

Teacher's Specialized Content Knowledge on the Concept of Square: A Vignette Approach

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Abstract

In learning geometry, the discussion about the definition of quadrilateral is a material that is difficult and not easily taught by the teacher. This study aims to explore the teacher's specialized content knowledge about square. This is a descriptive-qualitative research. The process of selecting subjects begins with searching prospective subject data according to the level of the teacher through a portfolio of 82 teachers in South East of Sulawesi: (33 First Teachers, 33 Young Teachers, and 16 Intermediate Teachers). The research subjects consisted of three teachers, namely: First Teacher, Young Teacher dan Intermediate Teacher with score > 50. Data were taken using vignette. The results show that there is a difference when the teacher is asked to define a square with when given a definition of a square. First Teacher is accurate when given a square definition with the symmetry and diagonal axis attributes; the side attribute is not accurate in giving arguments to the square definition. Young Teacher is inaccurate when given the definition of a square with side and angle attributes; accurate with symmetry and diagonal axis attributes; but it is not accurate when given a square definition. Regarding attributes of side; Intermediate Teacher revealed that the side and angle attributes are inaccurate but accurate with the symmetry and diagonal axis attributes but do not appear / are not used when asked to define a square. Specialized content knowledge First Teacher is better because it has been able to reconstruct concepts from a square, but Young Teacher and Intermediate Teacher are still influenced by concept images and figural concepts.

Keywords: Specialized Content Knowledge, Teacher, Rectangle, Vignette

Abstrak

Dalam pembelajaran geometri, pembahasan mengenai definisi segiempat merupakan materi yang sulit dan tidak mudah diajarkan oleh guru. Penelitian ini bertujuan untuk menggali *specialized content knowledge* guru tentang persegi. Penelitian ini merupakan jenis penelitian deskriptif-kualitatif. Proses pemilihan subjek diawali dengan penelusuran data calon subjek sesuai jenjang guru melalui portofolio sebanyak 82 guru di Sulawesi Tenggara: (33 Guru Pertama, 33 Guru Muda, dan 16 Guru Madya). Subjek penelitian terdiri dari tiga orang guru yaitu: Guru Pertama, Guru Muda dan Guru Madya dengan skor > 50. Data guru diambil menggunakan vignette. Hasil penelitian menunjukkan bahwa ada perbedaan ketika guru diminta mendefinisikan persegi dengan ketika diberikan definisi persegi. Guru Pertama akurat ketika diberikan definisi persegi dengan atribut sumbu simetri dan diagonal; atribut sisi tidak akurat dalam memberikan argumen terhadap definisi persegi. Guru Muda tidak akurat ketika diberikan definisi persegi dengan atribut sisi dan sudut; akurat dengan atribut sumbu simetri dan diagonal; tetapi tidak akurat ketika diberikan definisi persegi. Guru Madya terkait atribut sisi; atribut sisi dan sudut tidak akurat tetapi akurat dengan atribut sumbu simetri dan diagonal tetapi tidak muncul/tidak dipakai ketika diminta mendefinisikan persegi. *Specialized content knowledge* Guru Pertama lebih baik karena telah mampu merekonstruksi konsep dari persegi, tetapi Guru Muda dan Guru Madya masih dipengaruhi oleh concept image dan konsep figural.

Kata kunci: *Specialized Content Knowledge*, Guru, Persegi, Vignette

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INTRODUCTION

Concepts in mathematics learning, including geometry, are interpreted as abstract ideas that are used to classify objects, including examples or not examples. The concepts in mathematics are

generally composed of previous concepts. Teachers who understand the concept should be able to answer students' questions about the meaning behind symbol manipulation, explain why certain procedures can or cannot be used, and be able to explain the relationships between these concepts (Isiksal & Cakiroglu, 2011; Fuadiah & Suryadi, 2017). On the other hand, teachers who do not understand the concept tend to emphasize facts, rules (formulas), and procedures. This opinion implies that teachers are required to have knowledge of the content of the material to be taught and knowledge of the material is organized, represented, adapted to the interests and abilities of students to be presented in learning.

One of the content knowledge that must be possessed by a teacher is specialized content knowledge (SCK). SCK is described as special knowledge that only a mathematics teacher has. Ball et al. (2008, p. 400) stated that, "SCK is the mathematical knowledge and skills unique to teaching". SCK concerns knowledge that allows teachers to engage in specific tasks to teach knowledge not required in professions outside of teaching (Carrillo-Yañez et al, 2018). Examples of such knowledge are the knowledge needed to generate representations and use them to explain core mathematical concepts and knowledge to analyze student errors and interpret non-standard solution methods.

Furthermore Ball et al. (2008) explained, "SCK is mathematical knowledge not typically needed for other purposes than teaching. In looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general, as in our subtraction example, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed — or even desirable — in settings other than teaching". This requires a unique mathematical understanding and reasoning. SCK is mathematical knowledge specific to teaching (Hill et al. 2008). The difference between CCK and SCK can be exemplified by the question why $1/2 \times 1/2 = 1/4$. CCK shows that $1 \times 1 = 1$ and $2 \times 2 = 4$. So the result is $1/4$. In contrast to SCK, which explain that the meaning of fractions multiplication is to find a fraction of something, that is, $1/2$ of $1/2$ is $1/4$. In other words, SCK will be identified when it is able to answer the question: why is the product smaller than the multiplied number, while CCK does not have the capacity to explain this (Purnomo, 2017: 4). The word 'common' relates to the knowledge that mathematics teachers must have in common with other professionals who use mathematics disciplines, while 'specialized' refers to different teaching which must be well prepared by teachers to meet the demands of job requirements as teachers (Spyrilioti, 2012). Delaney et al. (2008) also argued that, "specialized content knowledge is mathematical knowledge and skill used by teachers in their work but not generally held by well-educated adults, e.g., knowing alternative algorithms for calculating $75-18$ ".

Specialized content knowledge from mathematics teachers regarding concept definition. Concept definitions, once defined in learning, will influence approaches to teaching mathematics, learning sequences, theorems and proof sets. As a result, definitions, and ways of presenting to students, form a relationship between concept images and concept definitions, forming an important

part of a person's knowledge structure that influences the learner's thought process (Vinner, 1991; 2020), so the teacher's difficulty in defining rectangles is not sufficient to justify content knowledge of the material, but it also needs to be seen when the teacher is given a definition so that further exploration of their knowledge. Teachers' knowledge when asked to define and when given definitions can provide better clues to the extent of teacher understanding and difficulties.

