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Ratios in Agriculture: A Rice Fertilization Context for Ratio Learning Trajectory

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Abstract

This study aims to develop a learning trajectory for ratio material using rice fertilization context. This study employed validation studies within a design research framework involving seventh-grade students at a junior high school in Belitang. Data was collected through student activity sheets, observations, and interviews. Design research consists of three stages: experimental preparation, experimental design, and retrospective analysis. The findings indicate that the learning trajectory for ratio material in the rice fertilization context includes identifying patterns and relationships between two quantities, determining the simplest unit ratios, representing ratios using ratios table, and solving real-world ratio-related problems. These results confirm that the designed learning trajectory serves as a practical reference for teachers in designing ratio learning that is meaningful, applicable, and relevant to the life experiences of students in agricultural areas, while supporting sustainable agriculture.

Keywords: Learning Trajectory; PMRI; Ratio; Sustainable Practices; Sustainable Agriculture.

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INTRODUCTION

Ratio is one of the materials that students must master because understanding this concept is important for the success of learning mathematics in grades 6-8 (Diba & Prabawanto, 2019; Lestari et al., 2019; Sari et al., 2024). The concept of ratio is widely used in various disciplines, integrated into other mathematical topics, and used to solve problems in daily life (Wahyuningrum et al., 2022). In mathematics, ratios play an important role in developing concepts and skills related to slope, constant rate of change, and similar forms, all of which are foundational to learning algebra (Dougherty et al., 2017). Ratios provide the foundation for understanding more complex mathematical concepts, such as probability, geometry, and fraction (Petit et al., 2020). In daily life, ratios are useful for adjusting recipes, comparing prices, interpreting map scales, and performing measurements (Sari et al., 2024). Thus, mastery of the ratio concept helps improve students' mathematical competence and strengthens their proportional reasoning, enabling them to identify relationships between two or more quantities (Sari et al., 2025).

A ratio is defined as the multiplicative comparison between two or more quantities or measures (Petit et al., 2020). Ratios can be expressed as part-to-whole, part-to-part, or as a comparison between two different types of measures (Lamon, 2020). Ratio learning can start by identifying patterns (Muttaqin et al., 2017). According to Walle (2008), teaching ratios can be effectively conducted through exploratory activities. Petit et al. (2020) stated that understanding ratios can be deepened through model representations such as ratio tables, tape diagrams, double number lines, coordinate planes, and equations. Several experts agree that ratio tables are effective representations to help students

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understand ratio concepts (Civak et al., 2022; Muttaqin et al., 2017; Sari et al., 2024; Sumarto et al., 2013; Utari et al., 2015). Solving ratio problems can be conducted by simplifying the ratio to $1{:}\,m$, where m is a whole number (Chaim et al., 2012; Wahyuni, 2022).

In Indonesia, students' ability to understand ratios is relatively low (Hodgen et al., 2024; Setyaningsih et al., 2018; Wijaya et al., 2019). This was reinforced by a preliminary survey conducted by researchers at a junior high school in Belitang. Based on interviews with mathematics teachers at the school, it was found that students struggled to understand ratios due to difficulties in grasping the underlying concepts. This finding aligns with Lamon's view that ratios are the most difficult material to teach, being mathematically complex and critical for successful mathematics learning (Petit et al., 2020). A study by Muttaqin et al. (2017) showed that students often struggle to develop a deep understanding of ratio concepts and tend to use formulas mechanically without truly understanding their meaning.

Specific problems faced by students in learning ratios include difficulty understanding basic ratio concepts, difficulty in solving contextual problems, insufficient depth of ratio material in textbooks, and the use of inappropriate solving strategies, such as addition or subtraction in situations that require multiplication or division (Andini & Jupri, 2017). Many students struggle to understand ratios because the learning approach is often disconnected from everyday experience, making it difficult for them to relate abstract mathematical concepts to real-life situations (Dougherty et al., 2017).

