Problem-solving Approach and Its Impact on Creative Thinking Ability of Prospective Mathematics Teachers

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Abstract

This study aims to investigate the implementation of a problem-solving approach and its corresponding impact on the creative thinking ability of prospective mathematics teachers. A qualitative case study approach was used in this study in the form of observations of learning and teaching processes for geometry topics through the use of a problem-solving approach and of a written test involving 20 prospective mathematics teachers, in one of the state universities in Bandung, Indonesia. The results showed that the implemented problem-solving approach influenced prospective mathematics teachers' creative thinking in solving problems. The use of different strategies showed prospective teachers' creative thinking ability in problem-solving processes. The effect of the problem-solving approach on prospective mathematics teachers can be investigated further to obtain a more comprehensive understanding of creative thinking ability.

Keywords: Creative Thinking Ability, Problem-solving Approach, Geometry Education, Prospective Mathematics Teachers

Abstrak

Penelitian ini bertujuan untuk menginvestigasi penerapan pendekatan pemecahan masalah dan dampaknya terhadap kemampuan berpikir kreatif mahasiswa calon guru matematika. Pendekatan kualitatif dengan desain studi kasus digunakan dalam penelitian ini dalam bentuk observasi proses pembelajaran topik geometri menggunakan pendekatan pemecahan masalah dan tes tertulis melibatkan 20 mahasiswa calon guru metamatika, di salah satu universitas negeri di kota Bandung, Indonesia. Hasil penelitian menunjukkan bahwa pendekatan pemecahan masalah yang diterapkan telah mempengaruhi kemampuan berpikir kreatif calon guru matematika dalam proses pemecahan masalah. Penggunaan beragam strategi pemecahan masalah menunjukkan kemampuan berpikir kreatif para calon guru matematika dalam proses pemecahan masalah**.** Selanjutnya, dampak dari penggunaan pendekatan pemecahan masalah terhadap kemampuan pemecahan masalah calon guru matematika dapat diinvestigasi lebih lanjut untuk memperoleh pemahaman yang lebih lengkap mengenai kemampuan berpikir kreatif.

Kata kunci: Kemampuan Berpikir Kreatif, Pendekatan Pemecahan Masalah, Pendidikan Geometri, Calon Guru Matematika

INTRODUCTION

Problem-solving is one of the abilities needed by everyone living in the 21st century, including prospective mathematics teachers, as a provision for their careers in the future (Jamaludin & Hung, 2017; Schoenfeld, 2013). For prospective teachers, this ability can be developed through various courses in the teacher education program, for instance, number theory, algebra, and geometry courses (Jupri, Gozali, & Usdiyana, 2020). It also can be developed through courses implemented using problem-solving approaches (Schoenfeld, 2016). The use of problem-solving approaches for the

learning and teaching processes in the teacher education program, however, is still limited (Barham, 2020). The implementation of a problem-solving approach in the learning and teaching processes, including for prospective mathematics teachers, can positively affect the ability in reasoning and critical and creative thinking (Birgili, 2015; Leikin & Pitta-Pantazi, 2013). These abilities are indispensable for them in their future careers as mathematics teachers who will face complex problems of the 21st century (Jamaludin & Hung, 2017).

Previous research on problem-solving abilities for prospective mathematics teachers shows various results. Capraro, An, Ma, Rangel-Chavez, and Harbaugh (2012) found that prospective mathematics teachers encountered difficulties in applying reasoning skills in the process of problem-solving open-ended problems. Cansoy and Türkoglu (2017) showed that there is a positive correlation between the problem-solving and critical thinking abilities of prospective mathematics teachers. Other studies on problem-solving, for the case of geometry topics in the Indonesian context, show that prospective mathematics teachers encountered difficulties in transforming visual-geometrical problems into symbolic mathematical models (Jupri, 2017), in problem-solving for the topics of similarity of triangles (Yuwono, 2016), and in proving geometry problems (Jupri & Syaodih, 2016).

Considering the above results, we wonder how the learning and teaching processes on problem-solving or on the use of the problem-solving approach for prospective mathematics teachers in Indonesia in the current Covid-19 Pandemic situation are implemented. Also, we wonder how the teaching and learning processes affect prospective mathematics teachers' mathematical thinking, particularly their creative thinking ability which is indispensable for problem-solving processes (Silver, 1997). Therefore, this study aims to investigate the implementation of a problem-solving approach and its corresponding impact on the creative thinking ability of prospective mathematics teachers.

