Jurnal Pendidikan Matematika
Volume 14, No. 2, July 2020, pp. 101-116
P-ISSN: 1978-0044, E-ISSN: 2549-1040, DOI: https://doi.org/10.22342/jpm.14.2.11261.101-116
Website: https://ejournal.unsri.ac.id/index.php/jpm
Accredited by SINTA 2: http://sinta2.ristekdikti.go.id/journals/detail?id=1811

Solving Fractions by Applying the Bar Model Concept with the Butterfly Method

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Abstract

This study aims to investigate an intervention in the application of the Butterfly Method Algorithm with the Bar Model Concept on the addition and subtractions of Fractions to Year 9 students in one of the Government secondary schools in Brunei Darussalam. The Butterfly Method is an alternative visual method for teaching fractions where the diagonal and horizontal multiplication of the denominators and numerators are employed by drawing the Butterfly. A mixed-method approach was used to explore the impact of the intervention with data gathered from the students' written pre- and post-tests and interview transcripts. The tests conducted before and after the intervention were used to analyze students' errors and misconceptions. The students' written analyses of the post-test results revealed that not all of the students applied the Butterfly Method. A few students were selected for interviews in order to gain deeper insights into how they developed the errors and misconceptions from both tests. Findings from the students' interview transcripts revealed they were not confident with the Butterfly Method, and they needed more time to be familiar with the concept. Another factor for not applying the Butterfly Method is due to students' confusion on subtracting fractions that has the same denominators as well as subtracting a proper fraction from a whole number. This study concluded that students who applied the Butterfly Method helped them to remember the new method from the intervention satisfactorily in comparison to those who lack the confidence in applying it.

Keywords: Fractions, Butterfly Method, Bar Model Concept, Addition, Subtraction

Abstrak

Tujuan penelitian ini adalah untuk menyelidiki intervensi dalam penerapan Algoritma Metode *Butterfly* dengan Konsep *Bar Model* pada penambahan dan pengurangan Pecahan ke siswa Kelas 9 di salah satu sekolah menengah kerajaan di Brunei Darussalam. Metode *Butterfly* adalah metode visual alternatif untuk mengajar pecahan di mana penggandaan penyebut dan pembilang *diagonal* dan *horizontal* digunakan dengan menggambar *Butterfly*. Pendekatan metode campuran digunakan untuk mengeksplorasi dampak intervensi dengan data yang dikumpulkan dari pra- dan pasca-tes tertulis dan transkrip wawancara. Tes yang dilakukan sebelum dan sesudah intervensi digunakan untuk menganalisis kesalahan dan kesalahpahaman siswa. Analisis tertulis siswa tentang hasil pasca-tes mengungkapkan bahwa tidak semua siswa menerapkan Metode *Butterfly*. Beberapa siswa dipilih untuk wawancara untuk mendapatkan wawasan lebih dalam tentang bagaimana mereka mengembangkan kesalahan dan kesalahpahaman dari kedua tes. Temuan dari transkrip wawancara siswa mengungkapkan mereka tidak percaya diri dengan Metode *Butterfly* dan mereka membutuhkan lebih banyak waktu untuk terbiasa dengan konsep tersebut. Faktor lain untuk tidak menerapkan Metode *Butterfly* adalah karena kebingungan siswa tentang pengurangan pecahan yang memiliki penyebut yang sama serta mengurangi pecahan yang tepat dari bilangan bulat. Penelitian ini menyimpulkan bahwa siswa yang menerapkan Metode *Butterfly* membantu mereka mengingat metode baru dari intervensi secara memuaskan dibandingkan dengan mereka yang kurang percaya diri dalam menerapkannya.

Kata kunci: Pecahan, Metode *Butterfly*, Konsep *Bar Model*, Penambahan, Pengurangan

How to Cite: Low, J., Shahrill, M. & Zakir, N. (2020). Solving fractions by applying the bar model concept with the butterfly method. *Jurnal Pendidikan Matematika, 14*(2), 101-116.

