Exploration of Creative Mathematical Reasoning in Solving Geometric Problems

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Abstract

Reasoning that is constructed from remembering is imitative reasoning, while the opposite is creative reasoning. This study aims to explore creative mathematical reasoning in solving geometric problems. Mathematical creative reasoning is reasoning that contains elements of novelty, plausibility, and mathematical foundation. This type of research is descriptive qualitative, which is explorative. The research subjects were the first-semester student in the mathematics education study program with 32 students. The results showed that from 32 students, there was only one student identified as having creative mathematical reasoning in solving geometry problems. Creative mathematical reasoning can be identified when the subject is able to reason algorithmically but is aware of problems so they cannot be resolved algorithmically so that they must form new reasoning, which consists of novelty, plausibility, and mathematical foundation. Creative mathematical reasoning arises after students make an algorithmic reasoning process, but find no solution. Novelty is the weakest indicator of creative mathematical reasoning, so it requires scaffolding to bring it up.

Keywords: Creative Mathematical Reasoning, Problem Solving, Geometry

Abstrak

Penalaran yang dikonstruksi dari mengingat adalah penalaran imitative sedangkan lawannya adalah penalaran kreatif. Penelitian ini bertujuan untuk mengidentifikasi dan mendeskripsikan penalaran kreatif matematis dalam pemecahan masalah geometri. Penalaran kreatif matematis adalah penalaran yang mengandung unsure kebaruan (*novelty*), masuk akal (*plausibility*) dan berdasar sifat-siafat matematis (*mathematical foundation*). Jenis penelitian adalah deskriptif kualitatif yang bersifat eksploratif. Subjek penelitian adalah mahasiswa semester pertama di program studi pendidikan matematika sejumlah 32 mahasiswa. Hasil penelitian menunjukkan bahwa dari 32 mahasiswa hanya ada 1 mahasiswa yang teridentifikasi memiliki penalaran kreatif matematis dalam memecahkan masalah geometri. Penalaran matematis secara kreatif bisa diidentifikasi ketika mahasiswa mampu bernalar secara imitative, yaitu algoritmis namun menyadari adanya permasalahan sehingga secara algoritmis tidak bisa diselesaikan sehingga harus membentuk penalaran baru. Penalaran yang terbentukmengandung novelty, masukakal (*plausible*) dan berdasarkan matematis (*mathematical foundation*) sehingga memenuhi penalaran kreatif matematis. Kesimpulan dalam penelitian ini adalah sebagian besar mahasiswa belum memiliki penalaran matematis yang kreatif, yaitu penalaran yang memenuhi *novelty*, *plausibility* dan *mathematical foundation*. Penalaran matematis kreatif muncul setelah mahasiswa melakukan proses penalaran algoritmis, namun tidak menemukan solusi. *Novelty* adalah indikator penalaran kreatif matematis yang paling lemah sehingga memerlukan *scaffolding* untuk memunculkannya.

Kata kunci: Penalaran Kreatif Matematis, Pemecahan Masalah, Geometri

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INTRODUCTION

Reasoning ability is important to be developed for students, especially prospective mathematics teacher students. Many things are related to one's reasoning abilities, which will later be used in life. Someone will need the ability to distinguish things that are appropriate and not appropriate, important, or not important when facing a problem in life. Besides, the ability to estimate something based on

available facts is also needed. Some of these abilities are a reflection of reasoning abilities. For mathematics education students who will later become teachers, the ability to reason is the main provision to teach mathematics to students.

The reasoning or logical thinking means thinking according to the correct rules and having a systematic, valid, and accountable nature. The reasoning process is a cognitive process in obtaining a conclusion that is following the correct rules or agreed-upon truth and can be accepted as truth that can be accounted for a problem faced. The importance of this reasoning ability so that it becomes the goal of school mathematics learning (Permendikbud, 2016). Reasoning skills can be developed in the learning process. The reasoning is the line of thought, which is adopted to produce assertions and reach conclusions(Lithner, 2008; Norqvist, Jonsson, Lithner, Qwillbard, & Holm, 2019). The reasoning is a line of thought, a way of thinking adopted to produce statements and obtain conclusions. The reasoning describes how someone's thinking about things or problems faced.