To investigate the learning and teaching processes of problem-solving, we propose to use Polya's problem-solving model as a framework. According to Polya (1973), there are four steps in a problem-solving process, consisting of understanding the problem, devising a plan, carrying out the plan, and looking back. In understanding the problem, we need to determine the unknown, the data, and the condition of the problem. In devising a plan, we need to connect the known data and the unknown and prepare problem-solving strategies. In carrying out the plan, we have to solve the problem using problem-solving strategies prepared in the previous step. Finally, in the step of looking back, we need to reflect on whether the solution makes sense, whether the problem can be solved using different problem-solving strategies, and whether the problem-solving process can be generalized for solving other relevant problems.

To study the creative thinking ability of prospective mathematics teachers, we propose to use four dimensions of creativity as a framework. The dimensions include flexibility, fluency, originality, and elaboration (Almeida, Prieto, Ferrando, Oliveira, & Ferrándiz, 2008; Dhayanti, Johar, & Zubainur, 2018). Flexibility refers to the ability to produce different mathematical ideas or different

solution strategies. Fluency concerns the ability to produce many solutions. Originality concerns the ability to produce a novel idea. Finally, elaboration means the ability to develop and describe ideas comprehensively.

METHODS

To reach the aim of this study, namely investigating the teaching and learning processes through a problem-solving approach for prospective mathematics teachers, a qualitative case study approach in the form of self-online classroom observations was carried out. The observations within the Mathematical Thinking Process course, taught by the first author, addressed geometry topics included two parts. The first part included an observation of the learning and teaching processes implemented using the problem-solving approach involving 20 students in a mathematics education program—as prospective mathematics teachers—in one of the state universities in Bandung, Indonesia. The teaching and learning processes lasted for 2 x 100 minutes (two meetings). The second part included an individual written online test on solving a geometry problem, administered after the two meetings, and lasted for 30 minutes. The problem used for the test has a problem-solving character as it requires students to use their repertoire of knowledge of school geometry and the solution procedure is not straightforward (Levav-Waynberg & Leikin, 2012a; 2012b). To prevent cheating activities, we require students to activate their zoom cameras during the test. Also, the written work of the students from the test must be uploaded and stored in a Google Classroom before the time of the test is up.

Data collected in this study included video-recorded meetings (via Cloud Zoom Meetings), lecture notes in the form of power-point presentation slides and corresponding additional notes on the slides, students' written work stored in the Google Classroom, and field notes. Data analysis was conducted as follows. First, the learning and teaching processes using the problem-solving approach were analyzed by using Polya's framework of problem-solving. Second, students' written work from the test was analyzed using the framework of the four creativity dimensions.

RESULTS AND DISCUSSION

This section describes the results of two parts of classroom observations: The learning and teaching processes and the results of a written test. These results are analyzed using the frameworks of the problem-solving approach and the four dimensions of creativity.

Learning and Teaching Processes for Prospective Mathematics Teachers

The topic of mathematics addressed in the learning and teaching processes concerns applying various sub-topics of plane geometry in solving a geometry problem. To address this, the lecturer

started the meeting by posing a geometry problem shown in Figure 1. For solving the problem, students were suggested to use the four steps of a problem-solving process of Polya's model (Polya, 1973). The lecturer informed the students what to do in each step of the problem-solving process. Students were given time for about 10 minutes to solve the problem.

Find the area of the triangle *ABC*!
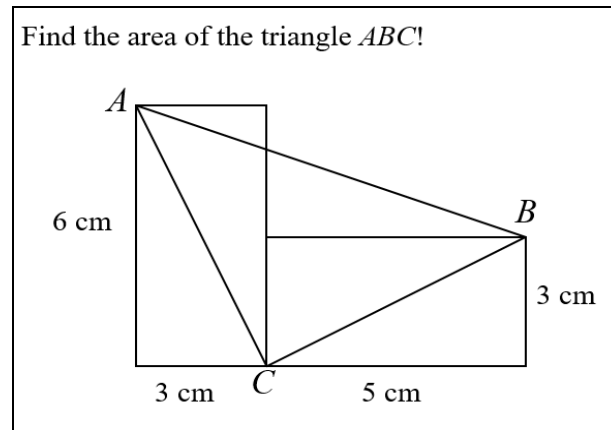
Figure 1. A geometry problem posed by the lecturer

Initially, one of the students proposed to solve the problem by applying the Pythagoras theorem for finding the lengths of *AC* and *BC*, next using the area formula of a right triangle to calculate the area of the triangle *ABC*. However, when the lecturer requested the student to verify whether the triangle *ABC* is a right triangle or not, the student realized that the triangle is not a right triangle. Therefore, he continued working to find appropriate strategies for solving the problem.