INTRODUCTION

The purpose of this study was to develop an intervention that teaches a method that is easy for students to understand by applying the Bar Model concept first and then relate it with the Butterfly

Received March 19, 2020; Revised May 11, 2020; Accepted June 24, 2020

Method algorithm to add and subtract fractions. The Butterfly Method is a visual and an alternative method for teaching the addition and subtraction of fractions where diagonal and horizontal multiplication of denominators and numerators are applied (Rosli, Han, Capraro, & Capraro, 2013). Meanwhile, the Bar Model concept is a technique where bars are drawn as a whole, divided into equal pieces, and defined by the denominator (Madani, Tengah, & Prahmana, 2018). The Bar Model algorithm helps students to visualize and a useful tool to understand the concept of addition and subtraction of fractions (Thirunavukkarasu & Senthilnathan, 2014). According to Duzenli-Gokalp and Sharma (2010), the use of visual images can help students to calculate the addition and subtraction of fractions with like and unlike denominators. Accordingly, the Butterfly Method will leave a mental picture of the algorithm that can be easily applied (Cardone, 2015; Miller & Obara, 2017).

The Bar Model and Butterfly Method to Develop Conceptual Knowledge of Fractions

Bar modeling is a technique of representing any Mathematics problems pictorially, which has been popularised by the Singapore Mathematics teaching method (Emeny, 2014). It was proposed that students learn in three stages, which are concrete, pictorial, abstract (McLeod, 2008; Bruner, 1996), and through Bar Models, it can support students with the pictorial stage.

A Bar Model is a diagram with the purpose of representing a mathematical problem which students find quite difficult to solve without visualizing it at the beginning (Thirunavukkarasu & Senthilnathan, 2014; Madani, Tengah, & Prahmana, 2018). It is a key feature of the Singapore Mathematics that used a problem-solving technique that was often referred to as the model approach (Yeap, 2010). Thirunavukkarasu and Senthilnathan (2014) also explained that drawing the model permits students to visually relate various types of information given in the problem to an unknown amount. This helps students to determine which mathematical expressions are useful in solving the problem. These researchers explained that the purpose of drawing the models is not to follow specific rules, but students need to understand the concepts and work out a strategy for finding the answer. For solving complex problems, several strategies are possible, and drawing the model allows the student to visualize a good strategy. The researchers commented further that drawing the Bar Model is a valuable tool for students to solve non-routine problems.

Mattock (2015) explained that in order for students to understand the concept of adding proper fractions with different denominators is to split the Bar Models equally according to the lowest common multiple. For example, adding, he further clarified that students need to understand the concept of splitting each of the Bar–Models equally without calculating the denominators for the lowest common multiple of the two fractions. The Bar Models cannot be split unequally, and students can visually understand the concept if the bars are not split equally for two of the given fractions. Feedback from the students stated the concept was easily understood through splitting the Bar Models equally, which makes it easier to visualize from the Bar Models.

The Butterfly Method is a visual and an alternative method for teaching addition and subtraction of fractions where the diagonal and horizontal multiplication of the denominators and numerators are employed as well as to work out a visual way by drawing the Butterfly on both of the fractions with different denominators. Cardone (2015) and a group of teachers from the online Mathematics community produced a book entitled 'Nix the Trick.' The objective of their contribution to teachers is to help students avoid memorized procedures and short-cutting tricks in favor of making connections with the concepts of understanding Mathematics. Students have the potential to develop rich conceptual understanding and the opportunity to discover new concepts. Cardone (2015) mentioned that students have no understanding of applying the Butterfly Method algorithm to add and subtract fractions. She explained the process of adding fractions by using the Bar Model before applying the algorithm of the Butterfly Method. According to Cardone (2015), fractions can be compared, added, or subtracted by shading the bars on the Bar Model, whereas the total bars represent the common denominator. The procedure is explained clearly with the application of the Bar Model, but no explanation is made with the relationship to the Butterfly Method.

By applying Bruner's constructivism theory, students will construct their understanding of adding and subtracting fractions by using the Bar Model concept. As far as is known, the study on the Bar Model concept in combination with the Butterfly Method algorithm on the addition and subtraction of fractions is very limited. Adopting the Bar Model concept as a reference for the new teaching strategy during the intervention perhaps may help to add further information to the existing literature on solving fractions. The conceptual knowledge of understanding the Bar Model expects to assist students in improving their procedural knowledge when applying the Butterfly Method. In order to achieve the purpose of this study, the following research question was addressed: How do the intervention lessons that align with the Bar Model and Butterfly Method, have an impact on Year 9 students' performance on the addition and subtraction of fractions?

METHODS

This study employed the mixed-method approach in order to explore the new teaching strategies of the intervention. The stages of the research framework are shown in Figure 1.