Reasoning and proof are important aspects of mathematics (NCTM, 2000). The reasoning is a person's way of thinking or thinking, while proof is the result of that thought (Lithner, 2008). Reasoning and proofing abilities can be developed through geometry courses. Geometry is a branch of mathematics whose applications are commonly found in mathematics and other sciences. Geometry requires students to have good and creative reasoning abilities (Jane, 2006; Lawson & Chinnappan, 2000). Someone who has a good understanding of ability will easily understand a concept and connect concepts creatively to reach a solution when dealing with problems. In geometry, students must be able to justify answers to the problems given (Verner, Massarwe, & Bshouty, 2019). It will show how students think about the problems faced.

Regarding the reasoning ability of students, referring to the results of TIMSS in 2011 and 2015 shows that in the cognitive domain the percentage of reasoning abilities of Indonesian students is still meager, namely 17% in TIMSS in 2011 and 20% in TIMSS in 2015 (Mullis, Martin, & Foy, 2015). This result is the lowest average percentage achieved by Indonesian students in mathematics. Also, several studies have shown that students' reasoning abilities are still low (Darta & Saputra, 2018; Octriana, Putri, & Nurjannah, 2019; Rohana & Ningsih, 2019; Siregar, 2018; Sukirwan, Darhim, & Herman, 2018). The low reasoning ability of students is one of which is determined by teaching activities by the teacher in the classroom, thus required teachers and prospective mathematics teachers. They can develop the reasoning of students in Indonesia.

At the level of higher education for prospective mathematics teacher students, one of the problems faced is the problem in geometry courses. The problem of geometry is a matter of geometry that requires solving by students, and the process of solving it does not yet have a clear procedure. Usually, geometric problems at an advanced level are problems in the form of non-routine problems that lead to proof, namely problems that require students to prove a matter based on known information.

Several studies have shown that students have unsatisfactory abilities in solving proof problems

(Maria Reiss, Heinze, Alexander, & Groß, 2008; Masfingatin, Murtafiah, & Krisdiana, 2017; Sentosa, 2013). Similar results were also revealed by Nursyahidah, Saputro & Prayito (2016), which states that a small proportion of students already have the ability to build evidence but are still in the stages of developing. It shows that the need for more attention so that students can solve the problem of proof well. Students do a series of reasoning processes so they can arrange a proof.

In reality, there are still often problems in the field. Students cannot make connections between statements with one another, unable to explain and check the validity of arguments in the proof of the theorem, so they cannot arrange proof of geometry theorem properly (Masfingatin, Murtafiah, & Krisdiana, 2018). It indicates that students have not been able to reason correctly. The reasoning is a process of linking some information that has been known and is being faced with finding conclusions.

The reasoning is also interpreted as the thought of someone who is realized in the form of a statement so that it reaches a conclusion in the solution of the task. This reasoning process does not have to be based on formal logic, so it is not limited to evidence, and may even be wrong as long as there are several types of reasons that make sense (for reasons) to support it (Lithner, 2008). The assignment in question is the student's work on geometry problems. The solution is the answer and motivation of why the answer is right. This solution often does not display the actual reason used to reach the answer, but rather an ideal summary. The problem referred to in this case, is a task or problem that is intellectually difficult for an individual(Lithner, 2008; Norqvist et al., 2019; Schoenfeld, 1985).

There are two types of mathematical reasoning, that is imitative reasoning and creative mathematical reasoning (Adawiyah, Muin, & Khairunnisa, 2017; Birkeland, 2019; Lithner, 2008). Imitative reasoning is a process of concluding a problem obtained from recalling prior knowledge. Imitative reasoning for an algorithm is called reasoning algorithmic (Lithner, 2008). Mathematical creative reasoning is a process of reasoning that is different from the structure of reasoning that a person already has (Bergqvist & Lithner, 2012; Olsson, 2017). Mathematical creative reasoning has a higher level of imitative reasoning and can make students better at mastering non-routine problems that are high-level (Boesen, Lithner, & Palm, 2010). Students will adjust new information with the existing cognitive structures to be adjusted when faced with a problem. If this adjustment process has in common, then the problem will arrive at the solution. However, if not, a more complex relationship must be sought so that the structure of knowledge possessed needs to be modified so that it is following new information. If this happens, and until completion, students may have reasoned creatively (Jonsson, Kulaksiz, & Lithner, 2016).

