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Spatial Reasoning Construction: The Way to Use It to Solve Geometric Problems

Lingga Nico Pradana[1], Octarina Hidayatus Sholikhah[2]

[1, 2]Department of Primary Education, Faculty of Teacher Training and Education, Universitas PGRI Madiun, Jl. Setiabudi No.85, Madiun, Indonesia
Email: nicopgsd@unipma.ac.id

Abstract

The aim of this research was to determine the spatial reasoning constructs employed by pre-service elementary school teachers when solving geometry problems. A total of 36 participants were invited to complete an online test, after which two selected individuals were engaged in solving the problems. Interviews were then conducted to accurately describe the process of solving geometric problems. The results showed the existence of two types of spatial reasoning constructions, namely series and parallel. Both construction types revealed the interrelationship between spatial reasoning and the problem-solving process. This research highlighted the significance of spatial reasoning as an integral component in solving geometric problems, emphasizing the need for further investigation into its distinctiveness. This could be achieved by incorporating advanced geometric concepts and materials into future research.

Keywords: Spatial Reasoning Construction, Geometric Problem, Spatial Visualization, Spatial Orientation, Mental Rotation

Abstrak

Penelitian ini bertujuan untuk mengonstruksi penalaran spasial calon guru sekolah dasar dalam menyelesaikan masalah geometri. Sebanyak 36 responden mengikuti tes penalaran spasial secara online. Dua peserta terpilih menjadi subjek penelitian kemudian dieksplorasi menggunakan masalah geometri. Untuk mendapatkan hasil yang akurat, wawancara dilakukan untuk menggambarkan bagaimana proses penyelesaian masalah geometri dilakukan. Hasil penelitian ini menunjukkan bahwa terdapat dua jenis konstruksi penalaran spasial yaitu konstruksi penalaran spasial series dan konstruksi penalaran spasial paralel. Kedua tipe konstruksi penalaran spasial tersebut menunjukkan bahwa aspek penalaran spasial saling terkait dalam proses pemecahan masalah. Penelitian ini menunjukkan bahwa aspek penalaran spasial bekerja dan membentuk konstruk dalam proses penyelesaian masalah geometri. Selanjutnya, sangat penting untuk dapat melihat lebih dalam mengenai penalaran spasial dengan melibatkan konsep dan materi geometri yang lebih tinggi.

Kata kunci: Konstruksi Penalaran Spasial, Masalah Geometri, Visualisasi Spasial, Orientasi Spasial, Rotasi Mental

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INTRODUCTION

Spatial reasoning is the ability to represent and use objects and their relationships in two or three dimensions (Lowrie, Logan, & Ramful, 2017; Ramful, Lowrie, & Logan, 2017; Yüksel, 2017). This cognitive ability encompasses three fundamental aspects, namely spatial visualization, mental rotation, and spatial orientation (Harris, Lowrie, Logan, & Hegarty, 2020; Lowrie, Harris, Logan, & Hegarty, 2019). Spatial visualization is the ability to describe situations based on the provided information (Haciomeroglu, 2016; Lowrie, 2016; Rabab'h & Veloo, 2015). On the other hand, spatial

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orientation involves the ability to position oneself within a given spatial context to obtain a comprehensive view of various situations (Diezmann & Lowrie, 2012; Liao, 2017). Mental rotation refers to the cognitive process of mentally envisioning the altered appearance of 2D and 3D objects after they have been rotated (Lowrie et al., 2017; Vandenberg & Kuse, 1978).

Spatial reasoning is closely related to problem-solving and is often explored through the application of geometry problems (Akayuure, Asiedu-Addo, & Alebna, 2016; Patkin & Fadalon, 2013; Pittalis & Christou, 2010). Despite its significance, the interrelationship among the three aspects of spatial reasoning is rarely discussed, with previous works primarily focusing on each aspect individually. For example, some investigations have concentrated on spatial visualization skills (Kurtulus & Yolcu, 2013), while others have examined the design of geometry-based activities to enhance spatial visualization and mental rotation (Bruce & Hawes, 2015; Cheng & Mix, 2014), and the exploration of spatial visualization process (Lowrie et al., 2017). It is crucial to note that the three aspects of spatial reasoning, namely visualization, mental rotation, and spatial orientation, are commonly examined independently, using distinctive instruments and methodologies tailored to each aspect. This emphasis on individual investigation can be attributed to previous research results. Unfortunately, this approach often overlooks the comprehensive examination of the interrelationships among these aspects, leading to a limited understanding of their interconnectedness. By acknowledging the importance of comprehensively exploring the interplay between visualization, mental rotation, and spatial orientation, the overall understanding of spatial reasoning and its practical applications can be enhanced. From a theoretical perspective, there exists a potential linkage among these three aspects of spatial reasoning.

