabola, the intersecting plane must be inclined until it intersects the base of the cone (E_1)."*	(2)

The excerpt above demonstrates that S1 has a strong understanding of how the plane's position and orientation influence the formation of ellipses and parabolas when intersecting with a cone. Furthermore, based on Figure 3 and Row 2 of the interview excerpt, it is evident that S1 meets the second indicator of spatial thinking, which involves exploring spatial objects, by describing the characteristics of plane, E_1, E_2, E_3).

S1's proficiency in activities related to the first three indicators enabled them to effectively utilize spatial objects to construct new knowledge, fulfilling the fourth indicator. Through this process, S1 identified the concepts of parabolas, ellipses, and hyperbolas by examining the interaction between cones and planes (U_1, U_2 dan U_3). However, in Figure 2, there is still an inconsistency in the image made by S1, namely how S1 uses solid lines and dotted lines (F_1). Therefore, the researcher reconfirmed this in the interview session. Because of these inconsistencies, researcher interviewed S1 to confirm his understanding about this matter. The following are excerpts of S1's interview.

R	:	*"Look at the picture you made, in first figure you use all thick lines (see Figure 2 (a)), in the second figure you use some of them with dotted lines (see Figure 2 (b)), different again from third figure which is all thick, but you omit some parts of the side of the base of the cone (see Figure 2 (c)). What is the meaning of your drawing?"*	(3)
S1	:	*"The dashed lines indicate the invisible sides of the cone and the planes. When creating the first drawing, I did not initially consider using a dotted line. In the second drawing, I attempted to provide a more complete depiction. However, in the third drawing, I intended to add the dotted lines at the end but ultimately forgot to include them."*	(4)

The interview excerpt indicates that S1 understands the use of dotted and thick lines to illustrate spatial objects (see Row 4). However, this understanding was not fully reflected in the spatial thinking test. While S1 demonstrated a conceptual grasp of how a plane intersects a cone to form a hyperbola and successfully built a mental image of it, accurately representing this understanding through drawing remained a challenge. This suggests that S1's verbal representation is well-developed, but further improvement is needed in visual representation skills.

The Spatial Thinking of High-Level Female Student (S2)

S2, a female student, scored in the high category for spatial thinking. This subsection explores the potential factors that contribute to the development of spatial thinking in high-achieving female students, using S2 as a case study. Refer to Figure 4 for S2's test results.

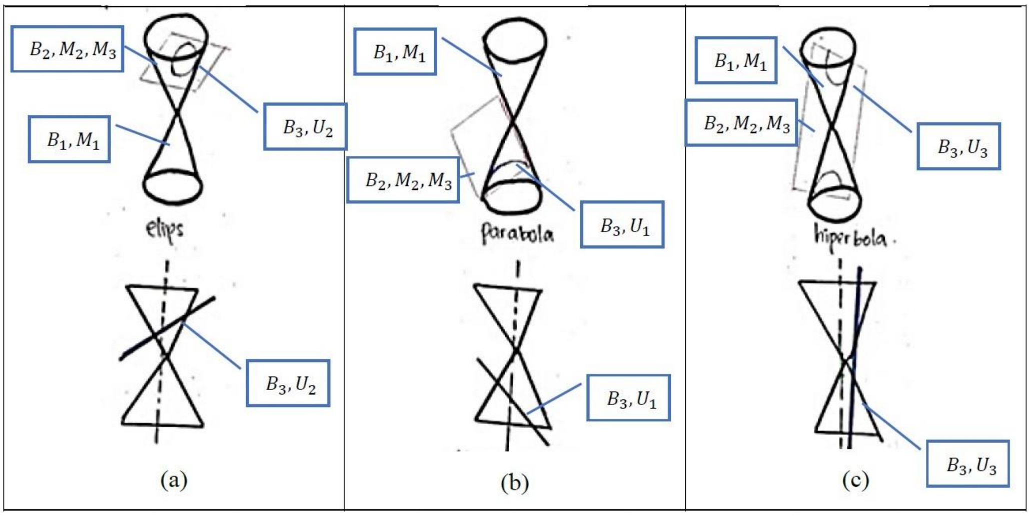
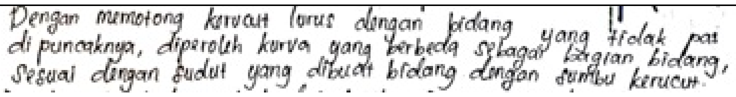


Figure 4. S2' image of a cone cut by a plane yielding (a) an ellipse, (b) parabola, and (c) hyperbola