After 10 minutes, a student solved the problem correctly and proposed his solution as shown in Figure 2 (only the steps of devising a plan and carrying out the plan are rewritten here for this article). The student hereafter is named Student1. Student1 explained that in the step of understanding the problem the unknown is the area of the triangle *ABC*, and the known data includes the lengths and the widths of the rectangles *AMCL* and *BKCN*. The steps of devising a plan and carrying out are shown in Figure 2. The looking back step was carried out by verifying whether each previous step is correct. Overall, Student1 can apply the four steps of Polya's problem-solving model correctly. By analyzing the solution of Student1, we see that he can view the figure within the problem from a different perspective, i.e., he sees *BKLA* as a trapezium with the parallel sides *AL* and *BK* and the altitude is *LK*. The ability to see the given figure from a different perspective, from the perspective of creativity, can be recognized as having flexibility in a problem-solving process (Almeida et al., 2008; Cenberci, 2018; Mrayyan, 2016). As the solution is unique and unusual, Student1 can also be recognized as having originality in the problem-solving process (Almeida et al., 2008; Mrayyan, 2016).
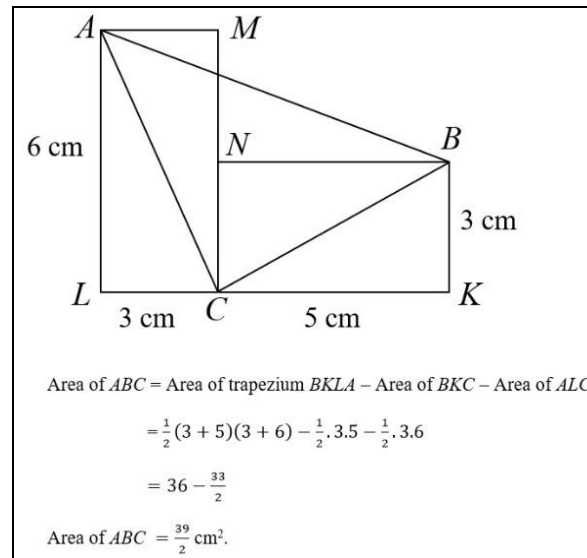
Figure 2. A proposed solution to the geometry problem by Student1

In commenting the Student1's work, which is in line with other studies (Dickman, 2016; Leong, Toh, Tay, Quek, & Dindyal, 2012; Taback, 1988; Tjoe, 2019), the lecturer explained that the looking back step can also be done by solving the problem using different strategies. Therefore, he encouraged students to find different solution strategies by providing additional time to work on the problem. After sufficient time was given, two different solution strategies from two students (Student2 and Student3) emerged and are rewritten (where the steps of understanding the problem and looking back are not rewritten) in Figure 3.
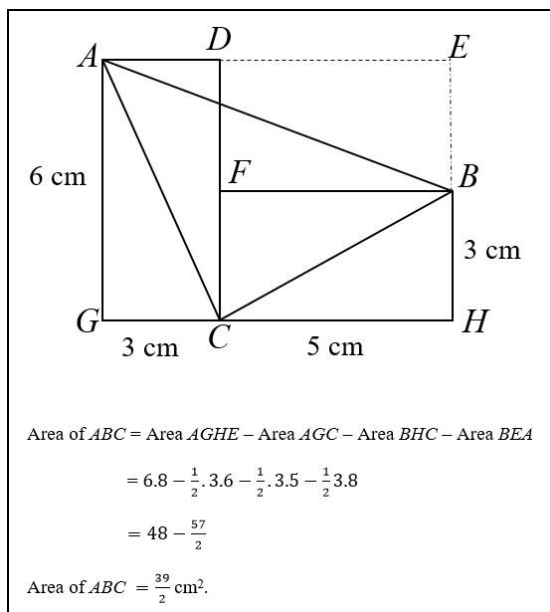
(a)
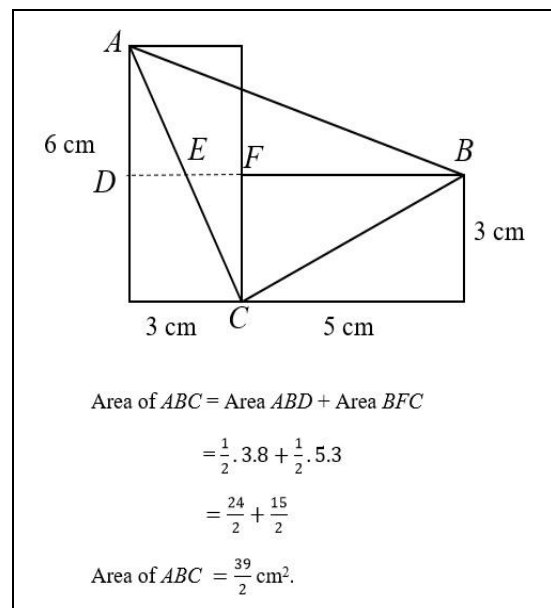
(b)