Spatial reasoning can be examined in terms of its relevance to the elementary school curriculum, with the three aspects aligning with its content (Lowrie et al., 2017). Consequently, numerous research has been conducted to train spatial reasoning skills among elementary school students. Previous research has implemented activities based on spatial reasoning, which has shown the potential in improving the spatial reasoning abilities of students (Bruce & Hawes, 2015; Francis, Khan, & Davis, 2016; Lowrie et al., 2017). The most important factor influencing spatial reasoning is age, hence Yüksel (2017) argued that during adolescence, individuals develop more complex spatial reasoning abilities. This provides an opportunity for further exploration in this domain.

By examining the content of spatial reasoning within the primary school curriculum and considering age factors, there arises an opportunity to explore pre-service primary school teachers. Therefore, these teachers need to incorporate spatial thinking instruction, as it enhances their ability to support the spatial development of students (Williams et al., 2010). Meanwhile, previous works have primarily focused on training students in spatial reasoning skills, it is equally crucial to investigate that of prospective primary school teachers. The urgency of understanding the abilities of these teacher candidates stems from the vital role that spatial reasoning plays in education. Notable, spatial reasoning is crucial for comprehending and effectively teaching concepts related to geometry, visualization, and problem-solving across various academic disciplines (Liao, 2017). By assessing the spatial abilities of teacher candidates, educational institutions can ensure that they possess the necessary skills to support the spatial development of students.

This current research aims to explore the spatial reasoning skills of prospective elementary school teachers. The exploration process primarily focuses on the aspects of spatial reasoning employed by these candidates in solving geometry problems. Furthermore, the research investigates the interrelationship among the three aspects of spatial reasoning and their application in solving geometric problems.

METHODS

Participant

In this research, an online spatial reasoning test was administered to 36 participants before their selection. The participants consisted of pre-service elementary school teachers in East Java with a mean age of 20.37 years and an age range of 19 to 21 years. Among the participants, there were 14 males and 22 females. The purpose of the test was to assess the initial spatial reasoning abilities of the participants, specifically in the aspects of spatial visualization, mental rotation, and spatial orientation. This aided the identification of participants who scored at least 90% in each aspect, and the results were shown in Figure 1.

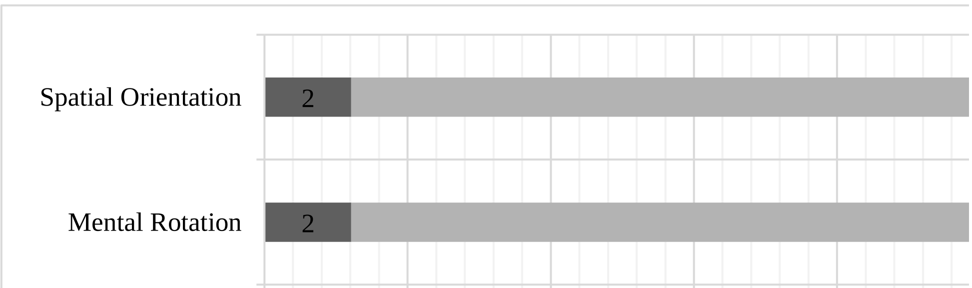

Figure 1. Results of spatial reasoning tests on each aspect

The results shown in Figure 1 showed that 7 data values exceeded 90% for each aspect. Among the participants, 4 individuals achieved scores over 90% in more than one aspect, while 1 participant did not participate. Based on this scenario, 2 participants (S1 female and S2 male) who scored more than 90% in all three aspects were selected for the research. They were then subjected to a spatial reasoning test based on geometric problems.

Materials

Two instruments were employed in this research of which the first was a spatial reasoning test used to evaluate the initial abilities of participants and to select the individuals involved. The second comprised a set of geometric problems designed for the participants to solve using spatial reasoning.

The problems given to the participants were shown in Figure 2.