This is where the Indonesian Realistic Mathematics Education (PMRI) approach plays an important role, emphasizing the use of real-life contexts to improve students' understanding through models and activities rooted in their experiences (Van den Heuvel-Panhuizen & Drijvers, 2014). PMRI helps students connect ratio concepts to real situations, strengthening their involvement and understanding through familiar contexts (Benson-O'Connor et al., 2019; Rawani et al., 2023). It highlights the importance of real-world context as the foundation for mathematics learning (Zulkardi et al., 2020). According to this theory, students can more easily internalize mathematical concepts when learning begins with concrete, familiar situations and gradually move to a more abstract understanding (Zulkardi & Putri, 2019). Gunawan & Hadi,(2024) and Tamur et al. (2021) also found that context-based PMRI learning is effective in improving students' mathematical abilities. Through the PMRI approach, students are guided to construct mathematical understanding based on learning experiences related to daily life (Zulkardi & Putri, 2020). There are three main principles in PMRI: (a) guided rediscovery and progressive mathematization; (b) didactic phenomenology; and (c) the use of models developed by students themselves (Fauziah et al., 2022). Zulkardi and Putri (2019) outlined five key characteristics of PMRI: (1) learning begins with real-world context, (2) the use of models to bridge abstract concepts and real situations, (3) the use of strategies developed by students, (4) interaction as an important component of the learning process, and (5) connections between the material studied and other mathematical topics. Furthermore, Van den Heuvel-Panhuizen and Drijvers (2014) identified six principles that reflect the characteristics of PMRI: the activity principle, reality principle, intertwinment principle, interactivity principle, interactivity principle, and guidance principle.

The integration of ratio learning with real-life agricultural contexts aligns with global efforts to achieve the Sustainable Development Goals (SDGs), particularly Goal 4 (Quality Education) and Goal 12 (Responsible Consumption and Production). The United Nations emphasizes that education should not only improve literacy and numeracy but also equip learners with the knowledge and skills needed to promote sustainable development, including sustainable agricultural practices (UNESCO, 2024). Embedding mathematical concepts such as ratio into meaningful agricultural contexts, such as rice fertilization, helps students improve their academic understanding while developing awareness of resource efficiency and environmental stewardship. This approach supports the development of context-aware, solution-oriented individuals capable of contributing to sustainable agricultural innovation in their communities (UNESCO, 2024).

Previous research has demonstrated the success of the PMRI approach in various contexts. For example, Rawani et al. (2023) developed a local instructional theory (LIT) that successfully increased students' understanding of geometric transformations by using local cultural contexts. Zulfah et al. (2024) showed that developing LIT for parallelogram material using RME improved students' problem-solving abilities. Several studies on ratio learning have also been conducted. Muttaqin et al. (2017) who studied the use of ratio tables and graphs in the East OKU context, found that a contextual approach could enhance students’ proportional reasoning abilities. Utari et al. (2015), who employed the Palembang cultural context in teaching ratios, found that students developed more effective problem-solving strategies when exposed to familiar contexts. These findings support the idea that designing local instructional theory based on contextual learning can overcome obstacles in learning abstract mathematical concepts (Nofriati et al., 2020; Rawani et al., 2023; Samura et al., 2024; Sumarto et al., 2013).

This study provides a novel approach in ratio learning by developing a learning trajectory for ratio material based on the context of rice fertilization. This study explicitly integrates a local context that is authentic and relevant to students' lives in an agrarian environment, especially in Belitang, the largest rice-producing area in South Sumatra, making it easier for students to practically understand ratio concepts. This context adds value by increasing students' motivation and engagement through personal and environmental connections, providing interdisciplinary insights between mathematics and agricultural science, and emphasizing the practical application of ratios in real-life decision-making (e.g., calculating fertilizer needs per hectare). Accordingly, the research question posed in this research is: How can a learning trajectory for ratio material be developed using the context of rice fertilization? The contribution of this research to Mathematics Education lies in offering new insights into the development of context-based learning trajectory, which can serve as a reference for teaching other mathematical concepts. This research provides a concrete example of how mathematics learning can be meaningfully connected to students' real-life experience, especially in agriculture.