Figure 3. Two different solution strategies: (a) Solution from Student 2; (b) Solution from Student 3

Figure 3(a) is presented by Student2 and Figure 3(b) is presented by Student3. Both solution strategies show flexibility. To a certain extent, particularly for the solution in Figure 3(a), it shows having an originality character (Almeida et al., 2008; Cenberci, 2018; Mrayyan, 2016). The ability to construct a rectangle *AEHG*, namely constructing lines *DE* and *BE*, shows flexibility in the sense of seeing the problem from a different perspective, and shows originality in the sense of constructing new lines to form a new plane figure (Koichu & Leron, 2015; Palatnik & Dreyfus, 2019). After this discussion, the learning and teaching processes were continued in the second meeting.
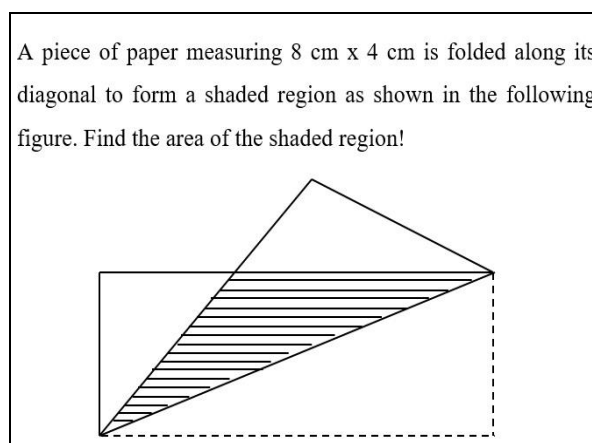
In the second meeting, the lecturer discussed the solution strategy presented in Figure 3(b). For this case, Student3 constructs a line segment *DF* such that the triangle *EFC* is congruent to the triangle *EDA*. Therefore, the area of these two triangles is the same. Before calculating the area of *ABC,* Student3 should explain why the triangle *EFC* is congruent to the triangle *EDA*. The ability to construct a new appropriate line to construct two congruent triangles concerns the dimension of originality (Palatnik & Dreyfus, 2019).

From the above description, the lecturer has guided mathematics education students using Polya's problem-solving model by providing sufficient time for students to find their solution strategies. It can be seen that this guidance has influenced students to find different strategies which can be characterized as developing creative thinking ability.

To assess student ability in the process of problem-solving of geometry problems, the lecturer administered a formative individual written assessment test. The results of this assessment are addressed in the next subsection.

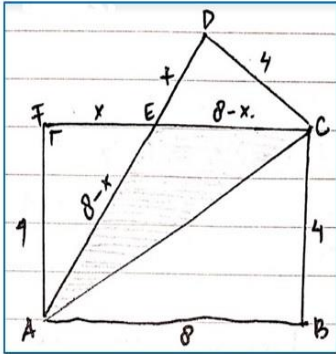### Analysis of Written Test on Solving a Problem
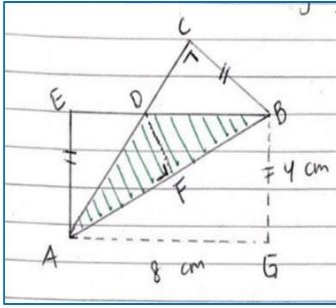
The written test required students to solve a geometry problem and lasted for 30 minutes in the second meeting. The problem for the test is presented in Figure 4.

A piece of paper measuring 8 cm x 4 cm is folded along its diagonal to form a shaded region as shown in the following figure. Find the area of the shaded region!