Draw a cuboid ABCD.EFGH. Then draw an isosceles triangle with the largest area on the side ABFE. Next, draw a square that is the same area and covers the sides of the EFGH. Draw a circle as wide as possible on the side ADHE. Next, draw the nets from the blocks that have been made. Suppose you are in front of the side ABFE, rotate the cuboid clockwise at 900. Draw the cuboid that is the result of the rotation!

Figure 2. Geometric problem

The indicators for every aspect of spatial reasoning, which were outlined in Table 1.

Table 1. Aspects and indicators of spatial reasoning

Aspect	Code	Indicator
Spatial visualization	SV1	Visualizing the results of folding/opening certain configurations
	SV2	Draw the sides of the shape into a flat shape
	SV3	Identify parallel or perpendicular lines in shapes
	SV4	Finds symmetry in an object
Mental rotation	MR1	Determine the result of the object's rotation
Spatial orientation	SO1	Determines the position of an object, relative to the observer
	SO2	Describe various kinds of situations based on information

Research Procedure

The first step of this research was to provide the participants with geometric problems. These problems were sent via email along with instructions for their completion. The responses of the participants were subsequently returned via email. The problem-solving approaches of the participants were analyzed and the spatial reasoning constructs employed by them were identified. These constructs encompassed the sequence of steps used in spatial reasoning and the nature of its process. Furthermore, online interviews were conducted to ensure robust data credibility. By combining the collected data, the spatial reasoning constructs used in solving the geometric problems were delineated.

Data Analysis

Data analysis was performed using interactive analysis techniques (Miles, Huberman, & Saldana, 2014). The analysis process commenced with data condensation, which was carried out during the research. The primary objective was to extract relevant and suitable data for defining spatial reasoning constructs. Subsequently, the data was presented through the creation of a spatial reasoning construction diagram and a comprehensive description of the spatial reasoning process. To examine the relationship between the three aspects of spatial reasoning, the specific aspects were identified based on the work results of the participants. This identification process aimed to determine the emerging aspects and also to observe any discernible patterns exhibited by the subject. These identified aspects served as the foundation for constructing spatial reasoning based on the three identified components. The produced spatial reasoning construction diagram showed the sequential order of spatial reasoning and its subsets. During the creation of the spatial reasoning construction diagram, data verification was conducted by combining the work results of the participants in solving geometric problems with online interviews. Each presented diagram was accompanied by terminology that represented the key findings of this research.

RESULTS AND DISCUSSION

In this research, the terminology of spatial reasoning constructs, its diagrams, and work results in solving geometric problems was introduced. Furthermore, it encompassed the excerpts from the interview transcripts of the two research participants.

S1's Geometry Problem Working Flow

During the process of solving geometry problems, participant S1 began by drawing the cuboid ABCD.EFGH as shown in Figure 3. Subsequently, S1 proceeded to draw the largest isosceles triangle on the side ABFE. The participant was then prompted to justify the process of drawing the triangle.

P : *Is this an isosceles triangle?*
S1 : *Yes*
P : *Why?*
S1 : *I make a midpoint of EF and connecting it to point A and B. So the length is same.*
P : *Should it be from the midpoint of EF?*
S1 : *Yes. Because, if it's not in the middle of EF, the triangle isn't isosceles.*
S1 : *Actually, it can be from the midpoint of AE, AB and FB. But I chose EF because it would have the same result.*
P : *Okay, then why does the triangle have the largest area?*
S1 : *Yes, cause the base and height are maximal.*
P : *Which base and height?*
S1 : *AB is the base. And the height is the same as the length of edge AE.*

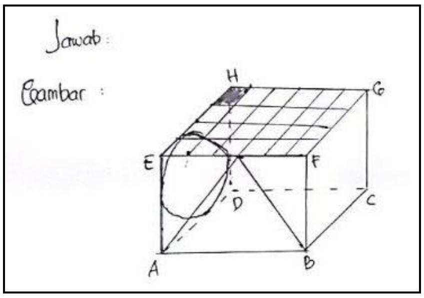

Figure 3. The cuboid drawn by S1

Based on Figure 3 and the conducted interviews, the largest isosceles triangle was drawn by identifying the midpoint on edge EF and connecting it with points A and B. The base of the triangle was defined as AB, and its height was measured along AE, indicating that the triangle had the largest area. This demonstration exemplified the aspects of spatial visualization (SV3 & SV4).