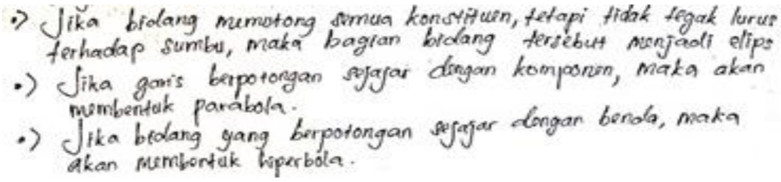
Based on the 3D drawing produced by S2 in Figure 4, it is evident that while S2 was able to accurately draw a double cone (B_1) and draw planes in various positions (B_2). S2 experienced significant difficulty in illustrating the intersection of a plane with the cone in three dimensions. Similar challenges were observed when attempting to draw the resulting ellipse, parabola, and hyperbola from the plane-cone intersections in 3D (B_3). To overcome this challenge, S2 included a two-dimensional drawing to better illustrate the relationship between the plane and the cone, reinforcing the researcher in S2's ability to build the necessary mental image of spatial objects. Despite difficulties in 3D representation, S2 effectively demonstrated proficiency in the first spatial thinking indicator by depicting a double cone, a plane, and their intersections in a 2D format, successfully representing a parabola, ellipse, and hyperbola.

Regarding the third indicator, S2 effectively manipulated spatial objects by accurately illustrating planes in different orientations and positions (M_2, M_3). This skill demonstrated S2's capability to perform geometric transformations, including rotation, translation, scaling, and reflection. Notably, S2 depicted a mirrored image by using the cone's vertex as the center point of reflection (M_1). Moreover, this ability was further demonstrated through the inclusion of a 2D drawing, which helped clarify S2's understanding of how the plane intersects the cone. The additional illustration reinforced the evidence of S2's spatial manipulation skills, confirming a strong understanding of the geometric transformations involved. Next, the focus shifts to S2's explanation to further evaluate their ability to explore spatial objects.


Dengan memotong kerucut lurus dengan bidang yang tidak pas di puncaknya, diperoleh kurva yang berbeda sebagai bagian bidang, sesuai dengan sudut yang dibuat bidang dengan sumbu kerucut.

English version:

When a cone is intersected by a plane that does not pass through its vertex, the resulting curve on the plane depends on the angle between the plane and the axis of the cone.


•) Jika bidang memotong semua konstituen, tetapi tidak tegak lurus terhadap sumbu, maka bagian bidang tersebut menjadi elips

•) Jika garis berpotongan sejajar dengan komponen, maka akan membentuk parabola.

•) Jika bidang yang berpotongan sejajar dengan benda, maka akan membentuk hiperbola.

English version:

a. If the plane intersects all the components but is not perpendicular to the axis, it will form an ellipse (E_1).
b. If the intersecting lines are parallel to the components, a parabola will be formed (E_2).
c. If the intersecting planes are parallel to the object, a hyperbola will be formed (E_3).

Figure 5. S2' explanation of the image of a cone cut by a plane yielding (a) an ellipse, (b) a parabola, and (c) hyperbola

S2 attempted to describe the characteristics of planes intersecting cones to form conic sections, however, the explanation lacked clarity. Initially, S2 wrote, "By cutting a cone straight to a plane that does not fit at its vertex, … (See Figure 5a)." This statement outlines the first condition for a plane to intersect a cone, resulting in an elliptical, parabolic, or hyperbolic section. This interpretation appears reasonable since, as shown in S2's drawing in Figure 4, no plane passes through the cone's vertex. Next, S2 stated, "If the plane cuts all of its constituents but is not perpendicular to the axis, an elliptical section will be formed (see Figure 5b)." However, S2 used ambiguous terms such as "constituents" and "axis" without clearly defining the geometric objects they refer to, leading to potential confusion. Additionally, the statement, "If the intersecting lines are parallel to the components, it will form a parabola," did not correspond with the accompanying image, and the term "components" lacked clarity. Similarly, S2 wrote, "If the intersecting planes are parallel to the object, it will form a hyperbola (see Figure 5c)," but the reference to "the object" was unclear. Given the frequent use of vague terminology in S2's explanation, an interview was conducted to clarify the intended meanings. The following is an excerpt from the interview with S2.

R	:	*"On your test sheet, you wrote "if the plane cuts all of its constituents, but is not perpendicular to the axis, then an elliptical section will be formed.". What you mean by this statement, please explain".*	(5)
S2	:	*"I mean to form an ellipse, the cone is intersected by a plane in the middle, obliquely. "Constituents," mean all the lateral surfaces of the cone (E_1)."*	(6)
R	:	*"Could you imagine the section result, if the plane were not oblique?"*	(7)
S2	:	*"To yields an ellipse shape, the plane needs to be oblique. If the plane is perpendicular to the altitude, the result was a circle."*	(8)
R	:	*"What did you mean by the "axis" in that statement?"*	(9)
S2	:	*"The cone altitude."*	(10)
R	:	*"Then you wrote, "if the intersecting lines are parallel to the components, it will*	(11)