METHODS

This research employed the design research method with a validation studies approach to address the problem formulation. This method aims to develop or validate theories regarding processes, strategies, or learning materials as solutions to educational problems, while improving the quality of learning through collaboration between teachers and researchers (Akker & Gravemeijer, 2010; Bustang et al., 2013; Gravemeijer & Cobb, 2006; Plomp & Nieveen, 2013). The validation studies approach in design research aims to develop a learning trajectory for ratio material using rice fertilization context. In this study, the rice fertilization context was used to help students understand the concept of ratios. This design research was conducted in three main stages: preparing for the experiment, implementing it in the classroom, and conducting a retrospective analysis (Plomp & Nieveen, 2013).

Stage I: Preparing for Experiment

At this stage, the researchers conducted literature studies and designed learning activities as well as learning instruments that will be used in the learning process. The literature review focused on the topic of ratios, the PMRI approach, and the analysis of the independent curriculum currently implemented in Indonesia. This stage resulted in the development of a Hypothetical Learning Trajectory (HLT), along with student activity sheets, learning tools, and assessment instruments. The HLT was designed to be flexible and subject to revision based on feedback during the research process. All instruments, including activity sheets, observations, and test questions, were validated through focus group discussions involving two mathematics education lecturers, four teachers, and three doctoral students in mathematics education.

Stage II: Design Experiment

At this stage, the researchers conducted trials of the designed learning trajectories in actual classroom settings to observe students' strategies and ways of thinking. The design experiment was conducted in two stages: a pilot experiment and a teaching experiment. The pilot experiment involved a small group of eight students with heterogeneous abilities to test the designed learning trajectory. Based on the results of this trial, the trajectory was refined and improved. The teaching experiment was then conducted in a full classroom setting to observe the actual implementation of the learning trajectory for ratio material using rice fertilization context.

Stage III: Retrospective Analysis

At this stage, the researchers compared the HLT with the actual learning trajectory observed during the design experiment stage. They identified the problem-solving strategies used by students, exploring possible alternative answers to design mathematical models that emerged during the learning process, such as the use of ratio tables to support understanding of ratio concepts. The analysis of observation and interview data provided complete insights into the study's findings. These findings highlight the role of the rice fertilization context in supporting students' understanding the concept of ratio.

Research Time and Participants

This research was conducted from August to November 2024 and involved seventh-grade students at a Junior High School in Belitang. The pilot experiment included eight students from class VII.3, selected through purposive sampling based on diverse ability levels: two-high ability students, two-low ability students, and four medium ability students. The classification of students is based on their daily grades, as recommended by teachers. The higher proportion of medium-ability students reflected the overall composition of the class. Next, the researchers divided the students into two groups consisting of four, each consisting of students with heterogeneous abilities. For the teaching experiment, all 31 students from class VII.4 were involved as research subjects.

Data Collection and Analysis

Data was collected through several techniques. The main instrument used was the student activity sheet, which contained four activities based on the rice fertilization context. These activities were designed to explore students' thinking process in relation to developed learning trajectory. In addition, direct classroom observations were conducted to record students' thinking strategies, interactions, and challenges encountered during the learning process. Semi-structured interviews were held with selected students to explore the reasoning behind their response on the activity sheets and to understand how the contextual approach supported their understanding of the ratio concept. To measure students' understanding after the learning process, a test that has been previously tested for validity and reliability, was administered following the teaching experiment. Documentation and field notes were also collected to support the validity of the findings.

The collected data were analyzed using a descriptive qualitative method by describing the results that emerged during the research process. To ensure data validity, methodological triangulation (through tests, observations, and interviews) and source triangulation (involving students and teachers) were conducted. In addition, data interpretation involves experts and peer reviewers to minimize subjectivity in data interpretation.

RESULTS AND DISCUSSION

Preparing for The Experiment

This research was conducted following the stages of design research, beginning with preparing for the experiment stage. At this stage, the researchers conducted a literature review, designed the necessary learning tools, developed the HLT and carried out expert validation. Based on literature review regarding the concept of ratio and PMRI approach, an HLT design was obtained along with a student activity sheet for understanding ratio material using rice fertilization context. The initial HLT design for ratio material is presented in Table 1.

