

Thinking Process of Concrete Student in Solving Two-Dimensional Problems

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Abstract

The purpose of this research was to find out the thinking processes of a concrete student in solving two-dimensional problems. The research method used is descriptive qualitative. The research subjects were two students taken using purposive sampling. The instrument used was the Test of Logical Operations and problem-solving tests. Stages of data analysis used are researching all data, making a cognitive classification of students, choosing concrete students to be used as research subjects, reviewing the results of concrete student work in solving mathematical problems, verify data and data sources that have been classified and transcribed in the presentation or exposure of data. The results showed that at the stage of understanding the problem and re-checking the answers, concrete students use the assimilation at the stage of planning to solve the problem of doing the disequilibrium. At the stage of carrying out a plan to solve a problem, concrete students carry out the accommodation. During this study, it was found that students' habits in mathematical problem-solving did not plan to solve problems, did not re-examine answers, and there were students' habits by interpreting the final results of problems. It can be concluded that the students' concrete thinking processes in solving two-dimensional problems vary according to the stages of problem-solving.

Keywords: Thinking Process, Cognitive Development, Concrete Students, Problem Solving, Two-Dimensional

Abstrak

Tujuan dalam penelitian ini adalah untuk mengetahui proses berpikir siswa konkret dalam memecahkan masalah dimensi dua. Metode penelitian yang digunakan adalah deskriptif kualitatif. Subjek penelitian sebanyak dua siswa yang diambil dengan menggunakan *purposive sampling*. Instrumen penelitian yang digunakan adalah *Test of Logical Operations* dan tes pemecahan masalah. Tahapan analisis data yang digunakan adalah meneliti semua data, membuat klasifikasi kognitif siswa, memilih siswa konkret yang akan digunakan sebagai subjek penelitian, meninjau hasil kerja siswa konkret dalam menyelesaikan masalah matematis, memverifikasi data dan sumber data yang telah diklasifikasikan dan ditranskripsi dalam presentasi atau paparan data. Hasil penelitian menunjukkan bahwa pada tahap memahami masalah dan memeriksa kembali jawaban, siswa konkret menggunakan proses berpikir asimilasi. Pada tahap merencanakan untuk menyelesaikan masalah melakukan proses disequilibrium. Pada tahap melaksanakan rencana untuk menyelesaikan masalah, siswa konkret melakukan proses berpikir akomodasi. Selama penelitian ini ditemukan kebiasaan siswa dalam memecahkan masalah matematis tidak melakukan perencanaan untuk menyelesaikan masalah, tidak melakukan pemeriksaan kembali jawaban, dan ada kebiasaan siswa dengan menginterpretasikan hasil akhir dari masalah. Dapat disimpulkan bahwa proses berpikir siswa konkret dalam memecahkan masalah dimensi dua berbeda-beda disesuaikan dengan tahapan pemecahan masalah.

Kata kunci: Proses Berpikir, Perkembangan Kognitif, Siswa Konkret, Pemecahan Masalah, Dimensi Dua

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INTRODUCTION

Mathematics is one of the sciences that underlies the development of science and technology (Amir, 2014). Also, mathematics plays an important role in several other fields of science, such as physics, engineering, and statistics (Masfingatn, 2013; Suratno, 2014). This is what underlies that

Mathematics must be given to students from an early age, so students are expected to master the concepts in Mathematics.

One of the abilities students must have in learning mathematics is the ability to solve problems (Hertiavi, Langlang, & Khanafiyah, 2010; Nuritasari & Anjani, 2019; Surya, Putri, & Mukhtar, 2017). This ability is the ability student of high ordering thinking skill (Abdullah, Abidin, & Ali, 2015; Nadapdap & Istiyono, 2017), where this ability is the main goal in learning mathematics in schools so it must be given, trained, and accustomed by the teacher to their students (Minarni, Napitupulu, & Husein, 2016; Purnomo & Mawarsari, 2014). In the mathematics curriculum, problem-solving is a very important part because, in the learning or completion process, students can gain experience using the knowledge and skills that have been needed to be applied to problem-solving that is not routine (Widodo & Turmudi, 2017). One method that can be used to solve mathematical problems is the method of Polya (1973), which consists of understanding the problem, planning to solve the problem, carrying out the plan, and checking the answers that have been prepared by students.