Some research on creative mathematical reasoning has been done (Bergqvist & Lithner, 2012; Jonsson, Kulaksiz, & Lithner, 2016; Hidayat, 2017; Hidayat, Herdiman, Aripin, Yuliani, & Maya, 2018; Hidayat, Wahyudin, & Prabawanto, 2018; Sukirwan, Darhim, & Herman, 2018). Bergqvist & Lithner (2012) have researched about creative mathematical reasoning at the presentation of teacher task completion. Jonsson, Kulaksiz, & Lithner (2016) states that learning mathematics using creative

mathematical reasoning and building students' solution methods is more effective. Hidayat, Herdiman, Aripin, Yuliani, & Maya (2018) states that learning mathematics using creative mathematical reasoning and building students' solution methods is more effective. Hidayat (2017) and Hidayat, Wahyudin, & Prabawanto (2018) researching about increasing students' creative mathematical reasoning abilities based on adversity quotient and inquiry learning. Sukirwan, Darhim, & Herman (2018) researching the quality of students' mathematical reasoning, according to Lithner (2008), where the quality of student reasoning still tends to lead to imitative reasoning, which is the process of concluding using routine procedures when solving a problem.

The results of research related to creative mathematical reasoning indicate that there are no researchers who have revealed more deeply about creative mathematical reasoning. Thus, this study aims to describe the creative mathematical reasoning of prospective mathematics teacher students in solving geometry problems. Reasoning creative mathematical student will be described based on indicators of reasoning by Lithner (2008) and Boesen, Lithner, and Palm (2010) which includes: (a) novelty (novelty), namely the formation of a sequence of reasoning that new or sequences are forgotten reappear, (b) makes sense (plausibility), when there are arguments in favor of the chosen settlement strategy, or implementation of the chosen strategy, supporting that the conclusion chosen is true or reasonable, and (c) is based on mathematical (mathematical foundation), namely arguments that are delivered based on mathematical properties.
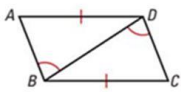
Research that reveals more in-depth about creative mathematical reasoning is important because it is beneficial for lecturers or education practitioners. Through this research, a mathematical description of students' creative reasoning can be known, which can be used to design learning to develop student creative reasoning. This research can be used as information for lecturers in determining the appropriate treatment for students to improve the quality of the process and learning outcomes of geometry, especially in solving problems related to proof. Also, the results of this study can be used as a reference for improving the design and implementation of learning in geometry courses.

METHODS

This research is descriptive qualitative research, which is explorative because the researcher wants to describe the creative mathematical reasoning of students in solving geometric problems. The research subjects were selected from 32 first semester students of the Mathematics Education Study Program of the Universitas PGRI Madiun Academic Year 2018/2019 who were given geometric problems related to proof. Based on the work of selected students, one student whose written work meets the mathematical creative reasoning criteria: flexibility, plausibility, and mathematical foundation.

Data collection techniques using test and interview methods. Reasoning tests are in the form of questions essay, which are non-routine questions about triangle congruence. The questions used as research instruments are somewhat different from the concepts students have learned and are rarely found in reference books (Jäder, 2019; Norqvist et al., 2019). This is intended to capture the creative reasoning of students. Test tasks that don't share important properties with the textbook mostly elicited creative mathematically founded reasoning (Boesen, Lithner, & Palm, 2010). Also, the test questions used are also exploratory. Problem number 1 is a question that requires students to reason creatively. The CMR approach can be seen as an alternative to enhance learning through students' own mathematical reasoning (Norqvist et al., 2019). Problem number 2 uses an imitative approach that is algorithmic, which is a problem that is algorithmically the same as what students encounter in the reference book or lecturer explanation (Jonsson, Norqvist, Liljekvist, & Lithner, 2014; Norqvist et al., 2019). The following is a matter of mathematical reasoning tests in this study.

1. Determine whether the information that is known is enough to prove that the folowing two triangles, are $\triangle ABD$ and $\triangle CDB$ congruent? Give an explanation.

2. An isosceles triangle ABC, with AB as the base side. If point D is located at AB. so CD is the angle bisector of $\angle ACB$. Explain why the $\triangle ADC \cong \triangle BDC$?