Table 1. HLT ratio material using rice fertilization context

No	Activity	Learning Purpose	Prediction of Students Response
1	Observe and explore patterns in subsidized fertilizer data based on rice field area	Students can understand the use of ratios in everyday life	- Students can understand the information presented in the subsidized fertilizer distribution table and understand the use of ratios in daily life. - Some Students cannot understand the information presented in the subsidized fertilizer distribution table and do not understand the use of ratios in daily life.
2	Calculating the amount of fertilizer subsidized for a 0.25 Ha rice field area	Students can identify the relationship between two quantities	- Students can calculate the amount of fertilizer subsidized for a 0.25 ha rice field. - Some students cannot calculate the amount of fertilizer for a 0.25 ha rice field due to difficulties with decimal division.
3	Make a ratio table of kg Urea fertilizer with kg Phonska fertilizer on land of 0.25 Ha to 1 Ha.	Students can create model representations using ratio tables	- Students can create a ratio table correctly - Some students have difficulty creating a ratio table correctly
4	Students discuss and write definitions of ratios and solve problems related to ratios	Students can understand the definition of ratios and solve ratio problems in everyday life.	- Students understand the definition of ratio and can solve ratio problems by calculating the amount of fertilizer with a decimal field area. - Some students understand the definition of ratio but have difficulty calculating the amount of subsidized fertilizer for fields with decimal areas.

In Table 1, the researchers identified that students needed to engage in four key activities to grasp the concept of ratios and apply them to real-life problem-solving. After formulating the HLT and designing the student activity sheet related to ratio material, the next step was to conduct a trial of the activity sheet during the design experiment stage.

Design Experiment: Pilot & Teaching Experiment

Implementation of Pilot Experiment

A pilot experiment was conducted to test the activity sheet designed to support students' understanding ratio material through rice fertilization context. The trial was conducted in small-class settings involving eight students from class VII.3 at a junior high school in Belitang. During the pilot experiment, the students were divided into two groups to allow for more intensive and focused testing of the Student Activity Sheets (SAS) on a small scale before implementation in a larger classroom. This grouping allowed the researchers to closely observe students' learning dynamics and responses, as well as to evaluate the effectiveness of the materials and activities. Each group consisted of students with heterogeneous abilities, intentionally selected to enrich group interactions and promote peer learning and support. Moreover, this setup provided a realistic picture of how the SAS would function in an actual classroom context, where students' abilities vary. Through this approach, the researchers were able to assess whether the SAS could effectively support the understanding of all students, not only those with higher academic performance.

At this stage, the researchers acted as a model teacher who implemented the learning process using the ratio activity sheets, while the mathematics teacher acted as an observer who monitored students' responses during the lesson. After the learning was completed, the researchers and four teachers reflected on the teaching process. Suggestions for improvement, based on the results of the retrospective analysis conducted by researchers and teachers regarding the use of ratio activity sheets, are summarized in Table 2.

Table 2. Summary of improvement suggestions for ratio learning

Activity on SAS	Improvement Suggestions	Purpose of Revisions
Activity of observing subsidized fertilizer distribution table data	- Present simpler and more patterned table data. - Add data sources presented in the table.	- Students find it easier to understand the relationship patterns between the two things being compared.
Calculating subsidized fertilizer for 0.25 Ha of land	- One additional question needs to be asked, such as calculating the amount of fertilizer for 1 Ha before asking about 0.25 Ha of land, ensuring that students do not have difficulty solving the problem.	- Makes it easier for students to solve problems and find units for each size of the rice field area.
Overall activity sheet	- The student activity sheet has too many questions, and the	- The activity sheets should be designed to align with the

Activity on SAS	Improvement Suggestions	Purpose of Revisions
	progression between questions is too abrupt.	available learning time and include appropriate adjustments to question difficulty, ensuring that they accommodate students with different cognitive abilities.

Based on the suggestions given by the teacher in Table 2, the researchers then revised the ratio activity sheet before implementing it in the experimental teaching stage. These revisions were necessary to prevent students from encountering the same difficulties and to ensure that the activity sheets could more effectively support students in understanding the concept of ratio. Researchers also revised the HLT according to the results of testing in the pilot experiment.