The facts show that the majority of students who have students' mathematical problem-solving abilities, both at the secondary and tertiary levels, are still relatively low (Akbar, Hamid, Bernard, & Sugandi, 2017; Ulya, 2015, 2016). Many factors cause that the ability of students to solve mathematical problems is not optimal, among them are the learning strategies used (Effendi, 2012; Puadi & Habibie, 2018; Sumartini, 2015), learning device used (Hasibuan, Saragih, & Amry, 2018; Peranginangin, Saragih, & Siagian, 2019; Siagian, Saragih, & Sinaga, 2019), the level of cognitive development of students have not yet reached the formal stage, so students have difficulty understanding abstract mathematical material (Fitriyani, Widodo, & Hendroanto, 2018; Widodo, Istiqomah, Leonard, Nayazik, & Prahmana, 2019), the ability of teachers to provide individual services is not maximal (Annamma, 2015; Elstad & Christophersen, 2017). Relating to the ability of teachers to provide individual services, teachers are expected to be able to clarify student thinking that is adjusted to student characteristics so that students' mindset becomes easier to understand (Fauziyah, Usodo, & Ekana, 2013; Ngilawajan, 2013; Wilkie, 2016). By classifying students' thinking, at least the teacher has provided individual services in the form of understanding students' thinking patterns according to their characteristics.

In learning mathematics, someone experiences a thought process because that person is doing mental activities, and people who are learning mathematics must be doing mental activities (Siswono, 2012). The thinking process is the entry of information that is then processed in a person's brain (Widodo & Turmudi, 2017). In the process of thinking, students at least make adjustments to the new scheme with the old scheme, or often referred to as the adaptation process (Kiosses, Ravdin, Gross, Raue, Kotbi, & Alexopoulos, 2015). This adaptation process consists of assimilation, accommodation, and abstraction (Sudarman, 2009). When solving problems, students do the thinking process so that they can determine answers. By knowing students' thinking processes, at least the teacher can guess the direction of the students' answers in solving mathematical problems.

Every individual is born with unique characteristics. Although individuals are born twins, the twinness of the individual lies only in his physical appearance, while the mindset possessed by individual twins is certainly very different. By knowing students' thinking processes in solving mathematical problems based on student characteristics, the teacher has indirectly provided services to heterogeneous individuals. One of the characteristics of students who rarely get the teacher's attention includes the characteristics based on cognitive development (Widodo et al., 2019). Cognitive development is divided into four categories, namely sensory-motoric, pre-operational, concrete operational, and concrete operational (Barrouillet, 2015; Inhelder & Piaget, 1964; Piaget, 2003; Siegler, 2016; Selvianiresa & Prabawanto, 2017). Concrete students are chosen to know the students' thinking process more because the results of previous studies indicate that junior high school students are not fully informal development or have not yet reached the formal phase (Widodo, Turmudi, & Dahlan, 2019). In other words, there are still junior high school students who are in the concrete phase, even though they are more than 12 years old (Ojose, 2008). Based on this, the purpose of this study was to determine the students' concrete thinking processes in solving rectangular problems.

METHODS

Following the problems to be studied, this type of research is included in a qualitative descriptive study. Qualitative descriptive research aims to reveal events or facts, circumstances, phenomena, variables, and circumstances that occur during research by presenting what happened (Creswell, 2009). In this regard, this study intends to describe students' thought processes in solving quadrilateral problems.

The subjects of this study were two eighth grade students of SMP Negeri 11 Yogyakarta in the even semester of the 2018/2019 school year taken by purposive sampling technique. Purposive sampling is sampling carefully chosen so that it is relevant to the research design (Creswell, 2012). The subjects to be used in research must first have a concrete cognitive development, and this is the reason researchers use purposive techniques in research sampling.