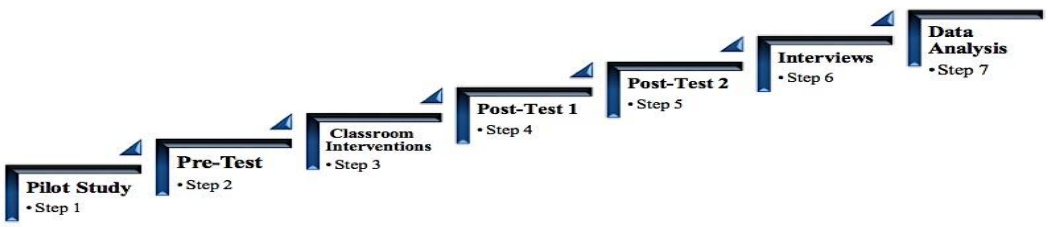

Figure 1. Research framework of the study

This first phase of the study employed the quantitative approach. Students' errors and misconceptions in the addition and subtraction of fractions were investigated using a modified research instrument from Idris and Narayanan (2011). Idris and Narayanan conducted a study on Form 2 students in Malaysia, and they discovered that the students developed all types of errors and the pattern of systematic errors in the operation of addition and subtraction of fractions. For this present study, the instrument was slightly revised with different questions but followed the same item levels. The revised instrument (the pre- and post-tests in Appendix 1) was used to investigate students' prior knowledge and procedures in attempting the questions, and to perceive if the Butterfly Method algorithm could be effectively applied to solve the different items that consisted of questions on adding and subtracting different types of fractions. In analyzing the test comparisons, descriptive statistics were utilized. Students' written analysis from the research instrument was used to identify how students develop the errors and misconceptions that may affect their conceptual understanding in computing fractions.

The second phase of the study adopted the qualitative approach through students' interviews. The interview transcripts were analyzed for similar themes and codes for reporting. This will help to shed light on why the misconceptions and/or the errors in order for the teacher to understand and support the development of the students' mathematical disposition. This approach was also used to investigate the impact of the intervention on improving students' performance in adding and subtracting fractions.

A total of 19 out of 22 Year 9 students from a Brunei Government secondary school participated in this study. There were three students who were not selected due to their infrequent attendance during the intervention lessons. The students were aged between 14 to 15 years old and are of mixed ability level based on their achievements from their previous examination results. The samples were chosen based on convenience, accessibility, and proximity.

The pre- and post-tests covers the addition and subtraction of different types of fractions (refer to Appendix 1). The test was designed in two sections where Section A tested students' knowledge on splitting the Bar Models, and Section B tested students' procedural knowledge of using the Butterfly Method. Calculators were not allowed during the 40-minute tests. Students were instructed to show the essential workings for each of the questions in Section B. After the intervention lessons, and two post-tests were conducted. The first post-test was conducted to analyze the data on the students' grades. The second post-test was conducted after one month to see if the students could recall the Bar Model concept and Butterfly Method, and to ensure the impact of the intervention and to identify if the students were able to retain the Butterfly Method.

A series of nine lessons were conducted after administrating the pre-test. The intervention lesson started with an activity on shading equivalent fractions on the squared-grid Bar Models. Students identified the equivalent fractions by folding a strip of paper and shaded the Bar Models. The second lesson introduced students to shade proper and improper fractions using the Bar Models with

brown colored paper as chocolate bars. This was used to relate the concept of Bar Models to the students' real-life objects. Lessons 2 to 5 introduced students to the Bar Model concepts on the addition and subtraction of fractions. The students practiced more on understanding the Bar Model concepts through group activities, quizzes, and classwork. From Lessons 6 to 8, the students were then introduced to the Butterfly Method. From these series of lessons, the students were taught the procedural method on the addition and subtraction of fractions. At the same time, the Bar Model concept was explained to ensure students could relate the new procedural method with the conceptual knowledge of shading the Bar Models. The Bar Model concept was drawn in rectangular bars where the bars are shaded and split according to the given fractions. Lesson 9 was used as a review lesson; students were given worksheets to practice adding and subtracting the different types of fractions. The students also applied the Butterfly Method while attempting the given questions and shaded the Bar Models. Subsequently, the students identified and related the concept of shading the Bar Model to the Butterfly Method.