Implementation of Teaching Experiment

The experimental teaching stage is conducted to determine the learning trajectory of ratio material through the implementation of revised student activity sheets. This stage was conducted in class VII.4 at a junior high school in Belitang. The mathematics teachers acted as the model teacher, facilitating students in the learning process using ratio material activity sheets. During the learning process, the researchers served as observers, monitoring the implementation of the lesson according to the previously prepared teaching modules. A change in roles between the researchers and the mathematics teacher was made to align with the objectives of each research stage. In the pilot experiment, the researchers acted as the model teacher to directly implement and observe the designed learning trajectory and student activity sheets (SAS), allowing for real-time evaluation and refinement. In the teaching experiment, the mathematics teacher assumed the role of model teacher to examine the practicality and effectiveness of the revised learning trajectory in a more authentic classroom setting. This shift also aimed to assess the sustainability of the learning design when implemented independently by teachers. To support this transition, the researchers conducted briefing sessions to explain the learning objectives, activity flow, and the use of the rice fertilization context. Guidance and examples were provided to ensure the teacher fully understood and could implement the learning trajectory as intended.

In this teaching experiment, students work on a student activity sheet consisting of four activities with a total of nine questions to be answered. The following is a description of each activity in the ratio activity sheet.

Activity 1: Observe and explore patterns in subsidized fertilizer data based on rice field area

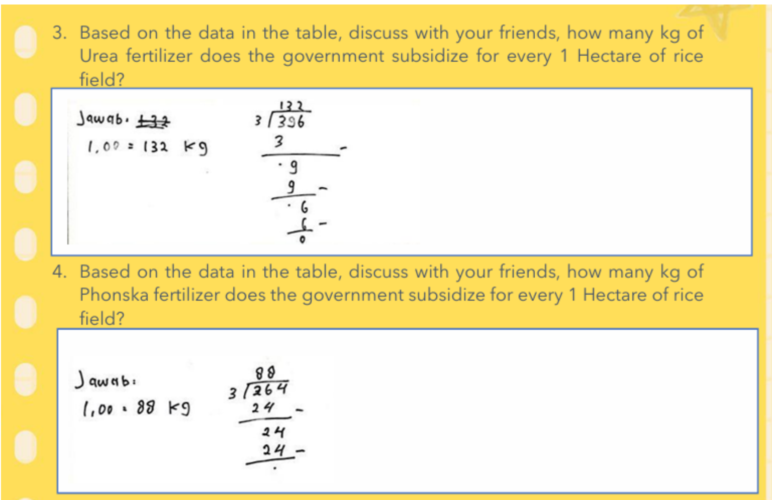
This activity aims to help students understand the use of ratios in daily life. Figure 1 describes the questions and students' answers for Activity 1.

Figure 1. Students' questions and answers in the first ratio activity

Figure 1 shows a student activity sheet using a rice fertilization context to explore the concept of ratio. Students can understand and analyze the information presented in the table. Through this table, they were guided to observe the relationship between the area of rice fields and the amount of fertilizer. This was evident from their ability to answer questions related to the types of fertilizers used in rice fertilization and to identify that urea was the most used type. In this case, students understood that fertilizer subsidies in agriculture apply the concept of ratios. The government considers the ratio between the area of rice fields owned by farmers and the amount of subsidized fertilizer. Through this activity, students can understand the concept of ratio as a comparison of two quantities.

Activity 2: Calculate the amount of Urea and Phonska fertilizer subsidized by the Government per 1 hectare of rice fields

Activity 2 aims to help students identify the relationship between two quantities: the area of rice fields and the amount of subsidized fertilizer. Figure 2 describes the questions and students' answers for Activity 2.


3. Based on the data in the table, discuss with your friends, how many kg of Urea fertilizer does the government subsidize for every 1 Hectare of rice field?

Jawab: 1,00 = 132 kg

3 / 396 = 132

4. Based on the data in the table, discuss with your friends, how many kg of Phonska fertilizer does the government subsidize for every 1 Hectare of rice field?