Content knowledge is an important factor in the teaching and learning process of mathematics. One of the characteristics of an expert teacher is that he not only knows many things about his discipline of knowledge, but also that his knowledge is organized systematically which reflects a deep understanding of the subject matter of his study (Anderson & Krathwohl, 2001: p.63; Stronge, 2018). Demirok & Baglama (2018: p. 508) stated, "content knowledge refers to the subject area which are taught or learned". For example, in teaching the material about rectangles, there are attributes that are important to learn, namely sides, angles and diagonals. According to Ball et al. (2008), this example is included in general material content because the content of this material can be easily answered by both teachers and students. Besides that, there are also definitions / definitions that need to be mastered in quadrilateral material. This example is specific material content, because it is material content that cannot easily be answered by students who know math content (Ball et al. 2008; Carrillo-Yañez et al, 2018).

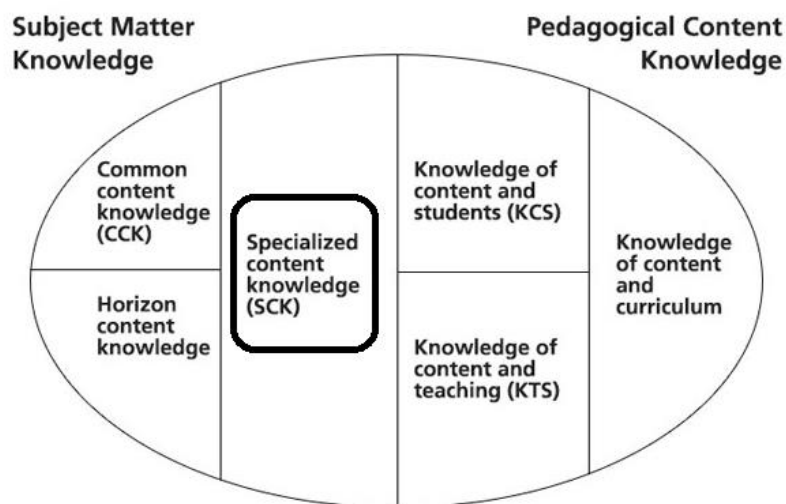


Figure 1. *Specialized Content Knowledge Framework* (Ball et al. 2008)

Figure 1 indicate the position of SCK among other types of knowledge proposed by Ball et al. (2008). In this framework, SCK is defined as mathematical knowledge not typically needed for other purposes than teaching. In looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general, as in our subtraction example, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed — or even desirable — in settings other than teaching”.

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knowledge specific to teaching (Hill et al. 2008; Carrillo-Yañez et al, 2018). Furthermore, Delaney et al. (2008) also argued that, "specialized content knowledge is mathematical knowledge and skill used by teachers in their work but not generally held by well-educated adults, e.g., knowing alternative algorithms for calculating 75-18".

Flores et al. (2013, p. 3056) also argues that, "specialized content knowledge is knowledge in terms that are purely mathematical and specific to the profession". Whereas Knapp et al. (2008, p. 258) states that "specialized content knowledge (SCK) is the way the mathematics arises in classrooms, such as for building representations". Herbst & Kosko (2012) explains that, "SCK is knowledge of mathematics used particularly in doing the tasks of teaching, such as, for example, the knowledge a teacher needs to use in writing the problems they will assign to students or figuring out whether a nonstandard approach would work in general". From these opinions, SCK is a unique mathematical knowledge and skill used by teachers for teaching purposes.

In mathematics learning activities, in general mathematics deals with abstract ideas that are arranged hierarchically and structured. The concepts in mathematics are built on the related concepts that underlie them. This condition implies that the teacher's SCK is very important because this knowledge is a bridge for the teacher to understand and apply it in learning. In addition, what is no less important is the experience and ability of individual teachers in mastering the material being taught.

In mathematics, concepts are closely related to definition. Definition is an expression to define a concept. With a definition, people can make illustrations, pictures or symbols of a defined concept and it becomes clearer what is meant by certain concepts (Purnomo, 2019; Soedjadi, 2000). The definition of composing the basics of mathematical thinking is very important in the formation of mathematical concepts, differentiating concepts from other concepts and expressing mathematical ideas (Türnüklü et al. 2013).

Although considered separate from each other, there are several relationships between concept definition and concept image. For example, concept images are formed as a result of individual experiences with concept definitions and concept examples. So, these images can increase as cognitive structures develop. Here, the formal definition is structured by the individual, creating a personal definition that is the foreground of the individual aspect of the formal definition. The definitions compiled by individuals for specific concepts (personal definitions), their formal characteristics in mind, transmissions and relationships between concepts are all important for conceptual understanding.

The definition of mathematical concepts, the structure that underlies the definition and the process of defining it becomes a fundamental component of mathematics teacher knowledge subject matter. This point of view follows several studies on the role of definition in secondary school mathematics (De Villiers, 1998; Zaslavsky & Shir, 2005; Buchbinder & Zaslavsky, 2019) as well as in professional development programs for mathematics teachers (van Dormolen & Zaslavsky, 2003).

Winicky-Landman & Leikin (2000) analyzed the role of equivalent mathematical statements that can serve as the definition of a mathematical concept.

Logical principles must be fulfilled in defining mathematical concepts, namely: (a) defining is giving a name: the statement used as the definition presents the concept name and this term (name) appears only once in the statement; (b) the definition specifies the necessary and sufficient conditions for the concept; (c) in defining a new concept, only previously defined concepts can be used; (d) the required set and sufficient conditions must be minimal (Fernández-León, 2019; Vinner, 1991).

Soedjadi (2000) suggests three kinds of definitions, namely analytic definitions, genetic definitions, and formula definitions. Analytic definition is a definition by mentioning the proximum (close family) and specific differentiation (special differentiator) genus. Genetic definition is a definition that shows or expresses the way a defined concept occurs or is formed. While the definition with formulas is a definition that is expressed by mathematical notations. Furthermore, Soedjadi (2000) argues that there are four elements of definition, namely: background, genus, defined terms, and attributes. For example, a kite is a quadrilateral that has two pairs of adjacent sides that are congruent, a pair of congruent sides that differ from one another. Background: flat shape, genus: quadrilateral, defined concept: kite, and attributes: two pairs of adjacent adjacent sides that are congruent, one pair of congruent sides different from another pair of congruent sides.