Figure 4. A geometry problem for a written test

From the written work of the 20 students of mathematics education as prospective mathematics teachers, 18 written work showed correct solutions. The other two written works provide incorrect solutions. In addition, two main different solution strategies were found: First, using geometry concepts; and second, using trigonometry concepts. Table 1 presents a brief version of two solution strategies (the texts have been re-written, and the figures are authentic from student work) and the corresponding number of students in each type of solution strategy.

Table 1. Two different solution strategies from the written work

Types of solution strategies	Number of students (Using the strategy)
Strategy I	
It can be shown that $\triangle AFE \cong \triangle CDE$. Let $EF = ED = x$, so $AE = CE = 8 - x$. Using Pythagoras, we obtain $(8 - x)^2 = x^2 + 4^2$. Therefore, $x = 3$ cm. Area $AEC = \frac{1}{2}.EC.AF = \frac{1}{2}.5.4 = 10$ cm².	14
Strategy II	
Let $\angle ABE = \alpha$. $\text{Tan}\, \alpha = \frac{FD}{BF} = \frac{AE}{EB} = \frac{4}{8} = \frac{BC}{AC} = \frac{DF}{AF}$. Therefore, $AF = BF = \frac{1}{2}AB = 2\sqrt{5}$. Then we obtain $DF = \frac{1}{2}BF = \sqrt{5}$. Area $ABD = \frac{1}{2}.AB.DF = \frac{1}{2}.2\sqrt{5}.\sqrt{5} = 10$ cm².	4

Strategy I is carried out by applying concepts of congruence of two triangles, the Pythagoras theorem, and the area of a triangle. The ability to see and show, for instance, that the triangle *AFE* is congruent to the triangle *CDE* can be considered that participating students are able to see a different view of the given figure in the problem. This indicates that they are showing flexibility in the problem-solving process (Almeida et al., 2008; Levav-Waynberg & Leikin, 2012a; 2012b).

Strategy II, used by only four prospective teachers, is carried out by applying concepts of trigonometry, the Pythagoras theorem, and the area of a triangle. In this case, students should see that two triangles are similar, so the tangent concept can be used (see Table 1). Similar to the previous

strategy, the ability of students to see and show, for instance, that the triangle *BAE* is similar to the triangle *BDF* can be considered that they can see the figure from a different perspective, and as such show flexibility in the problem-solving process (Levav-Waynberg & Leikin, 2012a; 2012b). Either Strategy I or Strategy II, if it is new to students, then the use of this strategy can be considered to show originality in the problem-solving process (Almeida et al., 2008; Gridos, Avgerinos, Mamona-Downs, & Vlachou, 2022; Palatnik & Dreyfus, 2019).

From the perspective of originality, in general, by considering different solution strategies that emerged from students' written work, the mathematics education students as prospective mathematics teachers seem to have shown creativity, particularly in the dimension of flexibility and to a certain extent in originality. This finding seems to be a direct effect of the learning and teaching processes in which the lecturer provided an opportunity to students to find different strategies for solving a problem.

CONCLUSION

Based on the description in the previous section, two conclusions can be drawn. First, the observed learning and teaching processes for prospective mathematics teachers on geometry topics emphasize the use of a problem-solving approach. The sequence of learning and teaching starts from providing a problem to providing an explanation of the problem-solving model as a general guideline for solving the problem, providing appropriate time for students to solve the problem individually, conducting classroom discussions, and drawing conclusions about a problem-solving process. The use of this approach is intended, inter-alia, for improving students' creative thinking ability. In practice, the problem-solving approach is implemented by providing an opportunity for students to do an investigative activity through the encouragement of finding different solution strategies for solving a problem. Considering this, for further research, it is worth investigating the impact of the use of the problem-solving approach toward a more comprehensive mathematical thinking process, including reasoning, critical and creative thinking.

Second, the findings of the use of different solution strategies in solving a problem indicate that mathematics education students have used their creative thinking ability in the problem-solving processes, particularly the use of flexibility and originality. This also indicates the qualitative impact of the use of the problem-solving approach on creative thinking ability. Other dimensions of creativity, including fluency and elaboration, seem not to be revealed yet in this study probably because the problems used in this study are not suitable enough to investigate these dimensions. Therefore, for further study, the four dimensions of creativity need to be studied so that a more comprehensive creative thinking ability of prospective mathematics teachers can be revealed. The results of the investigation of the effect of the problem-solving approach on creative thinking ability

can be used, for instance, by relevant stakeholders to prepare prospective mathematics teachers for the future.

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