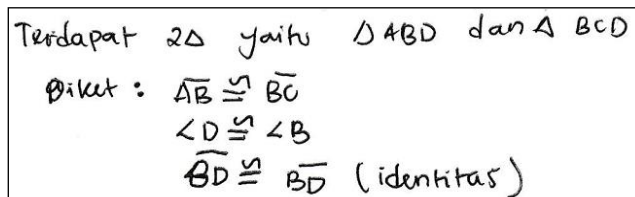**Figure 1**. The instrument used in this study

Test results were analyzed based on mathematical reasoning indicators. Test results that meet the criteria for creative mathematical reasoning are then conducted interviews based on test results to find out more about mathematical reasoning based on mathematical creative reasoning indicators, according to Lithner (2008). Data analysis uses reduction, presentation, and verification. The data of the written test results and the subsequent interviews were tested for validity by triangulation of techniques, which combines the results of the test data essay and interviews based on the test results.

RESULTS AND DISCUSSION

The research began with the provision of Essay tests to 32 first semester students of the Mathematics Education Study Program at the University of PGRI Madiun Academic Year 2018/2019. The results of the essay test show that there is only one student who has a different and almost complete answer. Therefore, the subject of this study was that one student, namely a male student with the initials NS. It shows that the mathematical reasoning ability of prospective math teacher students is still low (i.e., 3.125%), which can solve the given reasoning problems.

Result of Question Number 1

NS subjects begin by writing down what is known, namely: there are two triangles, namely ΔABD and ΔBCD. Also, NS writes a pair of congruent sides, a pair of congruent angles, and another pair of congruent sides of the two triangles. The written results of NS subjects are seen in Figure 2.

Translation

There are 2 triangle, are ΔABD and ΔBCD
Determine: $\overline{AB} \cong \overline{BC}$
                $\angle D \cong \angle B$
                $\overline{BD} \cong \overline{BD}$ (identity)

Figure 2. NS writes important information from the problem

Based on these written results, the researcher interviewed with NS to clarify the results of the written work.

P   : *What is known from the problem based on the picture?*
S1 (1): *Based on the picture, there are two triangles, namely, ΔABD ≅ ΔCDB, $\overline{AD} \cong \overline{BC}$, ∠1 ≅ ∠2; $\overline{BD} \cong \overline{BD}$. (NS explains by redrawing the geometry figure in the problem and adding the name of the angle ∠1 = ∠ABD and ∠2 = ∠CDB)*

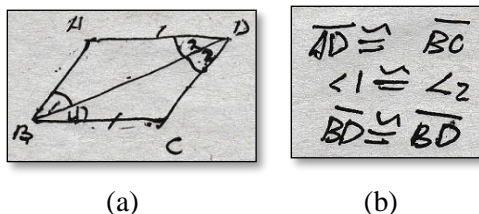   (a)   (b)

Figure 3. Subject draws object like a question

P   : *Why the $\overline{BD} \cong \overline{BD}$?*
S1 (2): *Because it is an identity or reflexive nature in line segments*

Based on written results and interviews, it is known that NS subjects understand problem one well. The subject is able to mention things that are known from the problem based on the picture presented. Also, the subject can also relate existing knowledge about the reflexive nature of line segments, to be able to mention that the allied segments are congruent.

The problem is whether the available information is enough to show that the two triangles, the ΔABD and ΔCDB, are congruent. The subject remembers two congruent triangles, a postulate or theorem that can be used to show that two triangles are congruent. The subject explained that the data that were known were not enough to show that the ΔABD and ΔCDB were congruent.

P   : *What is known to be able to show that $\Delta ABD \cong \Delta CDB$?*

S (3) : *In my opinion, the information presented is not enough. .. [CMR_1]*

P   : *Why?*

S (4) : *Because if we use the SSS postulate, it is less than one side, if we use postulate 13 (S-A-S) that the side is not the side of the wedge from and the side is not the side of the wedge if we use Postulate 14 (A-S-A)) less than one angle, and if we use the S-A-A theorem less one angle is known. ... [CMR_2]*

P   : *What about the S-Sd-S postulates? Is it not possible?*

S (5) : *... can't ma'am ... because S-A-S must be sequential (while explaining in the picture ... pointing at the sides - angles - sides), meaning that in order to use the S-A-S postulates, the angles that must be congruent are $\angle 3$ and $\angle 4$ or $\angle ADB$ on ΔABD and $\angle CBD$ on ΔCDB ...[CMR_2]*

P   : *But from what is known, there are two pairs of corresponding congruent sides, and the corresponding pair of angles are also congruent? Are you sure this cannot guarantee that the two triangles are congruent ?*

S(6)   : *(subject silent for a while)*

P   : *What if given two sides and an angle, then you are asked to construct a triangle, like the Figure 4.*
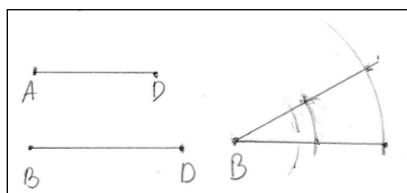