Definition is so important in mathematics that it can serve as a starting point for deductive arguments in axiomatic systems (Govender & Villiers, 2002). The definition of mathematics contains both necessary conditions and sufficient conditions (Govender & Villiers, 2002; Paksu et al. 2012). In flat geometry it is very important to identify the hypothetical conditions that exist in a statement in order to justify the conclusions to be made. The conditions in question are necessary and sufficient conditions. In this regard, Govender & Villiers (2002, p. 4) explain that for a condition in a given description (definition) to be sufficient, it must contain enough information (properties) to ensure that not only do we obtain the elements of the set we want to define, but only those elements (and not any others). However, normally we want to use as little information as possible, i.e. only as much as is really necessary. This shows that, a condition is called a necessary condition if the conditions are not fulfilled it results in not happening what is required. A condition is called a sufficient condition if the fulfillment of these conditions results in the occurrence of what is required.

Many experts view the importance of defining the concept of a quadrilateral (Zandieh & Rasmussen, 2010; Govender & De Villiers, 2004; Levenson et al. 2011). In defining the concept using a minimum of information. Govender & Villiers (2002, p. 4) explain, "A description (definition) which contains conditions (properties) that are sufficient is said to be correct. In a correct definition, all the conditions may be necessary or some of the conditions may be unnecessary, i.e. it is possible to have unnecessary conditions in correct definitions. A definition is incorrect if it contains an incorrect property or if it contains insufficient properties. A definition is incomplete if it contains necessary but insufficient properties. So an incomplete definition is also an incorrect one. A correct definition can

be either economical or uneconomical. An economical definition has only necessary and sufficient properties. It contains no superfluous information. On the other hand, an uneconomical definition has sufficient, but some unnecessary properties. In other words, it contains more information than necessary (redundant) information”.

From this description, in defining rectangular shapes formally, it is necessary to pay attention to the necessary and sufficient conditions. The definition is said to be wrong, if the definition contains wrong characteristics or does not contain necessary and sufficient conditions. The definition is said to be uneconomical, if the definition contains necessary conditions but does not meet sufficient conditions (excessive) or sufficient (sufficient conditions) but has some unnecessary characteristics or contains a lot of information than necessary (redundant). Meanwhile, the definition is said to be economical, if the definition contains necessary and minimal requirements. The definition of economics has only the properties which are necessary (necessary conditions) and sufficient properties (sufficient conditions). The economic definition does not contain redundant information.

A definition must be stated in the form of a sentence which contains "if and only if" or "reversible". To determine these definitions, it is necessary to study the conditions for the formation of a quadrilateral.

A square is a rhombus with a right angle. It can also be defined, a square is a rectangle with a pair of adjacent sides of the same length. This means that,

- If the rhombus is a square then one of the corners is right.
- If a rhombus has a right angle then the rhombus is square.
- If the rectangle is a square then a pair of adjacent sides is the same length.
- If a rectangle has a pair of adjacent sides the same length, then the rectangle is square.

Thus, it can be said: a quadrilateral is called a square if and only if the rhombus has a right angle. This definition is economical because it contains the necessary conditions, namely rhombus, a sufficient condition, namely that one corner is right. Because if one corner is right, it guarantees a square. See an example of square at figure 2.

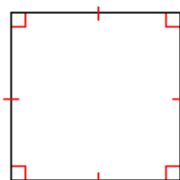


Figure 2. An example of square

In addition, it can also be said: a quadrilateral is called a square if and only if the rectangle whose sides are adjacent are the same length. This definition is economical because it contains a necessary condition, namely a rectangle, a sufficient condition, namely a pair of adjacent sides of the same length because if a pair of adjacent sides is the same length, it also guarantees a square.

Cognitive structure: Mental image and Concept image

Knowledge is the result of construction (formation) that occurs in the mind (cognitive structure) so that new knowledge is formed (concept image). Cognitive (mental) structures are the basic mental processes used to make meaning out of information. An individual's mental image (mental image) of a concept is a collection of all visual representations (including symbols) associated with the concept (Garnerr, 2011). In line with this statement, Tall & Vinner (1981) explains that "... the term concept image to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and process". This means that the concept image is seen as a whole cognitive structure associated with a concept, which consists of the mental image, properties and processes associated with the concept. Concept image in individual mind can be seen as a picture of individual understanding of a concept. Rosken & Rolka (2007, p. 184) illustrate the concept image (concept image), concept definition, and other related ideas as in the following figure.

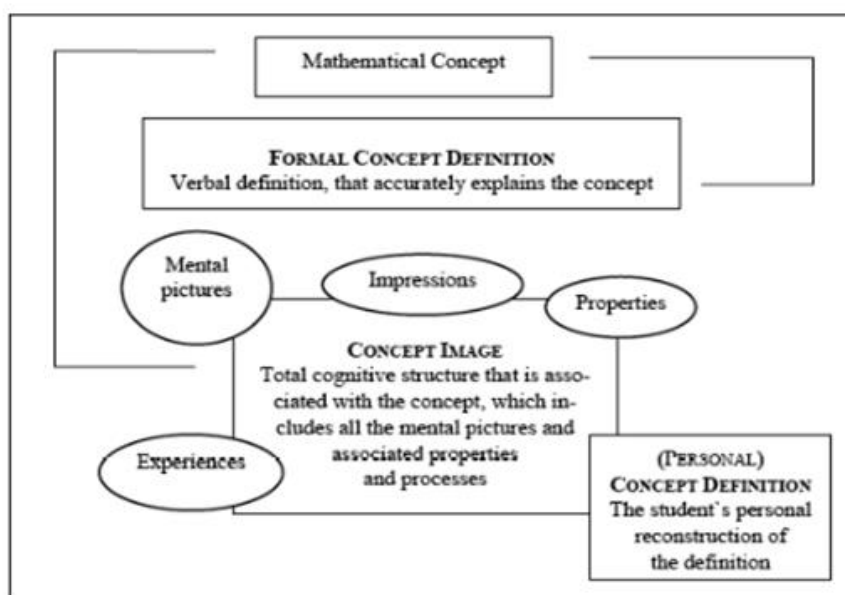


Figure 3. Concept image and concept definition (Rosken & Rolka, p. 184)

Furthermore, the definition of a person's concept, among others, is influenced by the concept image, experience related to the concept, concept characteristics, concept mental image and the reconstruction of the concept definition that someone makes (Rosken & Rolka, 2007).