RESULTS AND DISCUSSION

The students' overall results are shown in Table 1. The participants had shown improvement in the first post-test, which was administered after the intervention.

Table 1. The overall pre-test and post-test scores of the year 9 students

Type of Test	No. of Students	Mean Score ($\bar{X}$)	No. of Students who Passed the Test	Standard Deviation (*SD*)	Lowest Score	Highest Score
Pre-test	19	18.37	36.8 %	13.426	2	45
Post-test	19	28.05	73.7 %	8.243	14	44

There was a slight decrease in the mean score from the second post-test administered after one month, which drops to 24.6%. This shows that only a few of the students were able to retain the Bar Model concept and the Butterfly Method. After analyzing the students' responses from both the post-tests, not all of the students applied the Butterfly Method in the first and second post-tests. Still, only certain students could relate to the shading of the Bar Model concept with the Butterfly Method. Low-ability students are the majority who showed misconceptions using the Butterfly Method.

From the 19 participants, the highest score prior to the intervention was 45, and the lowest was 2 (from Table 1 above). However, the same participant had a slight decrease score of 44 in the first post-test and was able to score 46, which is slightly higher in the second post-test. The lowest score is 14 achieved by a low-ability participant that managed to display a slight improvement in the first post-test. The number of students who passed the first post-test displayed a higher percentage of

73.7%. Meanwhile, in the second post-test, the percentage of participants who passed the test dropped to 47.4%. The passing mark for the test is 50%.

The Impact of using the Bar Model with the Butterfly Method

The Year 9 students had no difficulty answering Questions 1 and 2 from section A. Section A consisted of 4 questions (refer to Appendix 1) where students are required to split and shade the given fraction bars. All the Year 9 students had no difficulty answering the first two questions from this section. From the pre-test, 42.1% of the students had incorrectly shaded the Bar Model while attempting Question 3, which investigated the addition of proper fractions with different denominators (. Meanwhile, after the post-test, the percentage dropped to 15.8%, where minor errors and misconceptions were observed from the students' written analysis. The students were able to relate the shading of the bar models visually to the addition of fractions while applying the Butterfly Method to add the two given fractions.

After the intervention, the mean scores for correctly shading the Bar Models in attempting Question 3 increased from 52.6% to 64.9%. There was a slight increase of 12.3% in the post-test. About 36.8% of the students incorrectly shaded the Bar Models in the pre-test. After the intervention, the percentage dropped to 21.1% for incorrectly shading the Bar Models. Question 4 represents the subtraction of fractions with different denominators. However, certain students still display misconceptions using the Butterfly Method to subtract the fractions.

The mean score for attempting Question 4 (which is the subtraction of two proper fractions with different denominators $\frac{1}{2} - \frac{2}{5}$) from the pre-test was 49.1%, and the post-test score displayed an increased score of 66.7%. There was a difference of 17.6% between the pre-test and post-test scores. Overall, the students had made improvements after the intervention for attempting Questions 3 and 4. The percentage decrease in incorrectly shading the Bar Models and increase in the percentage mean score after the intervention had helped to improved the students' performance in adding and subtracting of fractions with different denominators.

From the pre-test results, 21.1% of the students frequently added both the numerators and the denominators in the pre-test (refer to Figure 2 as one of the common errors). After the intervention, the misconceptions were reduced with the application of the Butterfly Method. The students did not tend to add both the numerators and the denominators together. When the Butterfly Method is applied, students cross-multiplied the numerator by the denominator of the other fraction. With reference to Figure 3, the impact of using the Butterfly Method on fraction addition had significantly reduced the students' misconceptions of adding both the numerators and denominators.