Figure 4. The subject explanation using the triangle construction

S(7)   : *The subject made a triangle construction from the data the researcher gave, by duplicating the BD side, B angle and AD side as in Figure 5 below.*
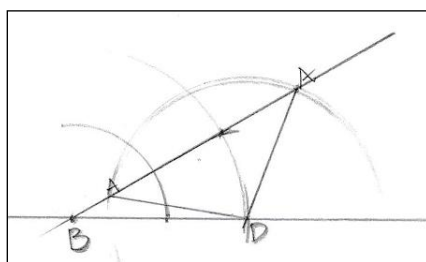

Figure 5. The construction of triangle

P   : *What does that mean?*

S(8)   : *The AD side intersects one side of the angle B at two points, meaning that AD can be constructed more than one without changing the angle B, so from what we know we can construct two different triangles ... so we cannot use S-S-A to prove two congruent triangles ... [CMR_1 & CMR_3]*

NS can give a statement; that is, the available information cannot be used to show that the ΔABD and ΔCDB are mutually congruent. NS can provide logical reasons (reasonable) that support

the arguments submitted (S (5)). NS clarified that there was no concept in the form of a postulate or a theorem appropriate to show the congruence of "ABD and" CDB. It shows plausibility.

Through scaffolding, NS can clarify that what is known is not enough to prove two congruent triangles, namely by constructing triangles with two pairs of congruent correspondences and a pair of congruent angles. It is in line with (NCTM, 2000) that "with guidance and many opportunities to explore, students can learn by the upper elementary grades how to be systematic in their explorations, to know that they have tried all cases, and to create arguments using cases." With a little guidance or question given (scaffolding), NS can explore an argument, namely, when NS constructs a triangle whose two sides have known lengths and an angle that is not the apex angle of both sides known earlier, NS can make more than one triangle, or in other words, the triangle is not single (Figure 5). This, in geometry, is a proof using counterexample. This shows that the completion of the NS meets the novelty.

The subject needs a question that leads her argument to the answer until this construction. It is in line with Lithner (2017) that if students do not have access to a solution method (recalled or given) to follow, only two possibilities remain for solving the task. One is to guess, but although guesswork can be a constructive part of problem-solving, it is almost never possible to solve a task only by guessing. The other possibility is to construct (part of) the solution, and this construction requires some guidance, some (explicit or implicit) argument to support the choices and conclusions. In general, the subject requires questions that can lead to the argument to the answer. NS explains the agency by using visualization in the form of a triangle construction. Similar results were delivered by Norqvist et al. (2019). Students that practice by CMR-tasks generally give more attention to the illustration. Students often use illustrations/visualizations in working on CMR questions.

The novelty that appears in this study is because the researcher provides scaffolding to appear that statements that have not been known beforehand by the subject. It is in line with Birkeland (2019), that to bring up novelty, students are given a little scaffolding. From a different point of view, the provision of scaffolding shows that NS is less able to connect mathematical concepts in problem-solving or commonly called mathematical connection capabilities. It is in line with the opinion that mathematical connections are the most difficult component of a student in reasoning (Sumarsih, Budiyono, & Indriati, 2018). New reasoning shows indicators of *novelty* in creative mathematical reasoning (Handayani, 2013; Lithner, 2008). The results of this study indicate that to bring up novelty, students still need scaffolding. It is in line with Hendriana, Prahmana, & Hidayat, 2018; Hidayat et al., (2018) that the creative mathematical reasoning abilities have not achieved optimally in indicator of novelty.

The argument is not just a show about the concepts previously known NS. Still, it can bring NS on the selected completion strategy and its implementation to bring NS on new reasoning, namely by submitting examples of deniers (counterexample). The reasoning, in this case, is how students carry out the process of transferring the knowledge they have to conclude, even though what they submit

does not meet the rules of formal proof. The same thing was said by Jonsson et al. (2014); Lithner (2008) and Norqvist et al. (2019).

Plausibility

Students in providing arguments have been supported by logical reasoning (S (4) and S (5)). It is seen when students state that there are no concepts (postulates or theorems) Side-Side Angles to show that two triangles are congruent. It shows an indicator of plausibility (Lithner, 2008; Norqvist et al., 2019). Students explained that the information provided in the problem was not enough to prove that the two triangles are congruent.