Based on this description, a strong content knowledge from a teacher will be able to have a positive influence on decision making related to teaching strategies. A teacher who has good content knowledge is expected to be able to construct mathematics concepts and learning materials well. Therefore, this study aims to study the specialized content knowledge of teachers, especially on square material.

METHODS

Type of Research

This study uses a qualitative approach with a descriptive-exploratory research type. This research was conducted in Southeast Sulawesi-Indonesia. The data was collected by giving a vignette to the teacher. The vignette instrument is a scenario that expresses special characteristics that are complemented by questions and submitted to research subjects for opinion or comment.

Research Subject

Determination of research subjects based on the following criteria: (a) qualified undergraduate education and (b) both come from higher education (Halu Oleo University). The data search from the mathematics teacher was carried out through observations and portfolios.

From the results of observation and portfolio analysis, 82 prospective subject teachers were determined according to the criteria and were given vignette A in Table 1.

Table 1. Participants

Level	Candidate of teacher		Total
	subject		
	Male	female	
First Teacher (GP)	10	23	33
Young Teacher (GM)	5	28	33
Middle teacher (GY)	9	7	16
Number	24	58	82

The results of the vignette A given to 82 prospective subject teachers were examined and 77 teachers got a score of more than 50 and 5 teachers got a maximum score of 50. Of the 77 prospective subject teachers were given a willingness questionnaire containing a description of: identity, tenure, education and training followed, as well as a willingness to participate in the study. The results obtained 12 teachers consisting of: First Teacher (2 male teachers, 2 female teachers); Young Teachers (4 female teachers) and Middle Teachers (4 male teachers). Each teacher level is taken by one teacher as the research subject, namely: one first teacher, one young teacher and one middle teacher.

The GP subject was a male teacher who completed his undergraduate education (S1) in 2015 with a cumulative grade point average = 3.50. GP subjects is 27 years old and teaches in grades VII, VIII and IX. The GM subject was a female teacher who completed undergraduate education (S1) in

2005 with a cumulative grade point average = 3.2. The 37 year old GM subject currently teaches in grades VII, VIII and IX. The subject of GY is a male teacher who completed his undergraduate education (S1) in 2000 with a cumulative grade point average = 2.9. The subject of GY is 53 years old currently teaching in grades VIII and IX SMP.

The collected data were then transcribed, reduced, validated, presented and concluded. The results of the definition were coded with three groups, namely: economic, uneconomical and wrong. The definition is said to be economical if the definition states the necessary and minimal conditions. The definition is said to be uneconomical if the definition states necessary and sufficient but excessive conditions. The definition is said to be wrong if the definition does not state the necessary and sufficient conditions.

Instrument

Data were collected through validated vignettes and declared fit for use (Sahidin, et al, 2018). Validation was carried out by three experts, one in the field of geometry at the State University of Surabaya and two experts in the field of mathematics education at the University of Halu Oleo Kendari.

Vignette A:

You are a junior high school math teacher will teach you square material.

Write down the definition of a square!

Vignette B:

Take a look at all the possible definitions of a square below.

- a. Build a quadrilateral that has all the same sides.
- b. A quadrilateral that has all the same sides and the same angles.
- c. A quadrilateral with four axes of symmetry.
- d. Rectangle with equal diagonal and perpendicular.

Give your explanation of all possible definitions for the square.

Scoring Rubric

Table 2. *Vignette A*

	Responds	Score
-	No answer	0
-	Wrong answer	0
-	Uneconomic / inaccurate answer (states necessary and sufficient terms but exaggerates the definition of a square)	0.5
-	Economical / accurate answer (states the necessary and sufficient terms with a minimum definition of a square).	1

Table 3. Vignette B

Responds	Score
- No answer	0
- Inaccurate answer (stating the reason the condition is necessary and the condition is sufficient definition of a square inaccurately)	0.5
- Inaccurate answers (stating the reasons for the necessary and insufficient definition of the square inaccurately)	1
- Accurate answer (states the reason for the necessary and sufficient definition of the square accurately)	

RESULTS AND DISCUSSION

Specialized content knowledge of the First Teacher

Vignette A

The following is the description of Specialized content Knowledge (PCK) on the square concept by the First Teacher.

Table 4. *SCK* of the First Teacher

<i>SCK</i>	%	Category
A rectangle with all sides the same length.	9.09	Economical/accurate
Rectangle with adjacent sides the same length.	9.09	
A rectangle whose four sides are the same length and the size of the four corners is 90 degrees.	33.33	Not economical /Inaccuarte
A rhombus whose four corners are right.	12.12	
A rectangle having all sides the same length and having four right angles.	12.12	
A rhombus with all the same angles.	6.06	
A two-dimensional plane formed by four equal sides, four equal angles (90 degrees) and two equal diagonals.	0.00	Incorrect
A quadrilateral where all sides are the same length and all the angles are equal.	9.09	No Response
A rectangle formed by four equal sides and four right angles.	9.09	
A quadrilateral that has four equal vertices and all four equal sides.	0.00	
No answer.	0.00	

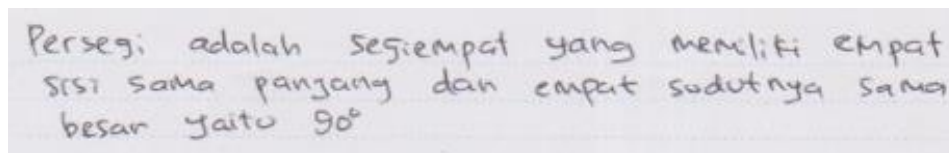
**Figure 4.** Definition of square by GP

Figure 4 shows the definition of a square proposed by the First Teacher, a square is a rectangle that has four equal sides and four equal angles, namely 90° . The definition put forward by GP is classified as uneconomical because the condition needs to be 'quadrilateral' not the closest genus; the sufficiently exaggerated condition 'has four equal sides and four equal angles, namely 90° '. Following are the results of the complete interview data with the First Teacher.

Table 5. Interview transcript

Excerpt of interview	
P:	Does your definition guarantee the definition of a square?
GP:	Yes, the definition guarantees the definition of a square.
P:	Try to explain, what is considered when defining a square?
GP:	The sides of the square must be the same length. Then all four corners are right. Actually the characters are the same as a rectangle. Only if earlier this rectangle has a parallel character with a right angle. A square is like that, only the difference is that a square must have four equal sides besides the right angles.
P:	Try to come up with a different definition of a square?
GP:	A square is a rectangle that has four equal sides. A square is a rhombus whose four corners are right.