Furthermore, a square that covered the side EFGH was drawn by paying attention to the beam edges. A line parallel to the beam edge on the same side was also drawn, representing it as a square that covered EFHG. The following excerpt from the interview explained the reasoning process.

P : *Is this a square? Looks like it's not a square drawn.*
S1 : *This is square sir. It is slanted because it adjusts the edge. When viewed from above it must look perpendicular.*

Based on Figure 3 and the interview excerpt, the aspect of spatial orientation (SO1) was demonstrated by drawing according to the perspective of the image. Even though the image was not perpendicular, it was still recognized as perpendicular and provided information on how it would appear when viewed from above.

Subsequently, the largest circle on the side ADHE was drawn. In Figure 3, the circle was positioned tangent to the cuboid edge on the side ADHE. It was explained that when drawing the cuboid, both the width and height were equal in length. Consequently, the diameter of the circle matched the length of the edge AD. In the section where cuboid nets were presented, the cuboid nets shown in Figure 4 were created. The figure showcased a cuboid net accompanied by drawings of triangles, squares, and circles. By accurately presenting the cuboid nets, the aspects of spatial visualization (SV1 & SV2) were demonstrated, and then S1 was asked to describe their thought process.

P : *How do you determine the position of triangle, circle, and squares on cuboid nets?*
S1 : *For the triangle, I imagine the sides of the cuboid open up and the triangle will rotate too. Then the position is reversed. If the nets fold again into the cuboid, the triangle will return to its original position.*
S1 : *To determine the position of squares and circles it is quite easy. Because both of them will stay on. It's just that the squares are made straight lines.*

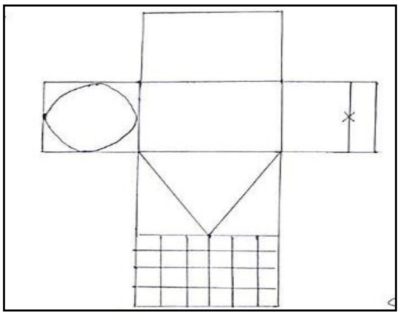

Figure 4. Cuboid nets by S1

By attentively observing the interview, S1 demonstrated the spatial visualization aspect and incorporated the mental rotation aspect (MR1) by rotating the triangle. Therefore, both aspects of spatial reasoning were involved simultaneously.

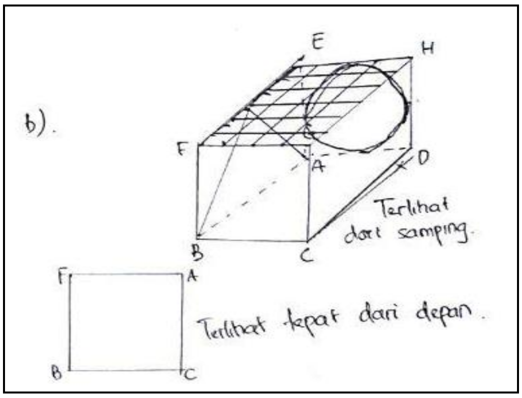

Figure 5. Rotation Results by S1

The final step in solving the problem required a 900-clockwise rotation, which was shown in Figure 5. This rotation was carried out by turning ABFE to the left and positioning the BCGF side in front. S1 successfully portrayed the rotation results and presented two different views in Figure 5, showcasing aspects of mental rotation (MR1) and spatial orientation (SO2).

After analyzing the performance of S1, the spatial reasoning construction shown in Figure 6 became apparent. The spatial reasoning was carried out sequentially to solve geometric problems. The key aspects of this process included spatial visualization, mental rotation, and spatial orientation. Among them, spatial visualization and orientation were the most frequently utilized aspects. This implied that the construction of spatial reasoning followed a linear sequence, with the three aspects being employed consecutively and without branching.