The research instrument used in this research study was the researcher himself as the main instrument. Besides, in this study, the supporting instruments used were the Test of Logical Operations (TLO) and mathematical problem-solving tests on the subject of the quadrilateral. TLO aims to confirm one's cognitive development (Inhelder & Piaget, 1964; Leongson & Limjap, 2003). This test was adapted from Leongson & Limjap (2003), where the indicator is classification, seriation, logical multiplication, compensation, proportional thinking (Ratio Probability), and correlational thinking. The problem-solving instrument test aims to determine the ability of students to solve mathematical problems on rectangular material. By solving quadrilateral problems, students must express what is in their minds to be written on the answer sheets that have been provided. The problem given to students is “ An ABCD parallelogram, it is known that Points P and Q are located

on BD so that DP and BQ are perpendicular to BD. If the length of AD = 13 cm, BD = 25 cm, and the area of the parallelogram are 125 cm², determine the length of PQ!"

To get research data, students are asked to convey what they think when solving mathematical problems, then interviewed. Data obtained at the time of the interview was recorded using a recorder. In this case, the method used to collect data is Think Out Louds or also known as Think Aloud. Think aloud is a method, where the subject is asked to voice his thoughts during problem-solving and ask him to repeat if there is something that needs to be said during the problem-solving process, in this case allowing the subject to say something or what he is thinking (Someren, Barnard, & Sandberg, 1994).

Stages of analysis of the data that has been obtained are done by (1) examining all data collected from various sources, (2) making a cognitive classification of types of students, namely concrete, transition, and formal, (3) selecting concrete students to be used as research subjects, (4) reviewing student concrete work in solving mathematical problem-solving questions, (5) verifying data and data sources that have been classified and transcribed in the presentation or exposure to data (Miles, Huberman, & Saldaña, 2013).

RESULTS AND DISCUSSION

Before students are given a problem-solving test, students are given a Test of Logical Operations first. This test aims to confirm the cognitive development of students. From the results of this test, it was found that students of SMP Negeri 11 Yogyakarta were divided into two groups, namely concrete and transition. Students with transition cognitive development are 28 people, while the rest are in concrete development. Following the objectives to be achieved in this research is knowing the students' concrete thinking processes in solving quadrilateral problems, then taken two students from three students who are in concrete cognitive development.

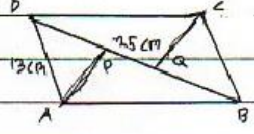
<p>Di ketahui : Panjang AD = 13 cm, BD = 25 cm dan luas jajargenjang tersebut adalah 125 cm²</p>	
<p>Di tanya : Tentukan panjang PQ!</p>	
<p>Jawab : </p>	$= 25 - 13$ $= 12 \text{ cm}$ $= \sqrt{12}$ $= 6 \text{ cm}$
<p>Jadi panjang PQ adalah 6 cm.</p>	

Figure 1. The answer of subject K1

As revealed in the previous section that to solve mathematical problems, students can use the stages of understanding the problem, plan to solve the problem, carry out the plan, and check the answers that have been prepared by students (Polya, 1973). Concrete student answers to the

mathematical problems that have been given can be seen in Figure 1 and Figure 2.

1. Diketahui = Sebuah jajargenjang ABCD $AD = 13\text{ cm}$, $BD = 25\text{ cm}$ luas jajargenjang tersebut adalah 125 cm^2
 Ditanyakan = Tentukan Panjang PQ!
 Jawab = $\frac{AB}{AD} = \frac{BD}{AP}$
 $\frac{AB}{13} = \frac{25}{AP}$
 $PQ = 25 \times 13$
 $= 325\text{ cm}$
 Jadi, Panjang PQ adalah 325 cm