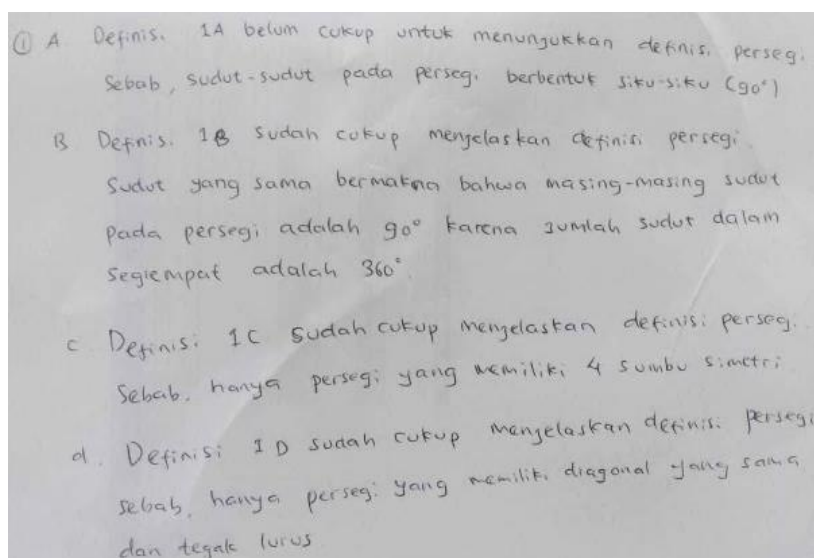
Vignette B**Figure 5.** GP's response on the definition of square

Figure 5 shows the First Teacher's response to vignette B.

Table 6. Interview transcript

Excerpt of interview	
P:	How do you respond to the definitions A, B, C and D given in question no.1.
GP:	If point 1A is not enough to show that the square only has equal sides. But it must have the same angle as well. The consequence is that the angles are equal, the angles must be 90 degrees. So this is still the same as 1B. Then if the 1C statement and 1D statement, it is the effect because the sides are the same length and the angles are right angles to form the 1C and 1D statements.

Specialized Content Knowledge of Young Teacher

The following is a description of the Specialized content Knowledge (SCK) of the square concept by Young Teachers.

Table 7. SCK of the young teacher

SCK	%	Category
A rectangle with all sides the same length.	3.03	Economical/accurate
Rectangle with adjacent sides the same length.	6.06	
A rectangle whose four sides are the same length and the size of the four corners is 90 degrees.	9.09	Not economical /Inaccuarte
A rhombus whose four corners are right.	18.18	
A rectangle having all sides the same length and having four right angles.	21.21	
A diamond with all the same angles.	15.15	
A two-dimensional plane formed by four equal sides, four equal angles (90 degrees) and two equal diagonals.	0.00	Incorrect
A rectangle that has 4 parallel and equal sides, each of which has angles of 90° .	9.09	No Response
A quadrilateral where all sides are the same length and all the angles are equal.	6.06	
A rectangle formed by four equal sides and four right angles.	6.06	
A quadrilateral that has four equal vertices and all four equal sides.	6.06	
No answer.	0.00	

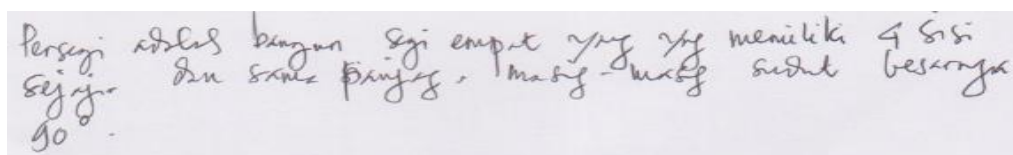


Figure 6. Definition of square by the young teacher

Figure 6 shows the definition of a square put forward by GM is a rectangular shape that has four parallel sides and the same length of each angle of 90 degrees. The definition put forward by GM is classified as uneconomical, because it states that the requirement for 'rectangular shape' is excessive and the sufficient condition 'has four parallel sides and the same length for each angle of 90 degrees' is excessive. Following are the results of interview data with GM.

Table 8. Interview transcripts

Excerpt of interview

P: Try to explain, what is considered when defining a square?

GM: The condition is that the size of each side is the same then the angle formed is also 90 degrees.

P: Try to come up with a different definition of a square?

GM: The definition of a square is like that.

Vignette B

The SCK-C of GM of a given definition

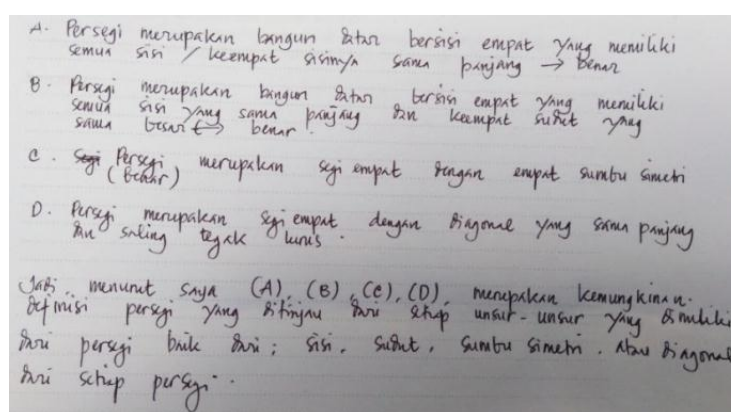
**Figure 6.** GM's response

Figure 6 shows GM's responses on vignette B.

Table 9. Interview transcript

Interview Transcript

P: How do you respond to the definitions A, B, C and D given in question no.1.

GM: The definition I put forward earlier is general or simple, but that definition can still be developed if we look at the elements we have. As for the definition I put forward earlier when viewed from the side and the angle. But if viewed from the axis of symmetry and diagonals, square shapes are also fulfilled. Like this part A (a flat shape with four sides) it is true that it belongs to a rectangular shape. Then part B (four sides have all the same sides and the same angles). Indeed, the sides and angles are the same. Then C (a quadrilateral that has four axes of symmetry), that's right. Then D (rectangle with the same diagonal and perpendicular) yes this is also true. There is indeed a square shape. So in my opinion there are several definitions of the square if we don't look at the elements it has.

Specialized content knowledge of Middle Teacher

The following is the *Specialized content Knowledge* (SCK) of the concept of square by the Intermediate teacher.