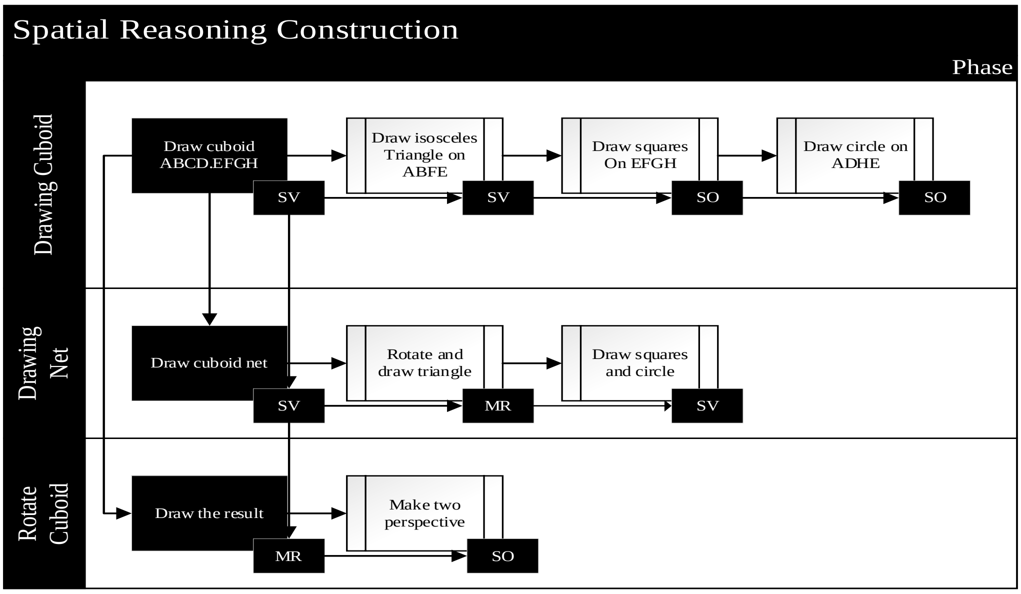

Figure 6. Spatial reasoning construct of S1

S2's Geometry Problem Working Flow

In solving geometry problems, S2 employed a distinctive approach in drawing the cuboid, which appeared cube-like, as shown in Figure 7. To verify this assertion, an inquiry was made about the method employed by S2 in drawing the cuboid.

S2 : *I chose to draw a cube because all the cubes are cuboid. Then it will make easier for me to take the next step.*

P : *Then how do you know that the triangle you make is the largest isosceles triangle?*

S2 : *Because ABFE is square. Therefore, AB and FB must be the same, so I immediately pulled the AF diagonal. So, the triangle is AFB. Since it's half the area of a square, yes it must be the largest triangle.*

P : *Then how to draw these squares?*

S2 : *I just need to draw on the EFGH side. I adjusted the number of lines with the size of side EFGH.*

P : *How about the circle? How long the diameter?*

S2 : *The diameter corresponds to the length of the cuboid edge. Since the side adhe is square, the diameter must be the same as the edge of the cuboid.*

P : *What happens if the diameter is greater than the edge of the cuboid or less than the edge of the cuboid?*

S2 : *If it is more than the edge then the circle will come out of the side ADHE. If it is less than, then the area is smaller. Does not the largest area.*

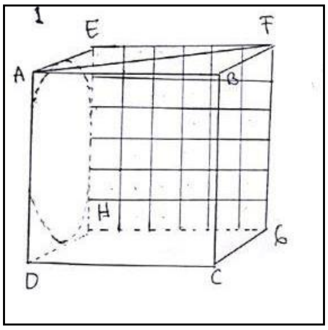

Figure 7. The cuboid drawn by S2

From the interview, S2 drew a cube and acknowledged that all the cubes were cuboids. Furthermore, the largest isosceles triangle was drawn on the side ABFE and its legs were represented by edges AB and FB, while AF served as the base. It was affirmed that the triangle was the largest because its area was half of the square (side ABFE). This indicated that the spatial visualization aspect (SV4) was shown by S2. In order to create squares on the side EFGH, perpendicular lines were drawn. This action was taken because, when viewed from the front, the side EFGH in Figure 7 was upright, showing the aspect of spatial orientation (SO1). Moreover, S2 understood how to determine the circle with the largest area in the circular image. The participants explained that, for the circle to be maximized, its diameter should align with the side of the cube. In case the diameter exceeded the length of the edge on the side ADHE, the circle would be beyond ADHE. On the other hand, if the diameter was smaller than the edge, the circle would not have the largest area. With this statement, S2 demonstrated the aspect of spatial orientation (SO1).