Jawab: 1,00 = 88 kg

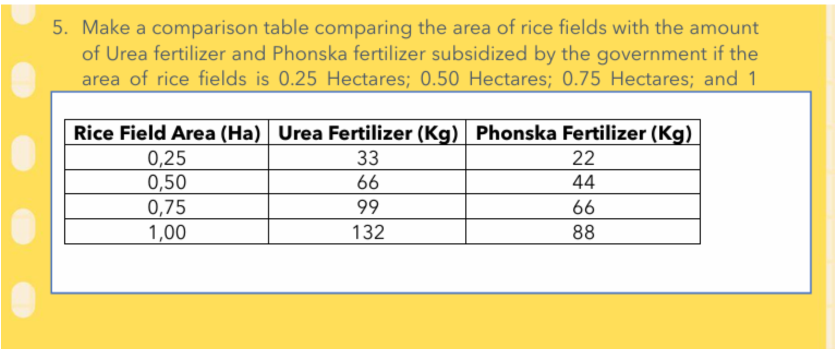
3 / 264 = 88

Figure 2. Students' questions and answers in the second ratio activity

In Figure 2, students can calculate the ratio between the area of land and the amount of subsidized fertilizer per hectare. Based on the data given in the table, students calculate the amount of subsidized fertilizer for 1 hectare of rice fields. Through this calculation process, they begin to understand that ratios involve multiplication and division relationships.

Activity 3: Make a ratio table of kg Urea fertilizer with kg Phonska fertilizer on land of 0.25 Ha to 1 Ha.

This activity aims to guide students in making the correct ratio table. In the ratio activity sheet, students are asked to make a table comparing the area of rice fields with the amount of subsidized fertilizer for areas of 0.25 hectares, 0.50 hectares, 0.75 hectares, and 1 hectare. Figure 3 presents the questions and students' answers for Activity 3.


5. Make a comparison table comparing the area of rice fields with the amount of Urea fertilizer and Phonska fertilizer subsidized by the government if the area of rice fields is 0.25 Hectares; 0.50 Hectares; 0.75 Hectares; and 1

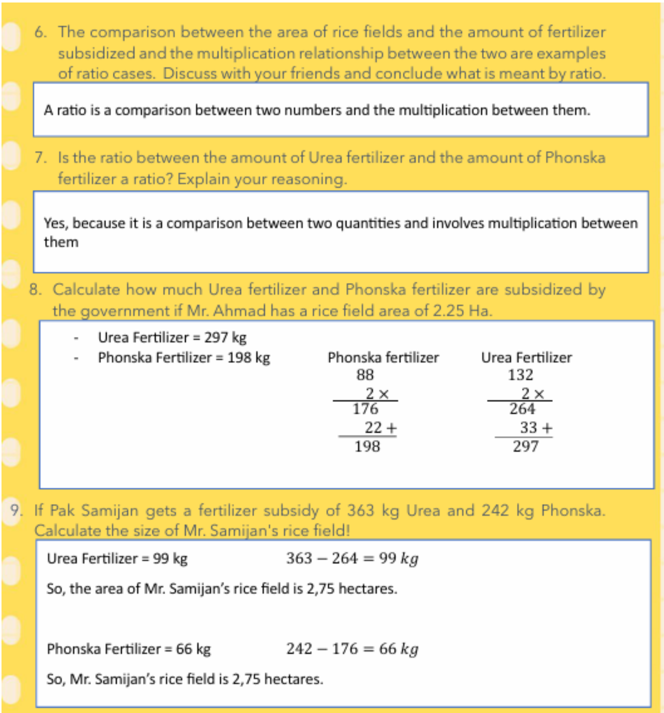
Rice Field Area (Ha)	Urea Fertilizer (Kg)	Phonska Fertilizer (Kg)
0,25	33	22
0,50	66	44
0,75	99	66
1,00	132	88

Figure 3. Students' questions and answers in the third ratio activity

Figure 3 shows that students can create model representations using ratio tables correctly. The table demonstrates a multiplicative relationship between the area of rice fields and the amount of fertilizer used. For example, when the rice field area doubles from 0.25 Ha to 0.50 Ha, the amount of Urea and Phonska fertilizer also doubles (from 33 kg to 66 kg, and from 22 kg to 44 kg). This table gives students a concrete example of how ratios are used in everyday life, particularly in agricultural practices. It also trains students to identify mathematical patterns and relationships from the presented data. Through this activity, students' understanding of the concept of ratio deepens, as demonstrated by their ability to create model representations using ratio tables. The ratio table clearly illustrates how the multiplicative relationship between the two compared quantities.