		form a parabola." What do you mean by this statement?"	
S2	:	*"The plane cuts the cone at the surface and the base of cone, and it needs to be parallel to the slant of the cone (E_2)."*	(12)
R	:	*"And for the hyperbola? You wrote in your paper, "if the intersecting planes are parallel to the object, it will form a hyperbola." What did you mean by "the object"?."*	(13)
S2	:	*"The plane cuts the double cone parallel to the cone."*	(14)
R	:	*"Looking at your drawings, it seems the planes are in different positions."*	(15)
S2	:	*"Oh yes, ma'am. The plane is oblique, but it cuts both cones—the top and the bottom—in my first drawing. In my second drawing, the plane is parallel to the altitude of the cone (E_3)."*	(16)
R	:	*"So, which of the two drawings shows the formation of a hyperbola?"*	(17)
S2	:	*"I think both."*	(18)

Based on the interview excerpt above, S2 was able to exploring the spatial object by describe the characteristics of the plane intersecting the parabola (E_1), ellipse (E_2), and hyperbola (E_3). However, S2 struggled to clearly express these ideas verbally, leading to the use of unfamiliar terms to explain the exploration results. For instance, S2 used the phrase "intersecting all constituents" to refer to the plane cutting through the lateral surface of the cone (Row 6). Similarly, the expression "not perpendicular to the axis" was meant to indicate that the plane was not perpendicular to the cone's altitude (Row 13, E_1). For the parabola, S2 explained that the plane must intersect the lateral surface and extend to the cone's base. S2 referred to the plane as a "component," suggesting that it should be parallel to the cone's slant height—one possible condition for forming a parabolic section (Row 12). Additionally, S2 seemed to assume that if the plane's inclination exceeded the slant height, it could also intersect the upper cone (Row 12, E_2). However, S2 was unable to provide a more in-depth analysis of this aspect. In the case of the hyperbola, S2 presented two interpretations: one where an inclined plane intersects both the upper and lower cones, and another where the plane is parallel to either the cone's lateral surface or its vertical axis (Row 14-18, E_3).

Lastly, in accordance with the fourth indicator, Figures 4 and Figure 5, along with the interview excerpts, demonstrate that S2 effectively utilized spatial objects, such as cones and planes, to solve the problems presented by the researcher. Through this process, S2 developed a conceptual understanding of ellipses, parabolas, and hyperbolas as conic sections, thereby constructing new knowledge within the study's context (U_1, U_2, U_3).

The Spatial Thinking of Low-Level Male Student (S3)

S3, a male student, received a spatial thinking score in the low category. This subsection explores the factors influencing the development of spatial thinking in low-level male students, using S3 as a case study. Refer to Figure 6 for S3's test results.

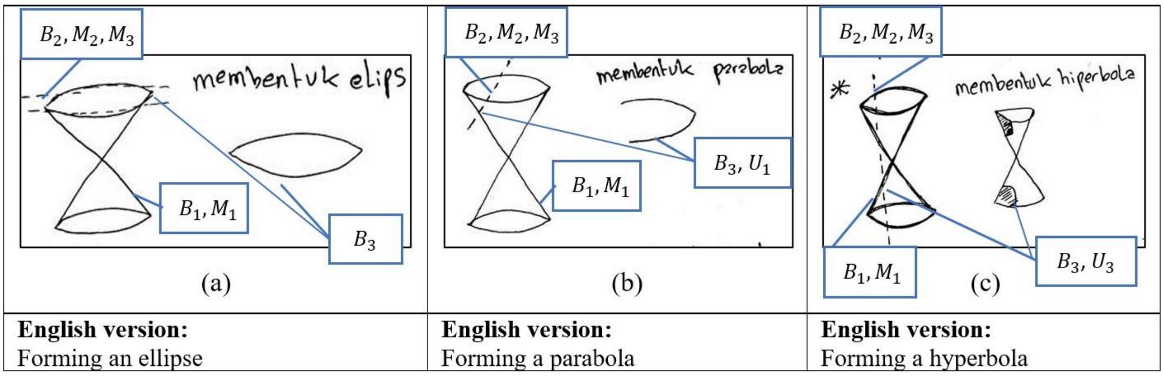


Figure 6. S3' image of a cone cut by a plane yielding (a) an ellipse, (b) parabola, and (c) hyperbola