When creating cuboid nets (shown in Figure 8), S2 began by drawing one of the known cuboid nets. Subsequently, triangles, squares, and circles were drawn on these nets. A unique approach that facilitated the drawing of the three components in the block was employed. An inquiry was further made regarding the thought of the respondents in implementing the technique.

P : *How do you make a cuboid net?*
S2 : *The way that I use was easy. I imagine side ADHE open and followed by ABCD and also BCGF. After that, just open CDHG down and ABFE up.*
P : *How about the position of triangle, circle, and squares?*
S2 : *These squares not going anywhere. Because when I rotate ADHE and the EFGH doesn't change. The circle doesn't change its shape. For the triangle, I just adjusted it because ABFE was lifted upwards, so the position was reversed.*

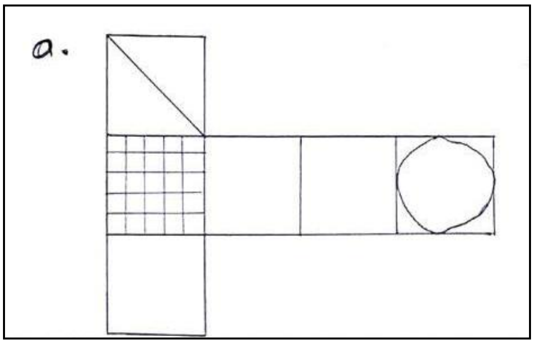

Figure 8. Cuboid nets by S2

Based on Figure 8 and the interview, S2 rotated ADHE until it was parallel to EFGH, leading to four parallel sides on the net. DCGH was then flipped downwards and ABFE upwards. This particular instance highlighted the ability of S2 to visualize spatial relationships (SV1) when drawing the net. In this process, the respondents imagined the rotation of ADHE until it became parallel to the EFHG, demonstrating the utilization of mental rotation (MR1) while constructing the net. Moreover, when drawing circles and triangles, adjustments were made to the shape and position of the ADHE and ABFE sides before sketching the circle and triangle on the respective side. This showcased the spatial visualization skills of S2 (SV2).

In the cuboid rotation segment, S2 employed a distinctive approach. These respondents recorded three different rotation results, differing in the selected clockwise direction. The rotation direction was determined by referencing the circular line shown in Figure 9. An inquiry was made about how these respondents accomplished these three rotation results.

P : *You have three cuboids as result of the rotation. How can you think that it will be more than one result?*
S2 : *First I positioned myself in front of ABFE. Then I felt the meaning of clockwise was broad. So, I imagined the position of the clock itself. First, the position of the clock is facing up, then I draw the rotation result. The two watch positions face right. And the three clock positions facing me.*
P : *So, you find three kinds of rotation in this case.*
S2 : *Actually, it could be more, because the position of the clock can vary. Unless the position has been determined, the result will be only one.*