Figure 2. The answer of subject K2

In the stage of understanding the problem, subjects K1 and K2 write what is known and what is asked of the problem. The subject can directly integrate his new perception into the scheme in his mind so that it can be said that the subject engages in the process of thinking assimilation. Assimilation is the process of integrating new perceptions, concepts, or stimuli into existing schemes or patterns (McReynolds, 2015; Nagai & Asada, 2015; Nuritasari & Anjani, 2019; Zhiqing, 2015). Assimilation is the process by which new stimuli from the environment are integrated into existing schemes (Adi, Meter, & Kristiantari, 2014; Bormanaki & Khoshhal, 2017; Lastiningsih, Mutohir, Riyanto, & Siswono, 2017). Assimilation is an individual process in adapting and organizing themselves with the environment or new challenges so that students' understanding develops (Blake & Pope, 2008; Kusmayadi, Sujadi, & Muhtarom, 2011; Lane, Ryan, Nadel, & Greenberg, 2015). Assimilation does not produce development or schemata, but only supports the growth of the schemata of subjects categorized to do the thought process of accommodation (Kusumawardani, 2017; McReynolds, 2015; Wahyudi, Waluya, Rochmad, & Suyitno, 2018). Assimilation occurs when the structure of the problem faced is following the scheme that is owned so that the structure of the problem can be integrated directly into the existing scheme.

For students in Indonesia, steps to plan problem solving are often not written on the answer sheet. But the step of planning is only in the minds of students, so it needs to be done with interviews or researchers can see from the flow of students' answers in solving mathematical problems. As with subject K1, the researcher needs to interview to find out the plan that was carried out to solve the problem.

- P : *How to determine the length of the PQ?*
 K1 : *Determine the length of PQ, I first determine the length of DP and BQ. Because this structure is a parallelogram (while pointing to a sketch of a picture of a parallelogram in the answer), the DP and BQ lengths are the same*
 P : *Okay, to find out the length of the DP or BQ how to do it!*

- K1 : *Hmmmm (paused for a long time)*
This APD Triangle is equilateral, sir. Really? (subject K1 asks himself whether the AOB triangle is equilateral or not).
- P : *If the equilateral triangle continues how, if this triangle is not equilateral how?*
- K1 : *If the APD triangle is the same side, the length of AP and PD is the same as the length of AD, which is 13. But if it is not the same side (K1's voice began to soften, and was silent for a long time)*

Subject K1, in planning to solve the problem, experienced a disequilibrium process or an imbalance between the old scheme and the new scheme. The old scheme, which was owned by K1 subjects turned out to be problematic when faced with this mathematical problem. Problems arise related to the initial understanding of subject K1 (initial scheme) related to the PPE triangle formed at the intersection of the BD line is a parallelogram with the AP lines that are perpendicular to each other. The subject of error occurred in understanding the nature of the triangle, namely, the sum of the three angles of a triangle is 1800. If subject K1 uses this property, then the subject can quickly decide that this triangle is not equilateral. Because in an equilateral triangle, one of the properties possessed is the three angles of 600. Besides, the subject did not use the variables that were known to the problem, namely the width of the network distance of 125 cm². With this variable, the subject is expected to know the length of the AP by using the similarity of two ABD triangles with a wide area.

On the K2 subject, researchers need to conduct interviews to find out the plans that are carried out to solve the problem.

- P : *How to determine the length of the PQ?*
- K2 : *I use congruence.*
- P : *Which one fulfills the similarity?*
- K2 : *ABD triangle and APD triangle (while pointing at a parallelogram sketch drawing on opaque paper in addition to the answer sheet)*
- P : *Next?*
- K2 : *Because the ABD triangle and APD Triangle are congruent, it applies $\frac{AB}{BD} = \frac{AD}{AP}$ or $\frac{AB}{AD} = \frac{BD}{AP}$*
- P : *Oh, I see. Are the ABD triangle and the APD triangle similar? What are the requirements of two similar triangles?*
- K2 : *Hmmm.....(long pause)*