Based on Figure 6, S3 was able to draw a double vertical right circular cone with coincident vertices (B_1). S3 attempted to draw planes with different slopes intersecting either the top or bottom of the cone. However, instead of explicitly depicting the plane, S3 used dotted lines to indicate where the plane cuts through the cone. Although the plane itself was represented as a single line, its orientation correctly aligns with the formation of an ellipse, parabola, and hyperbola. This suggests that S3 is capable of building a mental image of the plane and its intersection with the cone (B_2). Figure 6 illustrates that S3 recognizes the shapes of an ellipse, parabola, and hyperbola, as shown by the drawings of these curves outside the cone and plane. However, S3 appears to have a misconception about the ellipse. While the slicing direction seems appropriate for forming an ellipse, a closer examination of Figure 6 (a) reveals that S3 made a horizontal cut, which would produce a circle rather than an ellipse. Despite this misunderstanding, S3 still demonstrated the ability to construct a mental image of conic sections, including an ellipse, parabola, and hyperbola, based on the plane intersecting the cone (B_3). However, S3 faced challenges in accurately translating these mental images into drawings. Nevertheless, S3 demonstrated the ability to manipulate spatial objects, successfully sketching a double cone, which indicates an understanding of reflection using the apex as a mirror (M_1). Furthermore, by depicting the plane—albeit as a line—S3 demonstrates the ability to mentally rotate and translate it (M_2, M_3).

Although the test required an explanation of the solution steps, S3 did not provide a verbal response. Based on S3's test results, further investigation was needed to assess S3's understanding of planes and the conic sections formed. To gain clarity on this, the researchers conducted an interview, an excerpt of which is presented below.

R	:	*"What is the next part of the question?"*	(19)
S3	:	*"Draw a plane with a certain inclination that intersects the cone."*	(20)
R	:	*"How do you do that?"*	(21)
S3	:	*"The shape of a plane can be a square, rectangle, or another shape (B_3). Here, I had difficulty drawing the plane that intersects the cone, so I chose to draw a line instead of the square."*	(22)
R	:	*"Explain how the plane cut the double cone to yields an ellipse, parabola and hyperbola?"*	(23)

S3 : *"I cut the top cone horizontally to yields an ellipse shape (E_1). Then I cut the cone obliquely to yields a parabola shape (E_2), and cut the cone vertically to yields a hyperbola shape (E_3)."* (24)

Based on the interview excerpts above, S3 demonstrated a sufficient understanding of the concept of a plane by referencing various shapes commonly used to represent it in illustrations (Row 22, B_3). However, S3 mentioned having difficulty drawing the cone when it was cut, so the plane was represented using a simple line. Additionally, the researcher validated the second indicator of spatial thinking—exploring spatial objects—by analyzing S3's explanations of how the cone was cut to form an ellipse, a parabola, and a hyperbola.

During the interview, S3 described the process of obtaining different conic sections by cutting a cone. To form an ellipse, S3 stated that the cone was cut horizontally, as shown in Figure 6(a). For a parabola, S3 mentioned making an oblique cut but did not specify the characteristics of the cutting plane. Lastly, to create a hyperbola, S3 indicated that the cone was cut vertically (Row 24, $\mathrm{E_1, E_2, E_3}$). This serves as strong evidence that S3 demonstrates the ability to explore spatial objects, fulfilling the second indicator of spatial thinking. To further investigate S3's understanding of how a cone is cut to form an ellipse, the researcher conducted the following interview.

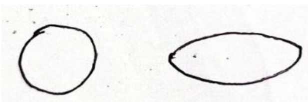
Figure 6(a) and the interview excerpt from Row 24 indicate that S3 holds a misconception about how a plane intersects a cone to form an ellipse. To explore the origin of this misunderstanding, the researcher conducted the following interview.

R : *If an ice cream cone is cut horizontally, what shape does it form?* (25)

S3 : *An ellipse or a circle.* (26)

R : *They are different. What is the difference between a circle and an ellipse?* (27)

S3 : *A circle looks like this, while an ellipse looks like this, Ma'am (shows a drawing).* (28)



Circle **Ellipse**

R : *Alright, let's go with that. So, if I cut the ice cream cone horizontally, what shape does it form?* (29)

S3 : *An ellipse.* (30)

R : *When you cut a carrot, the shape can vary depending on the cut—whether it's slanted or horizontal. If I cut a carrot horizontally, what shape is formed?* (31)

S3 : *If it's slanted, it's like a circle but elongated. An ellipse.* (32)

R : *And if you cut it horizontally, what shape is it?* (33)

S3 : *A circle.* (34)

R : *Correct. Now, returning to the cone problem, if the plane cuts the cone horizontally, what shape is formed?* (35)

S3 : *An ellipse, Ma'am.* (36)

R : *Why?* (37)

S3 : *Because when viewed from the front, it looks like an ellipse.* (38)

R : *But if you look from the top, what shape do you see?* (39)

S3	*:*	*A circle.*	(40)
R	*:*	*Exactly. So, what kind of cut produces an ellipse?*	(41)
S3	*:*	*An oblique cut.*	(42)

In the interview excerpt, the researcher used real-world cone-shaped objects, such as ice cream cones (Row 25) and carrots (Row 31), and asked S3 visualize the cutting process. However, when returning to the original problem, S3 mistakenly stated that a vertical cut would produce an ellipse (Row 35-36). S3 justified this by explaining that the result was viewed from the front (Row 38). When prompted to consider the top view, S3 then recognized that a horizontal cut would actually form a circle rather than an ellipse (Row 42). Test and interview results indicate that S3 struggles significantly with representing planes, especially when conceptualizing the formation of an ellipse. Therefore, it can be said that S3 is only able to meet two descriptors of the spatial thinking indicator (U_2, U_3).