Table 10. SCK by intermediate teacher

SCK	%	Category
A rectangle with all sides the same length.	3.03	Economical/accurate
Rectangle with adjacent sides the same length.	3.03	
A rectangle whose four sides are the same length and the size of the four corners is 90 degrees.	3.03	Not economical /Inaccuarte
A rhombus whose four corners are right.	3.03	
A rectangle having all sides the same length and having four right angles.	12.5	
A diamond with all the same angles.	18.75	
A two-dimensional plane formed by four equal sides, four equal angles (90 degrees) and two equal diagonals.	12.5	Incorrect
A build that has 4 equal sides and 900 corners.		No Response
A cascade where all sides are the same length and all the angles are equal.	18.75 12.5	
A rectangle formed by four equal sides and four right angles.		
A quadrilateral that has four equal vertices and all four equal sides.	0.00 0.00	
No answer.	0.00	

bangun datar yg memiliki 4 sisi yang sama panjang dan keempat sudutnya 90°

Figure 8. Definition of square by GY

Figure 7 shows the definition of a square put forward by GY, namely a flat shape that has four equal sides and four equal angles. The definition of a square put forward by GY is classified as uneconomical, because the condition needs to be 'flat' is not the closest genus and the sufficient condition 'has four equal sides and four equal angles' is excessive.

Table 11. Interview Data

Excerpt of interview	
P:	What do you know about the definition of squares?
GY:	A square is a shape that has four equal sides and four equal angles.
P:	Try to come up with a different definition of square?
GY:	I guess nothing else.

Vignette B

The SCK of GY with a given definition

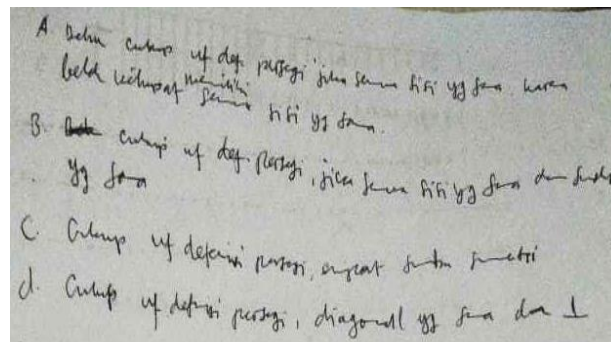
**Figure 9.** GY's responses

Figure 9 indicates the GY's responses on vignette B.

Table 12. Interview data

Excerpt of interview	
P:	How do you respond to the definitions A, B, C and D given in question no.1. The definition of A is not sufficient for the definition of a square. If only all sides were the same the rhombus would be like that too. If the definition of B is also not enough for the
GY:	definition of a square in my opinion, try adding the angles there. Then this definition of C can be for squares. Because only a square has four axes of symmetry. So enough for the definition of a square. Then part D is also sufficient for the definition of a square.
P:	Why is the usual definition of sides and angles while the axes of symmetry and diagonals are rarely used?
GY:	Just plainly like that. Because of this, if a real square looks like it has four sides and the four corners are right. I don't use the axes of symmetry and diagonals because I think the sides and angles are enough for me.

The three levels of teachers in this study are the First Teacher Subject, the Young Teacher Subject and the Intermediate Teacher Subject in defining a square through vignette using an analytic definition. Analytic definitions are definitions by mentioning the genus proximum and specific *diferensia* (Govender & De Villiers, 2002; Tiro, 2010; Akib, 2015). The different genus of proximum mentioned include: flat shapes, squares, lanes, rectangles, and rhombuses. Meanwhile, *diferensia spesifica* is the information behind the word 'yang'.

The results of this study also reveal that teachers are more familiar (often) using analytic definitions rather than genetic definitions and definitions with formulas. Genetic definition is a definition which expresses the occurrence or the way the defined concept is formed. Definitions with formulas, namely definitions expressed in mathematical sentences / in the form of a formula (Akib, 2015).

The First Teacher

Descriptively, the answers to the First Teacher Subject, 18.18% were classified as economical / accurate; 63.63% are classified as uneconomical / inaccurate and 18.18% are classified as wrong. This shows that the level of First Teacher is largely uneconomical / inaccurate in defining rectangles.

The subject of the First Teacher in defining a square through interviews shows accurate side attributes. But when given an example of the definition of a square with side attributes, the First Teacher did not accurately provide an argument against the definition. This is because his experience in classroom learning is more dominant in only referring to the teacher's textbook. Whereas in the applicable textbook / lesson (SMP curriculum) in Indonesia, the definition of a square generally uses the side and corner attributes.

The subject of the First Teacher was accurate when asked to define a square, but inaccurate when given the definition of a square with side and angle attributes. This inaccuracy is caused by understanding the attribute of the angle that the four angles understand as right angles. This is in line with the results of research by Haj-Yahya, A., & Hershkowitz, R. (2013) that "presentation of verbal definitions is given for a parallelogram based on side and angle properties but without a visual support".

The Subject Teacher First is accurate when given a definition of a square with the axis of symmetry attribute but not used when asked to define it. When the Subject Teacher First was asked to define a square with the axis attribute of symmetry it did not appear. This is because teachers do not usually use the definition of a square with the attribute of the axis of symmetry in teaching but the teacher understands the feature of a square related to the axis of symmetry. So that when given a definition, the teacher can give the argument accurately. This is in line with the results of research by Heinze, A., & Ossietzky, C. (2002), "Those based on diagonal and symmetry properties cannot accurately be interpreted without going through an analytical thinking process".

The subject of the First Teacher was accurate when given the definition of a square with the diagonal attribute but was not used when asked to define it. When the Subject Teacher First was asked to define a square with diagonal attributes it did not appear. This is because the teacher does not usually use the definition of a square with diagonal attributes but the teacher understands the characteristics of a square related to the diagonal. This is in line with the research results of Kose, N. Y., Yilmaz, T. Y., Yesil, D., & Yildirim, D. (2018).

Based on vignette data from the subject, the First Teacher explains that a square is a rectangle that has four equal sides and four equal angles, namely 90^0 . This definition is not economical because the expression 'and the four corners are equal, namely 90^0 is redundant. The word "quadrilateral" can use the word "rectangle" as the correct genus.

The Young Teacher

Descriptively, the answers of the Young Teacher Subjects, 9.09% were classified as economical / accurate; 72.72% were classified as uneconomic / inaccurate and 18.18% were classified as wrong. This shows that the Junior Teacher ladder is largely uneconomical / inaccurate in defining the quadrilateral.