Plausibility is also seen from the results of the following interview.

P : *If the information is added, $\overline{AD} \| \overline{BC}$ can it be concluded that ΔABD it is congruent with ΔCDB?*

S (9) : *Can...because if $\overline{AD} \| \overline{BC}$, then the opposite inner corner is congruent, that is $\angle 3 \cong \angle 4$...so that it can use Angular-Side-postulate. (subject writes the explanation as shown in Figure 6). [CMR_2]*
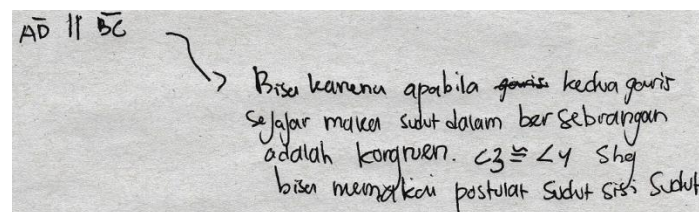

Figure 6. The subject explanation about angle-edge-angle postulate

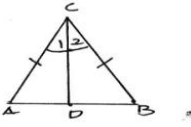P : *Aside from Corner-Side-Corner postulates, are there any others?*
S (10) : *It could be S-Sd-S too*

The result of the interview quote S (9) shows that the subject is able to make a logical statement by explaining the relationship that occurs if the information in the problem is added, namely AD // BC. It means that the statement subject is logical (plausible conveyed by the) because the arguments submitted are supported by explanations and conclusions obtained are true (Lithner, 2008; Norqvist et al., 2019).

The Student Result of Problem 2

In solving the second problem, NS presents problems mathematically and visually. The subject made a visualization of the ABC triangle with AB side as the base side and CA and CB sides as legs. The subject also painted point D and made a CD segment. Here are the results of the written work from problem 2.

Figure 7. The result of problem 2

Based on Figure 7 the subject can understand problem 2 well. The subject rewrites problem 2 with mathematical language and visualization. The subject can also relate the existing knowledge about isosceles triangles, to be able to describe the isosceles ABC triangle and its parts. Also, the subject understands the definition of the bisector angle, namely by making a picture that the CD divides ΔACB into Δ1 and Δ2 and gives congruent marks. The use of visualization in reaching this conclusion supports the NS subject as a person who has creative mathematical reasoning (Norqvist et al., 2019).

To show that ΔACD ≅ ΔBCD congruent subjects compose evidence using 2 columns, namely the statement column and the reason column and write the proof in the two columns as shown in the results of the work in Figure 8 below.

Translation

Proof

Statement	Reason
1. $\overline{CD}$ is bisector ∠ACB $\overline{CA} \cong \overline{CB}$	1. Determined
2. $\overline{CD} \cong \overline{CD}$	2. Identity
3. ∠1 ≅ ∠2	3. Definition of angle bisector
4. $\Delta ACD \cong \Delta BCD$	4. Postulat 13 (Side-Angle-Side)

Figure 8. The result of problem 2

Based on Figure 8, the subject can provide arguments based on problem two given. The subject also gave a logical reason for each statement given. The subject can apply the previously owned algorithm to problem 2. This means that the subject has algorithmic reasoning (Lithner, 2008), which is a process of drawing conclusions based on an algorithm that has been known before by the subject.

It is because problem 2 is algorithmic. Algorithmic reasoning is defined as a repetitive numerical task-solving method that uses algorithmic support (Jonsson et al., 2014). The subject can show the relationship between the nature of identity, the definition of the angle bisector, and the concept/postulate of the Side-Angle-Side Congruence. It indicates that the subject uses a mathematical foundation in solving problem 2 (Lithner, 2008).

Problem 2 is a problem that uses an approach imitative and is algorithmic so that the algorithm has been known beforehand by the subject. Therefore, the subject can arrange a thought process so that conclusions are obtained easily and the right answers.

Mathematical Foundation

NS reasoning is based on mathematics, namely the use of geometry constructions in proving the properties of geometry. The choice of method is based on the non-fulfillment of the postulate and theorem about the congruence of two corresponding triangles (on problem-solving 1). While solving problem two is also based on mathematical properties, namely the use of a theorem that has been proven to arrive at a conclusion. Thus the reasoning is done by subjects in questions 1 and 2 satisfies indicators mathematical foundation (Lithner, 2008).