The Spatial Thinking of Low-Level Female Student (S4)

S4, a female student, scored in the low category for spatial thinking. This section explores the factors influencing the development of spatial thinking in female students with lower proficiency, using S4 as a case study. Refer to Figure 7 for S4's test results.

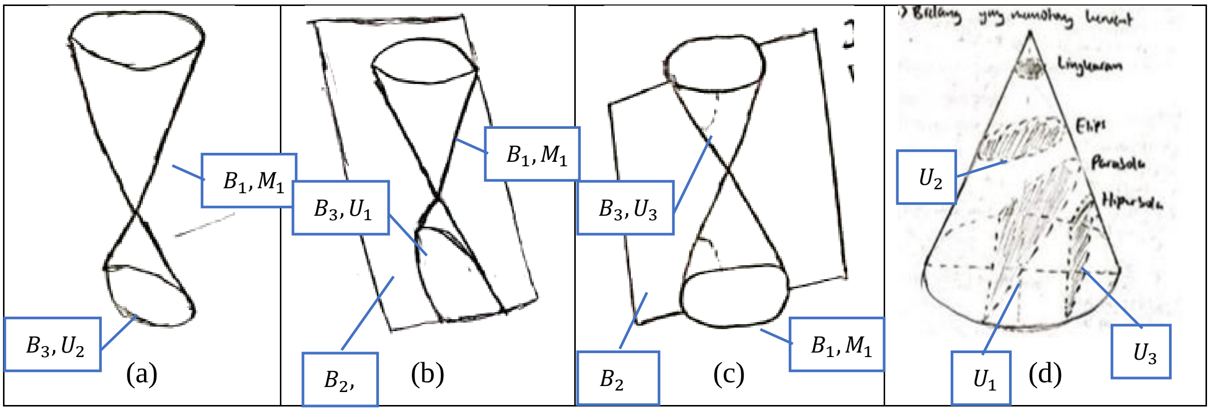


Figure 7. S4' image of a cone cut by a plane yielding (a) an ellipse, (b) parabola, (c) hyperbola, and (d) additional image

The test results indicate that S4 created three distinct drawings to represent the intersection of a plane with a double cone, producing an ellipse, a parabola, and a hyperbola (see Figures 7(a), (b), and (c)). To further clarify these representations, S4 added additional sketches. In Figure 7(a), S4 depicted a double cone with the lower cone truncated to form an ellipse but omitted the intersecting plane. In Figure 7(b), S4 included an intersecting plane cutting through a double cone, resulting in a parabola; however, the portion of the cone behind the plane was not illustrated. In Figure 7(c), S4 drew a complete double cone intersected by a vertical plane, adding a small hyperbola on one side of the plane using a broken line. These figures suggest that S4 encountered difficulties in accurately depicting complex spatial objects, particularly planes intersecting cones. To compensate, S4 created an additional drawing

(Figure 7(d)), illustrating a single cone with representations of a circle, ellipse, parabola, and hyperbola, but without explicitly including the intersecting plane.

The following presents the results of the data analysis on S4's spatial thinking. Figure 7, especially Figure 7(c), illustrates S4’s ability to accurately depict a pair of vertically aligned right circular cones with coincident vertices (B_1). Figures 7(b) and Figure 7(c) further indicate S4's understanding of the concept of a plane and its representation. However, the stiffness in the drawings and the absence of the cutting plane in Figure 7(a) suggest that S4 faces challenges in accurately depicting a plane intersecting the cone. (B_2). Despite these challenges, S4 still exhibits the ability to construct a mental image of the conic sections, as reflected in the drawings of the resulting shapes: ellipse, parabola, and hyperbola (B_3). This confirms that S4 successfully meets the first spatial thinking indicator, which involves constructing mental images of spatial objects based on the given information.

Concerning the third indicator, which involves manipulating spatial objects, S4 demonstrated partial proficiency in meeting the criteria. While successfully drawing opposing cones showing his ability to rotating or reflecting the cone (see Figure 7(c), M_1), S4 was not proficient in performing the rotation or reflection of the plane, as shown in Figure 7(b), Figure 7(c). The planes that generate a hyperbola and a parabola have nearly the same inclination, suggesting that S4 faced difficulties in accurately performing the rotation or reflection of the plane. Alongside the spatial object drawings, S4 also explained the steps taken to create them.