Activity 4: Students discuss and write definitions of ratios and solve problems related to ratios

This activity aims to help students understand the correct use of ratios and solve ratio problems in daily life. Figure 4 presents the questions and students' answers for Activity 4.


6. The comparison between the area of rice fields and the amount of fertilizer subsidized and the multiplication relationship between the two are examples of ratio cases. Discuss with your friends and conclude what is meant by ratio.

A ratio is a comparison between two numbers and the multiplication between them.

7. Is the ratio between the amount of Urea fertilizer and the amount of Phonska fertilizer a ratio? Explain your reasoning.

Yes, because it is a comparison between two quantities and involves multiplication between them

8. Calculate how much Urea fertilizer and Phonska fertilizer are subsidized by the government if Mr. Ahmad has a rice field area of 2.25 Ha.

- Urea Fertilizer = 297 kg
- Phonska Fertilizer = 198 kg

Phonska fertilizer: 88 × 2 = 176; 176 + 22 = 198

Urea Fertilizer: 132 × 2 = 264; 264 + 33 = 297

9. If Pak Samijan gets a fertilizer subsidy of 363 kg Urea and 242 kg Phonska. Calculate the size of Mr. Samijan's rice field!

Urea Fertilizer = 99 kg $363 - 264 = 99\ kg$

So, the area of Mr. Samijan's rice field is 2,75 hectares.

Phonska Fertilizer = 66 kg $242 - 176 = 66\ kg$

So, Mr. Samijan's rice field is 2,75 hectares.

Figure 4. Students' questions and answers in the fourth ratio activity

Based on the previous activity, students and their group of friends were asked to discuss the definition of ratio. In Figure 4, students wrote the definition of ratio and were able to explain a comparison referred to as ratio. Next, students are asked to solve a ratio problem involving the amount of fertilizer subsidized for a known area of rice field. Figure 4 shows that students can solve real-life ratio problems. Students use multiplication operations to solve the problems and refer to the previously created ratio table for support. Overall, students show a good understanding of the concept of ratio and its application in a real-world context, such as fertilizing rice fields. They were able to explain the concept of ratio verbally and apply appropriate mathematical calculations to the given problem. The four activities given effectively supported students in understanding the concept of ratios and solving related problems in everyday life.

Based on observations made by researchers during the teaching experiment stage, students appeared enthusiastic while working on the ratio activity sheet. The use of the rice fertilization context helped students better understand the concept of ratio in a more practical and applicable manner. This is reinforced by the following interview excerpt.

R : *"After learning about ratios using the student activity sheet with the rice fertilization context, do you think the concept of ratios became easier to understand?"*

S : *"Yes, Ma'am. I understand it better now because the examples are real and easy to imagine. For example, when calculating the ratio between urea and phonska fertilizer, I could clearly see how the quantities relate based on what the rice field needs."*

R : *"Can you explain the definition of a ratio based on your understanding?"*

S : *"Yes, Ma'am. A ratio is a comparison between two related things using multiplication."*

R : *"What do you mean by "related multiplicatively"? Can you explain with an example?"*

S : *"What I mean by "related multiplicatively" is, for example, if we use 132 kg of urea and 88 kg of Phonska to fertilize 1 hectare of rice field, then if the field is 2 hectares, we just multiply everything by two. So, we would need 264 kg of urea and 176 kg of Phonska. Like that, Ma'am."*

R : *"Okay, do you think learning through this kind of context helps more than just using regular ratio problems?"*

S : *"Yes, Ma'am. Because now I understand what the ratio is used for. It's not just numbers—it's used to calculate real fertilizer needs in a rice field. It makes more sense, and I can remember it better."*

The interview results show that the use of real contexts, such as fertilizing rice fields, in student activity sheets supports a deeper understanding of the concept of ratio. Students reported that the concept of ratio became easier to understand because the examples were relevant and concrete. Students can explain ratio as a comparison between two multiplicatively related quantities and provide examples of its application on a larger scale. In addition, students stated that context-based learning is more meaningful than conventional ratio problems, as it provides a practical understanding of the function of ratios in everyday life, making the concept easier to remember.