The subject of the Young Teacher in defining the square through the interview showed no appearing / unused side attributes. But when given an example of a square definition with side attributes, the Subject of the Young Teacher did not accurately provide an argument against the definition because experience in classroom teaching was more dominant in referring to the teacher's textbook. In the applicable textbook / lesson (SMP curriculum) in Indonesia, the definition of a square generally uses the attributes of sides and corners.

The Young Teacher subject was accurate when asked to define a square but not accurate when given the definition of a square with the side and angle attributes. This inaccuracy is caused by understanding the attribute of the angle that the four angles understand as right angles.

The Young Teacher subject was accurate when given the definition of a square with the axis of symmetry attribute but not used when asked to define it. When the Young Teacher Subject was asked to define a square with the axis attribute of symmetry it did not appear. This is because teachers do not usually use the definition of a square with the attribute of the axis of symmetry in teaching but the teacher understands the feature of a square related to the axis of symmetry. So that when given a definition, the teacher can give the argument accurately.

The Young Teacher subject was accurate when given the definition of a square with the diagonal attribute but was not used when asked to define it. When the Young Teacher Subject was asked to define a square with the diagonal attribute it did not appear. This is because the teacher does not usually use the definition of a square with diagonal attributes but the teacher understands the characteristics of a square related to the diagonal.

Based on the vignette data from the Young Teacher Subject, it was explained that a square is a rectangular shape that has 4 parallel and equal sides, each of which is 900. This definition is not economical because the phrase '4 sides are parallel and the length of each corner is 900'. redundant. The word 'wake up' also doesn't need to appear anymore but it is enough to use the word 'quadrilateral'.

The Intermediate Teacher

Descriptively, the answers of the Middle Teacher Subjects, 6.06% were classified as economical / accurate; 68.56% classified as uneconomic / inaccurate and 12.5% classified as wrong. This indicates that the Intermediate Teacher level is largely uneconomical / inaccurate in defining rectangles.

The subject of the Intermediate Teacher in defining the square through the interview showed that there were no side attributes. But when given an example of a square definition with side attributes, the subject of the Intermediate Teacher did not accurately provide an argument against the definition because experience in classroom teaching was more dominant in referring to the teacher's textbook. In the applicable textbook / lesson (junior high school curriculum) in Indonesia, the definition of a square generally uses the attributes of sides and corners.

The Intermediate Teacher subject was accurate when asked to define a square but inaccurate when given the definition of a square with the side and angle attributes. This inaccuracy is caused by understanding the attribute of the angle that the four angles understand as right angles.

The Intermediate Teacher subject was accurate when given the definition of a square with the axis of symmetry attribute but not used when asked to define it. When the Intermediate teacher is asked to define a square with the axis attribute of symmetry not appearing. This is because teachers do not usually use the definition of a square with the attribute of the axis of symmetry in teaching but the teacher understands the feature of a square related to the axis of symmetry. So that when given a definition, the teacher can give the argument accurately.

The Intermediate Teacher subject was accurate when given the definition of a square with the diagonal attribute but was not used when asked to define it. When the Intermediate Teacher Subject is asked to define a square with the diagonal attribute it does not appear. This is because the teacher does not usually use the definition of a square with diagonal attributes but the teacher understands the characteristics of a square related to the diagonal.

Based on the vignette data from the subject, the Intermediate Teacher explains that a square is a flat shape that has 4 equal sides and 90^0 four angles. This definition is not economical because the phrase '4 equal sides and four 90^0 angles' is redundant. The word 'flat shape' also does not need to appear anymore but simply use the word 'quadrilateral'.

The results of this study indicate that teacher level is not dominant in the specialized content

knowledge of the square concept. This is influenced by the conceptual analysis of the definition of the square. For this reason, the teacher needs to explore the relationship between the concept of the quadrilateral with the prerequisite concepts and concept examples. So that it can help strengthen teacher understanding in teaching square concepts in the classroom.

CONCLUSION

First Teachers, Young Teachers and Intermediate Teachers define a square through analytic definitions. The definition of a square put forward by the First Teacher, Young Teacher, and Intermediate Teacher is uneconomical. The First Teacher was influenced by concept reconstruction. The First Teacher was inaccurate with side attributes; side and corner attributes; accurate when given the definition of a square with the attributes of symmetry and diagonal axes but does not appear / is not used when asked to define a square. The Young teachers are influenced by the concept image. Young Teachers related inaccurate side attributes; not accurate with side and angle attributes; accurate with the attributes of the axes of symmetry and diagonals but does not appear / is not used when asked to define a square. The Intermediate teacher is influenced by the concept image. The Intermediate Teacher is regarded to perform inaccurate side attributes; not accurate when with side and angle attributes; accurate with the attributes of the axes of symmetry and diagonals but does not appear / is not used when asked to define a square.

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REFERENCES

- Akib, I. (2016). *Implementation of Robert Gagne's Learning Theory in Learning Mathematical Concepts (An Alternative to Teaching and Learning Mathematics Concepts)* [in Bahasa]. Makassar: PT. Berkah Utami. ISBN: 978-602-8187-54-1.
- Anderson, L. W. & Krathwohl, D. R. (2001). *Framework for Learning, Teaching and Assessment* [in Bahasa]. Alih Bahasa: Agung Prihantoro. Yogyakarta: Pustaka Pelajar.
- Ball, D. L., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education* 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>.
- Buchbinder, O., & Zaslavsky, O. (2019). Strengths and inconsistencies in students' understanding of the roles of examples in proving. *The Journal of Mathematical Behavior*, 53, 129-147. <https://doi.org/10.1016/j.jmathb.2018.06.010>.

- Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., ... & Ribeiro, M. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236-253. <https://doi.org/10.1080/14794802.2018.1479981>.
- Demirok, M. S., & Baglama, B. (2018). Examining technological and pedagogical content knowledge of special education teachers based on various variables. *TEM Journal*, 7(3), 507-512. <https://doi.org/10.18421/TEM73-06>
- Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). "Mathematical knowledge for teaching": Adapting US measures for use in Ireland. *Journal of Mathematics Teacher Education*, 11(3), 171-197.
- De Villiers, M.D.. (1998). To teach definitions in geometry or to teach to define? In A. Olivier, & K. Newstead (Eds.). *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 248–255).
- De Villiers, M.D.. (2009). *To Teach Definitions in Geometry or Teach to Define?*. Retrieved from www.researchgate.net/publication/255605686
- De Villiers, M.D. (1994). The role and function of a hierarchical classification of the quadrilaterals. *For the Learning of Mathematics*, 14(1), 11-18.
- Fernández-León, A., Gavilán-Izquierdo, J. M., González-Regaña, A. J., Martín-Molina, V., & Toscano, R. (2019). Identifying routines in the discourse of undergraduate students when defining. *Mathematics Education Research Journal*, 1-19.
- Fuadiah, N. F., & Suryadi, D. (2017). Some Difficulties in Understanding Negative Numbers Faced by Students: A Qualitative Study Applied at Secondary Schools in Indonesia. *International Education Studies*, 10(1), 24-38. <https://doi.org/10.5539/ies.v10n1p24>.
- Garner, B. K. (2011). *Getting to. Got It!"—Helping Struggling Students Learn How to Learn*, 1. ISBN-13: 978-1416606086.
- Govender & De Villiers. (2002). *Constructive Evaluation of Definitions in a Sketchpad Context*. AMESA Durban South Africa.
- Haj-Yahya, A., & Hershkowitz, R. (2013). When visual and verbal representations meet the case of geometrical figures. *Proceedings of PME* (vol. 37, pp. 409-416).
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371– 406. <https://doi.org/10.3102%2F00028312042002371>.
- Heinze, A., & Ossietzky, C. (2002). "...Because a square is not a rectangle" students' knowledge of simple geometrical concepts when starting to learn proof. In A. Cockburn & E. Nardi (Eds.), *Proceedings of The 26th Conference of the International Group for the Psychology of Mathematics Education* (vol. 3, pp. 81-88).
- Herbst, P., & Kosko, K. (2012). *Mathematical Knowledge for Teaching High School Geometry*. North American Chapter of the International Group for the Psychology of Mathematics Education. Springer: Research Trends in Mathematics Teacher Education (pp.23-46). https://doi.org/10.1007/978-3-319-02562-9_2.

- Isiksal, M. & Cakiroglu, E. (2011). The nature of prospective mathematics teachers' pedagogical content knowledge: The case of multiplication of fractions. *Journal Math Teacher Education*, 14(3), 213-230. <https://doi.org/10.1007/s10857-010-9160-x>.
- Knapp, A., Bommer, M., & Moore, C. (2008). Lesson study as a learning environment for mathematics coaches. *Proceedings of the 32nd International conference for the Psychology of Mathematics Education* (Vol. 3, pp. 257-263).
- Kose, N. Y., Yilmaz, T. Y., Yesil, D., & Yildirim, D. (2018). Middle school students' interpretation of definitions of the parallelogram family: Which definition for which parallelogram?. *International Journal of Research in Education and Science*, 5(1), 157-175.
- Levenson, E. Tirosh, D., & Tsamir, P. (2011). *Preschool Geometry. Theory, Research, and Practical Perspectives*. Rotterdam: Sense Publishers.
- Paksu, Asuman Duatepe., Pakmak, Gul Sinem., & Iymen, Esra. (2012). Preservice elementary teachers' identification of necessary and sufficient conditions for a rhombus. *Procedia Social and Behavioral Science*, 46(2012), 3249-3253. <https://doi.org/10.1016/j.sbspro.2012.06.045>.
- Purnomo, D. (2019). Characteristics of students' metacognition process in solving calculus problems [in Bahasa]. *Paradigma: Jurnal Filsafat, Sains, Teknologi, dan Sosial Budaya*, 25(1), 1-15. <https://doi.org/10.33503/paradigma.v25i1.477>.
- Prahmana, R.C.I., Hendrik, Sopaheluwakan, A, van Groesen, B. (2008). Numerical Implementation of Linear AB-Equation Model using Finite Element Method, Technical Report. Bandung: LabMath-Indonesia
- Prahmana, R.C.I. (2012). Designing of Number Operation Learning Using the Traditional Game of "Tepuk Bergambar" for Three-Grade Students of Elementary School [in Bahasa]. Unpublished Thesis. Palembang: Sriwijaya University.
- Rosken & Rolka. (2007) *The Role of Concept Image and Concept Definition for Student's Learning Integral Calculus*. The Montana Mathematics Enthusiast 3.
- Sahidin, L., Budiarto, M. T., & Fuad, Y. (2019). Developing vignettes to assess mathematical knowledge for teaching based conceptual. *International Journal of Instruction*, 12(3), 551-564.
- Soedjadi, R. (2000). Tips for Mathematics Education in Indonesia (constrain the present situation towards the hope in the future) [in Bahasa]. Jakarta: Directorate General of Higher Education, Ministry of National Education.
- Stronge, J. H. (2018). *Qualities of effective teachers*. ASCD.
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational studies in mathematics*, 12(2), 151-169.
- Tiro, M. A. (2010). *Effective Ways To Learn Mathematics* [in Bahasa]. Makassar: Andira Karya Mandiri.
- Türnüklü, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers (IUMPS): The Journal*, 1, 1-13.

- Van Dormolen, J., & Zaslavsky, O. (2003). The many facets of a definition: The case of periodicity. *Journal of Mathematical Behavior*, 22(1), 91–106. [https://doi.org/10.1016/S0732-3123\(03\)00006-3](https://doi.org/10.1016/S0732-3123(03)00006-3).
- Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. O. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 65–81). Dordrecht: Kluwer.
- Vinner, S. (2020). "Concept development in mathematics education." *Encyclopedia of Mathematics Education*, 123-127.
- Winicky-Landman, G., & Leikin, R. (2000). On equivalent and nonequivalent definitions I. *For the Learning of Mathematics*, 20(1), 17–21.
- Yeo, K. K. J., (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceeding of the 31th Annual Conference of the Mathematics Education Research Group of Australasia. Merga* (pp.621-627).
- Zaslavsky, O., & Shir, K. (2005). Students' conceptions of a mathematical definition. *Journal for Research in Mathematics Education*, 36(4), 317–346. <https://doi.org/10.2307/30035043>.
- Zandieh & Rasmussen, (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *Journal of Mathematical Behavior*, 29(2), 55-75. <https://doi.org/10.1016/j.jmathb.2010.01.001>.
- Zandieh & Rasmussen. (2004). A dynamic approach to quadrilateral definitions. *Pythagoras*, 58, 34-45.