→ Elips : Posisi pisau untuk memotong yaitu miring tidak sampai ujung maka berbentuk elips.	**English version:** Ellipse: When the knife is positioned at an oblique angle without reaching the corner, it forms an ellipse (E_2).
→ Parabola : Posisi pisau untuk memotong yaitu miring sampai ke ujung atau alas kerucut akan membentuk parabola.	**English version:** Parabola: When the knife is positioned at an oblique angle and cuts through the cone until it reaches the base or a corner, it forms a parabola (E_1).
Jika dua ujungnya berimpit, dan posisinya vertikal atau tegak lurus, maka bidang potongnya berbentuk hiperbola.	**English version:** If both endpoints coincide and the plane is either vertical or perpendicular, the intersection results in a hyperbola (E_3).

Figure 8. S4’ explanation of the image of a cone cut by a plane yielding (a) an ellipse, (b) a parabola, and (c) hyperbola

Although S4's drawing during the test was inadequate, S4 effectively explained how the characteristics of a given plane determine the resulting cross-sectional shapes (see Figure 8, E_1, E_2, E_3). In the explanation, S4 stated that to produce an ellipse, the plane (or knife) must cut obliquely but not reach the corner of the cone (E_2). This differs from a scenario where the cut reaches the corner or base of the cone, which would result in a parabola (E_1). Meanwhile, to obtain a hyperbola, the plane must cut both cones vertically (E_3). Notably, S4 referred to the term "knife" instead of "plane," suggesting a potential misunderstanding of the correct terminology. Nevertheless, S4's explanation provides strong evidence of fulfilling the second spatial thinking indicator—exploring spatial objects. Additionally, it highlights S4's ability to utilize spatial objects to develop new knowledge about conic sections (U_1, U_2, U_3).

Although S4's explanation effectively described the process of drawing a plane intersecting a double cone to form an ellipse, parabola, and hyperbola, the researcher conducted an additional interview to validate the analysis and identify any challenges S4 encountered.

R	:	*Could you explain the answer you drew?*	(43)
S4	:	*The question instructed to create a vertical cone and a horizontal plane with a certain slope that intersects it. So, as I understand it, when the surface of the cone is cut at a specific angle, it forms an elliptical slice.*	(44)
R	:	*How do you cut it?*	(45)
S4	:	*The cone is cut obliquely, cutting through the lateral surface of the cone.*	(46)
R	:	*Next, how do you obtain a parabola?*	(47)
S4	:	*The direction of the plane's intersection cuts through the base and the lateral surface, whereas for an ellipse, it only cuts the lateral surface.*	(48)
R	:	*How does the plane cut the cone to produce a hyperbola?*	(49)
S4	:	*For a hyperbola, the plane must cut through two cones whose vertices are coincident. It cuts vertically, along the height of the cone or perpendicular to the base.*	(50)
R	:	*Did you encounter any difficulties in solving this problem?*	(51)
S4	:	*Yes, there was, ma'am. I was confused about drawing it completely because some parts are visible, and others are not (those behind the plane).*	(52)

Based on the interview, S4 exhibited a clear understanding of the problem but struggled to accurately represent complex spatial objects. This difficulty stemmed from the challenge of distinguishing between visible and hidden parts of the object, particularly the section concealed by the plane.

After analyzing the data from the four subjects, notable differences were identified in their spatial thinking processes across both high and low categories. The following table provides a concise summary of the spatial thinking characteristics of each subject.

Table 3. A Brief Summary about Student's Spatial Thinking

Indicator	High		Low	
	Male	Female	Male	Female
Building a mental image of a spatial object based on the	Capable of visualizing a double cone, its intersecting plane, and the resulting conic sections (ellipse, parabola,	Capable of forming a mental image of a double cone and its intersecting plane, as demonstrated through a 3D drawing. Also able	Capable of constructing a mental image of a double cone, as evidenced by the drawing. Able to describe the mental	Capable of clearly visualizing a double cone and accurately representing the plane