Retrospective Analysis

Retrospective analysis was conducted by comparing HLT with data from the design experiment stage. Based on the comparison, it is known that the learning trajectory for ratio material consists of: 1) identifying patterns and relationships between two quantities, 2) determining the simplest unit ratios, 3) representing ratios using table ratios, 4) solving ratio-related problems in real contexts. Interviews and observations results show that students find it easier to understand ratios because the context used was real and relevant to their lives. Students were not only able to explain the definition of ratio as a comparison between multiplicatively related quantities but also can apply it on a larger scale and solve real-world problems.

In this study, learning began with the presentation of fertilizer subsidy data that followed a specific pattern. This aligns with the theory proposed by Muttaqin et al. (2017), which states that ratio learning can start by identifying patterns. Furthermore, students were asked to observe and explore subsidized fertilizer data, consistent with Walle's (2008) view that the ratio concept can be introduced through exploration activities. Through this exploration, students can identify the relationship between two quantities: land area and amount of fertilizer. Thus, this activity helps build students' initial understanding of the ratio concept as a comparison between two quantities.

After understanding the pattern or relationship between the two compared quantities, students then converted the presented data into the simplest form of comparison. Students calculate the ratio of fertilizer per hectare to understand the multiplicative relationship between the area of land and the amount of fertilizer. They simplify the ratio into a more comprehensible form, 1 hectare to the corresponding number of kg of fertilizer. This supports the theory proposed by Wahyuni (2022) and Chaim et al. (2012), which states that understanding ratios can be facilitated by simplifying them to $1: m$ form, where m is an integer. Through this simplification process, students were able to better understand the ratio between one hectare of rice field and the amount of fertilizer used.

In the next activity, students created a model representation using a ratio table. Through this process, they learned that ratios involve multiplicative and divisive relationships: if a quantity is divided or multiplied by a certain number, the other quantity changes proportionally. From this, students understood that a ratio is a comparison between two quantities with a multiplicative relationship. This study identified two advantages of using ratio tables. First, the ratio table was built using real data on fertilizer subsidies and linked to land size (0.25 ha to 1 ha). By creating the table based on varying land areas, it served not only as a mathematical tool but also as a representation for calculating efficiency, estimating fertilizer needs, and evaluating proportional use in various land sizes. Second, the ratio table functioned as a bridge to problem-solving strategies in context. It extended beyond conceptual understanding and became a practical tool for solving real-world problems related to fertilizer subsidies. Instead of relying on formulas, students referred to the ratio tables they had constructed to develop solution strategies. The findings of this study support the research by Civak et al. (2022); Muttaqin et al. (2017); Sari et al. (2024); Sumarto et al. (2013); and Utari et al. (2015), which concluded that ratio tables are appropriate tools to help students understand ratios.

Through the ratio activity sheet, students not only understand mathematical concepts, but also learn about agricultural practices, particularly the rice fertilization process, which has not previously been used specifically in ratio learning. This finding supports the theory of Zulkardi and Putri (2019), which states that one of PMRI characteristics is a connection between the material studied and other topics. By linking the ratio concept to real contexts, students not only understand abstract mathematical concepts but also learn their practical applications, such as calculating fertilizer needs based on land area, recognizing types of fertilizers, and adjusting quantities based on efficiency and appropriate proportion. This supports Zulkardi and Putri's (2019) statement that students more easily internalize mathematical concepts when they begin with familiar, concrete situations, then gradually progress toward a more abstract understanding. Thus, learning ratios through the context of rice fertilization provides a meaningful and relevant learning experience, especially for students from agricultural environments.

CONCLUSION

The learning trajectory of ratio material using rice fertilization consists of four activities: identifying patterns and relationships between two quantities, determining the simplest unit ratios, representing ratios using ratios table, and solving ratio-related problems in real contexts. The findings of this study are particularly relevant to education in agrarian areas, as they incorporate practices familiar to students. The main limitation of this study lies in its focus on the specific context of rice fertilization and a single mathematical topic of ratio, limiting its applicability in non-agrarian environments and generalization to other mathematical concepts. Teachers can use this context-based learning trajectory as a reference for designing more meaningful, applicable, and contextual instruction to enhance students' understanding of ratios. In addition, further research could explore the effectiveness of similar methods at different educational levels or in various cultural settings or develop learning trajectories for other mathematical concepts utilizing alternative, locally relevant contexts.

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