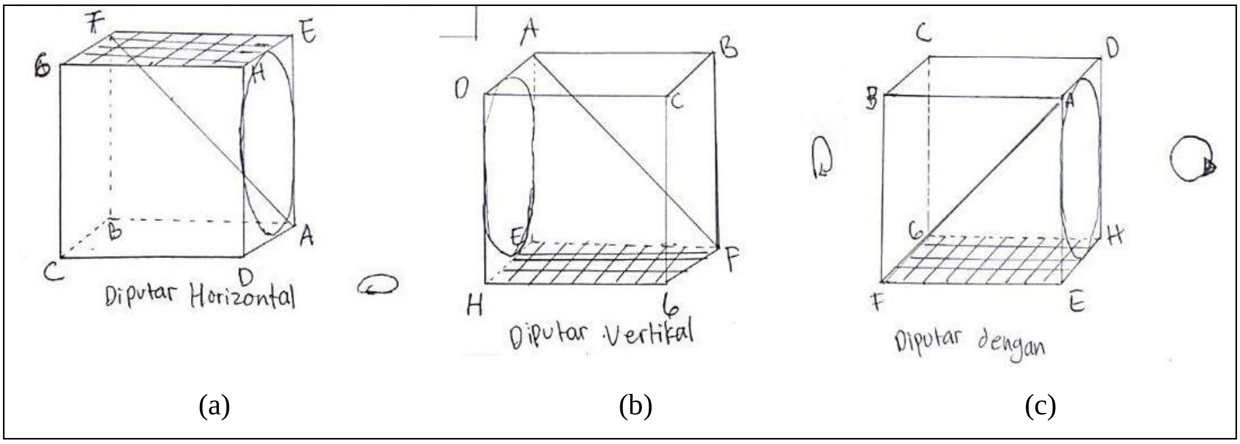

Figure 9. Rotation results by S2

According to Figure 9 and the interview, S2 exhibited a broader perspective beyond the immediate problem. Various potential conditions were presented based on the position of the clock, which determined the rotational conditions (clockwise). These respondents demonstrated a spatial orientation aspect (SO2). Additionally, three rotations were performed based on specified rotational conditions (clockwise), effectively showcasing the aspect of mental rotation (MR1).

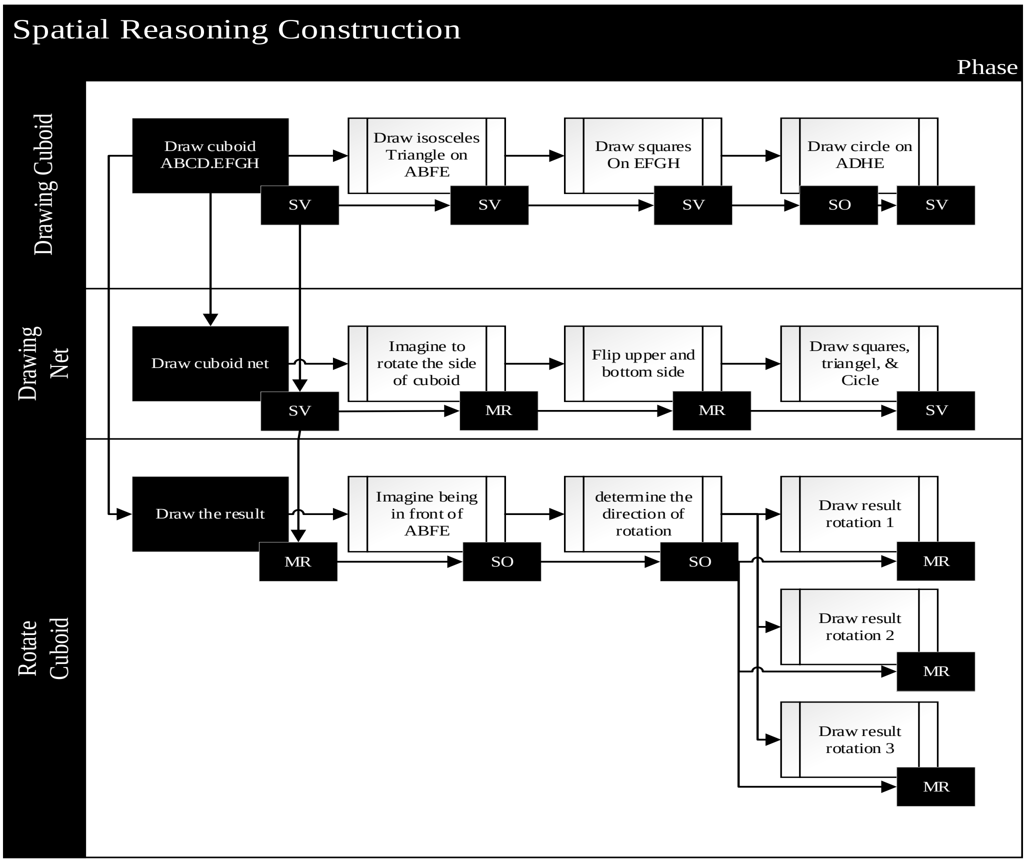

Figure 10. Spatial reasoning construct of S2

Figure 10 showed the construction of spatial reasoning based on the performance of S2. The figure showed that spatial reasoning comprised three aspects, namely spatial visualization, mental rotation, and spatial orientation. Within the cuboid rotating section, the respondents demonstrated spatial orientation and exhibited three types of mental rotation, and also acknowledged the existence of additional rotations, indicating that engaging in spatial orientation opened up numerous possibilities for mental rotation and led to various kinds of spatial visualization. Therefore, the construction of spatial reasoning occurred simultaneously, since spatial orientation triggered the emergence of various mental rotation options and spatial visualization.