Subject K2, in planning to solve problems, also experienced a disequilibrium process or the imbalance between the old scheme and the new scheme. The old scheme which had been owned by K2 subjects turned out to be problematic when faced with this mathematical problem. Problems arise related to the initial understanding of the K2 subject (old scheme) related to congruence. Thought of the K2 subject allegedly concluded that $\frac{AB}{BD} = \frac{AD}{AP} = \frac{AB}{PB}$. When subjects are asked about the terms of a similar triangle, the subject is unable to explain it to the researcher. Almost is the same as subject K1, subject K2 does not use the variable that is known in the problem, namely the breadth area of 125 cm². With this variable, the subject is expected to know the length of the AP by using the similarity of two ABD triangles with a wide area. In connection with these results, then at the stage of planning to

solve the problem of concrete subjects doing disequilibrium process. Disequilibrium is the process of imbalance between assimilation and accommodation (Barrouillet, 2015; Ward, Pellett, & Perez, 2017; Worsley & Blikstein, 2015). This imbalance can occur because the subject is not able to process accommodation between new knowledge with old knowledge (Labouvie-Vief, 2015).

In the step of implementing the problem-solving plan, subjects K1 and K2 can implement the program that was prepared in the previous stage, although the planning compiled by the two subjects does not reflect to solve the problem at hand. As happened to subject K1 using the concept of the equilateral triangle and subject K2 by using the concept of congruence in solving the problem at hand. Because the subject can implement the plan that was made in the previous stage, the subject is categorized to do the accommodation thinking process.

Accommodation is the process information of new schemes to adjust to the stimulus received, so it is necessary to change the old scheme to fit the problem structure (Barrouillet, 2015; Bormanaki & Khoshhal, 2017; Hendrowati, 2015). The accommodation process can occur because the new experience is not at all compatible with the existing scheme (Ibda, 2015), in other words, it can occur if a person is unable to assimilate a new knowledge obtained with an old knowledge (Masfingatin, 2013; Widyastuti, 2015).

At the last stage or the stage of re-checking the answers, students are expected to look back at the answers to make sure that the results of the answers to these problems are correct. This step is important to check whether the results obtained are following the provisions, and there is no contradiction with what was asked (Suryana, 2015; Utomo, 2012). Steps that can be used by students to do the re-checking stage include questioning themselves related to the process of the answers obtained, whether they are correct or not, and matching the results obtained with the questions asked (Annisa, 2014). Although this stage has the smallest weight among the other stages, this step is still important to do by students to make corrections or correct errors such as concept errors, procedural errors, and calculation errors that have been done in the previous stages (Utomo, 2012). Besides, this stage can be used to minimize errors that might appear in the previous stage (Herman, 2000).

At the stage of checking the answers, both K1 and K2 subjects did not write down the step of checking the answers. But both subjects were able to interpret the answers obtained to conclude an outcome of the process. This interpreting process is the final part of the process of questioning yourself related to the process of answers that have been obtained. By interpreting a result from the process of solving a problem, students believe in the final result that has been obtained, so students are thought to have done the process of checking the results of the answers even without writing them on the answer sheet. This is in line with the opinion that states that interpreting the answers obtained by students is one way that can be done by students in the stage of re-checking the answers (Wahyudi & Budiyo, 2011). In this regard, the concrete subject of the process of thinking assimilation in re-examining the answers that have been made.

CONCLUSION

Based on the results of the research and discussion that has been done, it can be concluded that students are concrete in understanding the problem and re-checking the answers that have been done to do the assimilation thinking process. At the stage of planning to solve a problem, a concrete student performs disequilibrium, and at the stage of implementing a plan, a concrete student performs an accommodation thought process. The findings show that students' habits in solving mathematical problems have never done the stages of planning to solve a problem, and the stage of re-examining the answers found that students' habits only interpret the final results of the whole problem-solving process. These results indicate that students' habits in solving mathematical problems still use a three-step pattern that is known, asked, and answered. This shows that problem solving can be considered as a heuristic in learning mathematics. In connection with these results, it is necessary to prepare learning tools that can accommodate the problem-solving process into a heuristic in learning.

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