Indicator	High		Low	
	Male	Female	Male	Female
information provided	and hyperbola), as demonstrated through clear drawings and written explanations. However, there are minor challenges and inaccuracies in using dotted lines to represent hidden parts of the figures.	to represent conic sections (ellipse, parabola, and hyperbola) using both 3D and 2D illustrations. Experiences challenges in depicting complex spatial objects in 3D but compensates by effectively explaining them through 2D drawings.	visualization of a plane intersecting the cone to form conic sections (parabola and hyperbola) through explanations in the interview session. The plane was represented in 3D using lines, but there were challenges in depicting complex 3D objects, and no written explanation was provided.	intersecting the cone to form conic sections (ellipse, parabola, and hyperbola) through written explanations and interview responses.
Exploring spatial objects	Demonstrates the ability to explore spatial objects by describing the characteristics of a plane intersecting a cone to form conic sections (ellipse, parabola, and hyperbola), with written explanations further clarified through the interview session.	Demonstrates the ability to explore spatial objects by describing the characteristics of a plane intersecting a cone to form conic sections (ellipse, parabola, and hyperbola) during the interview session. However, the written explanation lacks clarity and includes unfamiliar mathematical terms.	Demonstrates the ability to explore spatial objects by describing the characteristics of a plane intersecting a cone to form parabolas and hyperbolas, based solely on interview responses. However, struggles to accurately describe the plane forming an ellipse, exhibiting a misconception about the ellipse as a conic section.	Demonstrates the ability to explore spatial objects by describing the characteristics of a plane intersecting a cone to form conic sections (ellipse, parabola, and hyperbola) through written explanations and interviews.
Manipulate spatial objects (rotation, translation, reflection, etc)	Demonstrates the ability to manipulate spatial objects by drawing rotated, reflected, or translated cones and planes, providing written explanations further clarified	Demonstrates the ability to manipulate spatial objects by drawing rotated, reflected, or translated cones and planes, with explanations given during the interview.	Demonstrates the ability to manipulate spatial objects by drawing rotated, reflected, or translated cones, as evidenced by written explanations and interviews. The manipulation of the	Demonstrates the ability to manipulate spatial objects by drawing rotated, reflected, or translated cones. Successfully manipulates the

Indicator	High		Low	
	Male	Female	Male	Female
	through interviews.		plane is reflected in the direction of the drawn lines and further supported by interview responses. However, struggles to accurately depict the plane in a 3D illustration.	plane, as evidenced by written explanations and interviews, but faces challenges in flexibly depicting the plane in drawings.
Utilizing spatial objects to construct new knowledge	Utilizes cones and planes with varying slopes to construct the concepts of parabolas, ellipses, and hyperbolas, demonstrated through drawings and written explanations, further clarified through interviews.	Utilizes cones and planes with varying slopes to construct the concepts of parabolas, ellipses, and hyperbolas through drawings and interviews.	Utilizes cones and planes with different slopes to construct the concepts of parabolas and hyperbolas through drawings and interviews but were unable to identify the concept of an ellipse.	Utilizes cones and planes with varying slopes to construct the concepts of parabolas and hyperbolas, through drawings, written explanations, and interviews.

In this study, students in the high category of spatial thinking demonstrated all four spatial thinking indicators, which is supported by their drawings, explanations, and additional interviews. These students demonstrated a strong ability to externalize their mental representations into accurate visual depictions. This aligns with a prior study involving 56 participants (28 males and 28 females, average age 21) who completed both paper-based and computer-based spatial ability tests. The researchers used five cognitive factor tests to assess participants' spatial abilities and a computer-based visualization task to evaluate their mental imagery skills. The visualization test required participants to analyze three orthogonal projections (back, bottom, and right) of both geometric and realistic objects and select the correct object from four possible answers, each shown with slight rotations. Participants were required to form a mental image of the object after viewing the projections and were not allowed to revisit them. The test aimed to assess spatial comprehension and mental rotation without the need for advanced training. Findings revealed a small yet significant correlation between visual memory and performance. One analysis categorized participants into two groups—high spatial ability (HS) and low spatial ability (LS)—and found that individuals in the HS group demonstrated greater accuracy in solving 3D geometry problems (Velez et al., 2005). Notably, high-category students excelled in building complex mental images, such as the detailed mental image of double cone that cut by a plane yielding an ellipse.

Still consistent with the findings from Velez et al. (2005) that participants with high spatial ability are better at constructing mental representations of complex objects. This supports the idea that spatial thinking extends beyond visualizing basic geometric shapes to include the ability to mentally manipulate and comprehend more complex and detailed structures. Additionally, previous research suggests that students with strong spatial thinking skills are more proficient in articulating their problem-solving processes in mathematics (Forndran et al., 2019). Then this in line to other study that stated students of high conducted the symbolic representation process at the comprehension level of visual representation level (Utomo & Syarifah, 2021). However, high spatial thinkers exhibited minor challenges, such as inconsistencies in using thick or broken lines and difficulties with more complex 3D drawings. Despite these issues, their overall ability to visualize and articulate spatial concepts remained well-developed. One of the causes that emerged in this study was the inaccuracy or negligence of students when drawing. This was proven by students being able to explain their mistakes in the interview session.