Spatial reasoning played an essential role in solving geometric problems (Pittalis & Christou, 2010). This research exemplified how spatial reasoning served as a bridge to solve such problems. Through spatial reasoning, multiple representations were generated, aligning with the definition of spatial reasoning, as the ability to represent and utilize objects and their relationships in two or three dimensions (Bruce & Hawes, 2015; Joh, 2016; Lowrie et al., 2017). This phenomenon led to an emerged creative problem-solving (Gero, 2015), thereby raising the question of whether problems were resolved by selecting a single option or by exploring all available options.

A distinctive operationalization of the spatial reasoning aspect was demonstrated, and it eventually functioned as sequential steps in solving geometric problems. This scenario set the current research apart from the previous ones that primarily focused on measuring individual aspects of spatial reasoning (Bruce & Hawes, 2015; Harris et al., 2020; Hartatiana, Darhim, & Nurlaelah, 2017). This research showed the inherent unity among the three aspects of problem-solving.

Previously, the process of solving geometry problems by demonstrating spatial reasoning had been carried out. However, recent research focused on specific problems related to each aspect, such as the spatial visualization process when arranging cubes (Kurtulus & Yolcu, 2013; Wulandari, Sa'Dijah, Irawan, & Sulandra, 2019). Mental rotation training involved the use of a rotation test in evaluating this phenomenon (Akayuure et al., 2016; Gold, Pendergast, Ormand, Budd, & Mueller, 2018; Hawes, Moss, Caswell, & Poliszczuk, 2015). The difference in this scenario was that the present research employed geometry problems to explore all aspects of spatial reasoning.

Furthermore, this research identified and analyzed spatial reasoning constructs based on different aspects. There were two types of spatial reasoning constructs, namely series and parallel. The series involved the systematic use of sequential aspects of spatial reasoning. The position of the spatial orientation served as a guiding principle for determining rotations and visualizations. Meanwhile, the parallel involved the utilization of more complex aspects of spatial reasoning, allowing for a comprehensive of geometry problems. Solving these problems yielded diverse results, as spatial orientation provided various perspectives on the situation and acquired relevant information. In this case, the presentation of geometric problem-solving was not limited to a single approach.

The spatial reasoning constructs obtained from this research enabled pre-service elementary school teachers to enhance their thinking by examining problems from different viewpoints. This approach fostered creativity in problem-solving, in line with the previous research findings (Gero, 2015; Schoevers, Kroesbergen, & Leseman, 2019). By solving geometry problems, pre-service elementary school teachers were able to practice and improve their spatial reasoning skills. Consequently, teachers who possessed series and parallel skills could effectively integrate spatial reasoning into their teaching practice or curriculum (Bruce, Sinclair, Moss, Hawes, & Caswell, 2015; Mulligan, Woolcott, Mitchelmore, & Davis, 2017).

The influence of gender on spatial reasoning had also been investigated. While some research stated that there were no significant gender differences (Gilligan, Flouri, & Farran, 2017; Lowrie et al., 2017; Patkin & Fadalon, 2013; Pradana, Sa'dijah, Sulandra, Sudirman, & Sholikhah, 2020), the present work suggested subtle differences in construction between men and women. The sample size was not extensive, hence, the need for further exploration of this issue.

CONCLUSION

In conclusion, the construction of spatial reasoning was based on emerging aspects of spatial reasoning and the patterns that arose through sequences and repetitions. This research showed that when solving geometric problems, aspects of spatial reasoning functioned in distinct ways. This distinctiveness was manifested through two constructions of spatial reasoning, namely series and parallel. The series construction involved utilizing aspects of spatial reasoning in order to present a single type of representation. Meanwhile, the parallel involved utilizing aspects that led to multiple representations. An important point in this regard was that the spatial orientation aspect played a crucial role in enabling various visualizations and rotations.

Considering the identified constructs of spatial reasoning, the utilization of geometric problems in exploring spatial reasoning was essential. Consequently, the creativity of pre-service elementary school teachers could develop alongside their spatial reasoning abilities. This approach served as a natural training method for acquiring proficient spatial reasoning skills. The discovery of spatial reasoning constructions marked the beginning of comprehending the model in terms of its aspects. This research necessitated the involvement of more pre-service elementary school teachers to further enhance the understanding of the uniqueness of the capability. The geometric content used was only in the aspect of basic education, hence the need to expand its scope.

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