Based on the results of the completion of the subject on problems 1 and 2 and the results of the analysis, it was concluded that the NS subject had creative mathematical reasoning because it met the novelty, plausibility, and mathematical foundation (Lithner, 2008). The reasoning by NS is new, namely the discovery of a counterexample. In this case, the decision making process by basing on the pattern of obtaining conclusions, namely by constructing triangles with known provisions, so finding an example of denial is an inductive creative process. This is in line with Adawiyah et al. (2017) that create patterns is defined as using a correlation of pattern to analyze the situation in a unique way. Another finding is that NS was able to solve problem two well and smoothly so that it concluded. It means that someone who has creative mathematical reasoning also has reasoning imitative, namely algorithmic reasoning.

CONCLUSION

The mathematical creative reasoning ability of prospective teacher-students is still very low, especially in geometry courses, so the quality of student reasoning is still dominated by algorithmic reasoning. Mathematical creative reasoning is higher reasoning than algorithmic reasoning. When students can reason creatively, they have imitative reasoning, which is algorithmic reasoning. Mathematical reasoning creatively can be identified when the subject can reason algorithmic but is aware of the problem so that algorithmic can not be solved that requires students should form the reasoning of new consisting of novelty, plausibility, and based on a mathematical foundation. Of the

three indicators of creative mathematical reasoning, novelty is the weakest indicator that requires scaffolding to bring it up.

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REFERENCES

Adawiyah, R., Muin, A., & Khairunnisa, K. (2017). Mathematical Inductive-creative reasoning , a theoretical study. *Advances in Social Science, Education and Humanities Research (ASSEHR)*, *57*, 247–251. https:// dx.doi.org/10.2991/icmsed-16.2017.53.

Bergqvist, T., & Lithner, J. (2012). Mathematical reasoning in teachers' presentations. *Journal of Mathematical Behavior*, *31*(2), 252–269. https://doi.org/10.1016/j.jmathb.2011.12.002.

Birkeland, A. (2019). Pre-service teachers' mathematical reasoning - how can it be developed?. *The Mathematics Enthusiast*, *16*(1–3), 579–596. Retrieved from https://scholarworks.umt.edu/tme/vol16/iss1/29.

Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educ Stud Math*, *75*, 89–105. https://doi.org/10.1007/s10649-010-9242-9.

Darta, & Saputra, J. (2018). Indicators that influence prospective mathematics teachers representational and reasoning abilities. *Journal of Physics: Conference Series*, *948*(1). https://doi.org/10.1088/1742-6596/948/1/012053.

Handayani, A. D. (2013). Creative mathematical reasoning [in Bahasa]. *Jurnal Pengajaran MIPA*, *18*(2), 161–166. https://doi.org/http://dx.doi.org/10.18269/jpmipa.v18i2.

Hendriana, H., Prahmana, R. C. I., & Hidayat, W. (2018). Students' performance skills in creative mathematical reasoning. *Infinity Journal*, *7*(2), 83. https://doi.org/10.22460/infinity.v7i2.p83-96.

Hidayat, W., Wahyudin, & Prabawanto, S. (2018). Improving students' creative mathematical reasoning ability students through adversity quotient and argument driven inquiry learning. *Journal of Physics: Conference Series*, *948*(1). https://doi.org/10.1088/1742-6596/948/1/012005.

Hidayat, W. (2017). Adversity quotient and creative mathematical reasoning of senior high school students in argument-driven inquiry learning on derived functions material [in Bahasa]. *KALAMATIKA Jurnal Pendidikan Matematika*, *2*(1), 15. https://doi.org/10.22236/kalamatika.vol2no1.2017pp15-28.

Hidayat, W., Herdiman, I., Aripin, U., Yuliani, A., & Maya, R. (2018). Adversity Quotient (AQ) and creative mathematical reasoning of pre-service teacher [in Bahasa]. *Jurnal Elemen*, *4*(2), 230. https://doi.org/10.29408/jel.v4i2.701.

Jäder, J. (2019). Task design with a focus on conceptual and creative challenges. In *Eleventh congress of the european society for research in mathematics education, Utrecht University* Netherlands: Utrecht (p. hal-02423408).

Jane, M. . (2006). *Developing geometric reasoning*. Washington DC: GED Mathematics Training

Institute.