On the other hand, students with lower performance showed near-completion of most spatial thinking indicators, as indicated by their drawings and the interviews. They struggled with more intricate mental building, such as envisioning the details of a cone intersected by a plane at various angles. However, the main issue they faced was the difficulty in externalizing their mental imagery into a precise visual representation. This challenge was evident in the discrepancy between their internal mental picture (showed by explanation and interview) and the final visual representation (showed by their drawing image). Additionally, misconceptions emerged, such as misinterpreting a circle as an ellipse resulting from a plane intersection. This was primarily due to their limited flexibility in shifting perspectives while visualizing spatial objects. Their difficulty in altering viewpoints hindered their ability to accurately conceptualize and represent complex 3D structures. This is in line with study by Unal et al. (2009), that study the differences between high, middle and low spatial thinking of preservice teacher. This study found that individuals with low spatial thinking abilities struggled to create drawings that could aid in analyzing figures and identifying necessary conditions. Additionally, participants with weaker spatial skills often faced difficulties in forming accurate mental images, particularly when dealing with complex objects featuring hidden surfaces, multiple vertices, or intricate edges (Velez et al., 2005). Other studies also stated students had difficulty in multi-step spatial reasoning, spatial language understanding, and 2D to 3D transformation (Hasanah et al., 2024).

The challenges faced by students, regardless of their spatial ability level, can stem from various factors. These include cognitive load, discrepancy between mental image and motor control, language-verbal dissociation, attention management issues, and a lack of practice in externalization. Recent research shows that students acquire knowledge more effectively and immediately when presented with highly detailed visuals accompanied by minimal text. In contrast, conditions with low visual detail and high text content increase cognitive load (Zhou et al., 2024). Another contributing factor is the mismatch between mental imagery and motor control. Students need to be accustomed to dealing with spatial objects. Lack of training in spatial thinking can be one of the obstacles in solving mathematical problems that require spatial thinking. Some research stated that spatial training can be given to student and can enhance their spatial and mathematics performance (Lowrie et al., 2018; Sorby, 2016). In addition, the use of learning aids such as GeoGebra can help students improve their visualization skills (Baiduri et al., 2024; Dintarini, et al., 2024).

The differences between male and female students in spatial thinking when solving the given problems became one of the researcher's concerns. In visualizing the mental images of male and female students, both were successful in building mental images of the requested spatial objects, namely double cones, planes and their intersections. However, male students exhibited inconsistencies in using dashed or solid lines to represent spatial objects and demonstrated limited written exploration of their characteristics. While female students face challenge in inaccurately drawing conic sections in 3D images, but in return female students were able to explain in 2D form and verbal representation. In addition, female students always used solid lines in drawing the requested spatial objects, indicating difficulty in drawing details in 3D. Previous research also tried to investigate differences between men and women in spatial reasoning, where male subjects showed a broader perspective in spatial reasoning. Although, researchers in this case said that there were not enough subjects to differentiate between men and women. So further research regarding differences in spatial thinking based on gender needs to be carried out (Pradana & Sholikhah, 2023).

Understanding the spatial thinking patterns of male and female students can provide lecturers' insight into organizing geometry classes with the existing possibilities. Lecturers might be able to implement differentiated instruction in the classroom (Geel et al., 2019; Pozas et al., 2020). Launching a geometry course involves an initial assessment of the spatial cognitive capacities inherent in both male and female students, enabling instructors to tailor educational approaches accordingly. This diagnostic phase serves as a foundation for the design of versatile pedagogical frameworks, amenable to universal application or nuanced differentiation targeted at each gender cohort. Instructors are encouraged to explore diverse instructional media crafted to augment spatial reasoning abilities, alongside the implementation of specialized frameworks adept at providing simultaneous interventions for individuals exhibiting diminished spatial cognitive aptitudes. This deliberate and comprehensive understanding seeks to optimize learning outcomes, with the overarching objective of equipping students with the requisite skills to excel in the geometry curriculum, fostering the achievement of commendable academic performance.

CONCLUSION

The highlighting the need for further research into additional factors contributing to this challenge. Students with lower spatial thinking abilities struggle to construct and represent complex mental images. Additionally, the study highlights gender differences in spatial thinking. Male students tend to provide limited written explanations of spatial object characteristics, while female students face challenges in accurately drawing 3D objects but compensate with effective 2D representations and verbal descriptions. These findings suggest that implementing differentiated instruction based on students' spatial cognitive abilities could improve geometry education, enhance learning outcomes, and support academic success in this field.

This study has several limitations that present opportunities for further research. For example, it primarily utilizes a qualitative approach. Future studies could build on these findings by adopting a quantitative approach, using this research as a theoretical foundation to enhance generalizability across diverse groups.

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DECLARATIONS

Author Contribution	:	MD: Conceptualization, formal analysis, Investigation, Methodology, Visualization, and Writing- original draft; MTB: Conceptualization, Methodology, Project administration, Supervision, Validation, and Writing – review and editing; YF: Conceptualization, Methodology, Project administration, Supervision, Validation, Data Analysis, and Writing – review and editing.
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