Jonsson, B., Kulaksiz, Y. C., & Lithner, J. (2016). Creative and algorithmic mathematical reasoning: effects of transfer-appropriate processing and effortful struggle. *International Journal of Mathematical Education in Science and Technology*, *47*(8), 1206–1225. https://doi.org/10.1080/0020739X.2016.1192232.

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *Journal of Mathematical Behavior*, *36*, 20–32. https://doi.org/10.1016/j.jmathb.2014.08.003.

Lawson, M. J., & Chinnappan, M. (2000). Knowledge connectedness in geometry problem solving. *Journal for Research in Mathematics Education*, *31*(1), 26–43. https://doi.org/10.2307/749818.

Lithner, J. (2008). A Research Framework for Creative and Imitative Reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/sl0649-007-9104-2.

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM - Mathematics Education*, *49*(6), 937–949. https://doi.org/10.1007/s11858-017-0867-3.

Masfingatin, T., Murtafiah, W., & Krisdiana, I. (2018). Mathematics pre-service teacher ability in solving problems of geometry theorema proof [in Bahasa]. *Jurnal Mercumatika: Jurnal Penelitian Matematika dan Pendidikan Matematika*, *2*(2), 41–50. https://dx.doi.org/10.26486/jm.v2i2.272.

Mullis, I. V. S., Martin, M. O., & Foy, P. (2015). *TIMSS 2015 international results in mathematics*. Boston: International Association for the Evaluation of Educational Achievement.

NCTM. (2000). Six principles for school mathematics. In *National Council of Teachers of Mathematics* (pp. 1–6).

Norqvist, M., Jonsson, B., Lithner, J., Qwillbard, T., & Holm, L. (2019). Investigating algorithmic and creative reasoning strategies by eye tracking. *Journal of Mathematical Behavior*, *55*(April), 100701. https://doi.org/10.1016/j.jmathb.2019.03.008.

Nursyahidah, F., Saputro, B. A., & Prayitno, M. (2016). Mathematical reasoning ability of junior high school students in learning of lines and angles with geogebra [in Bahasa]. *Suska Journal of Mathematics Education, 2*(1), 13-19.

Octriana, I., Putri, R. I. I., & Nurjannah, N. (2019). Students' mathematical reasoning in learning of number patterns using PMRI and LSLC [in Bahasa]. *Jurnal Pendidikan Matematika*, *13*(2), 131–142. https://doi.org/10.22342/jpm.13.2.6714.131-142.

Olsson, J. (2017). *GeoGebra, enhancing creative mathematical reasoning*. Institutionen för tillämpad utbildningsvetenskap. Retrieved from http://www.diva-portal.org/smash/get/diva2:1085687/FULLTEXT01.pdf.

Permendikbud. (2016). Minister of Education and Culture Regulation Number 21 of 2016 concerning Basic and Secondary Education Content Standards [in Bahasa]. Jakarta: Permendikbud.

Reiss, K. M., Heinze, A., Renkl, A., & Groß, C. (2008). Reasoning and proof in geometry: Effects of a learning environment based on heuristic worked-out examples. *ZDM - International Journal on Mathematics Education*, *40*(3), 455–467. https://doi.org/10.1007/s11858-008-0105-0.

Rohana, R., & Ningsih, Y. L. (2019). Students' statistical reasoning in statistics method course. *Jurnal Pendidikan Matematika*, *14*(1), 81–90. https://doi.org/10.22342/jpm.14.1.6732.81-90.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando: Fla: Academic Press.

Sentosa, C. A. H. (20013). Overcoming student difficulties when conducting formal mathematical proof [in Bahasa]. *Jurnal Pengajaran MIPA*, *18*(2), 152–160. https://doi.org/http://dx.doi.org/10.18269/jpmipa.v18i2.

Siregar, N. (2018). Reviewing mathematical reasoning abilities of junior high school students through interviews based on geometry task [in Bahasa]. *Mosharafa: Jurnal Pendidikan Matematika*, *5*(2), 128–137. https://doi.org/10.31980/mosharafa.v5i2.268.

Sukirwan, Darhim, D., & Herman, T. (2018). Analysis of students' mathematical reasoning. *Journal of Physics: Conference Series*, *948*(1). https://doi.org/10.1088/1742-6596/948/1/012036.

Verner, I., Massarwe, K., & Bshouty, D. (2019). Development of competencies for teaching geometry through an ethnomathematical approach. *Journal of Mathematical Behavior*, (May), 100708. https://doi.org/10.1016/j.jmathb.2019.05.002.