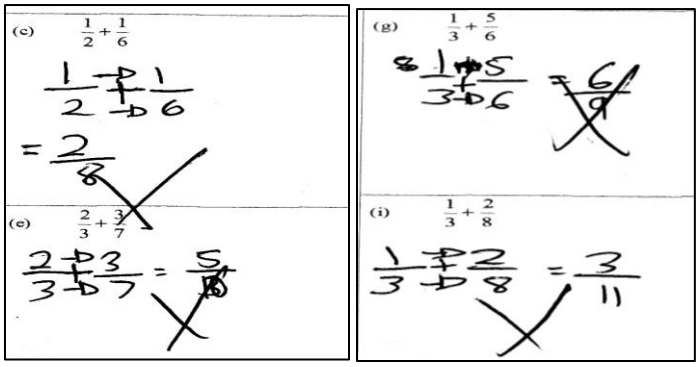

Figure 2. Students' incorrect responses from the pre-test

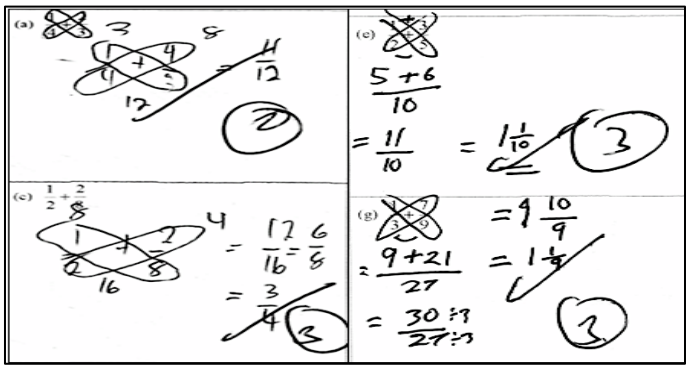

Figure 3. Students' correct written responses when applying the Butterfly Method

The Interviews with Students

A total of nine students were selected for the interviews. A brief demographic profile of the students is listed in Table 2. The students were selected based on their post-test percentage mean scores and the occurrences of the errors, which was analyzed from their written responses. The students were classified into three groups, according to the specified ranking, such as the low performance, average and high performance. The students were interviewed in the Mathematics Resource room during their break time. The interviews were conducted approximately 8 to 10 minutes, either individually or in pairs. However, the male students who were interviewed in groups of four lasted approximately 20 to 30 minutes. All interview sessions were audio-recorded and transcribed.

Table 2. Students' demographic details

Student ID	Gender	Achievement
ST1 and ST7	Male	Low
ST8	Male	Average
ST12, ST14 and ST16	Female	Average
ST13, ST15 and ST17	Female	Low

Note: 'ST' stands for Student. For example, ST1 refer to Student Number 1.

Theme 1: Students' responses for not applying Butterfly Method to the subtraction of fraction

When asked during the interview, "Why did you not use the Butterfly Method to all the questions?", there were three students (ST1, ST7, and ST8) who did not apply the Butterfly Method to all the questions from Section B in their post-test written analysis. For ST1, he preferred the formal method of calculating the Lowest Common Multiple (L.C.M) of the two fraction denominators. Meanwhile, ST7 started drawing the Butterfly Algorithm, consumed so much time to apply to all the questions, and ST8 explained that he had forgotten the Butterfly Method algorithm and had no confidence in applying it.

Whereas for the second group, all the girls gave the same views and responses for not applying the Butterfly Method only to the subtraction of fractions, yet they applied the Butterfly Method to questions on fractions addition. The reasons for the students not applying the Butterfly Method to the subtraction of fractions were due to no denominator present underneath the whole number, and the fractions had the same denominators. A sample of the students' written work is shown in Figure 4.

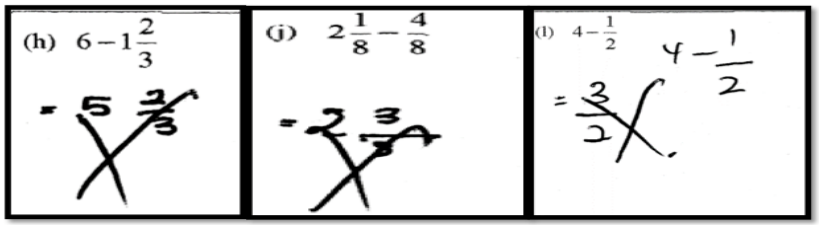

Figure 4. The students who did not apply the Butterfly Method

The students' written analysis from the post-test indicated the misconception of subtracting the second fraction of the whole number. Both Questions 5 (*h*) and 5 (*l*) contain whole numbers, and students thought that the whole number did not have any denominator. They found it difficult to apply the procedure of cross-multiplying the numerators with the denominators, even though they were told to put the number 1 underneath the whole number during the intervention. The students over-generalized the rules and misconceptions from the addition of fractions, which they treated both the numerators and denominators as two separate whole numbers. From these earlier misconceptions of fractions, students applied the same procedure and misconception to the subtraction of fractions (Dhlamini & Kibirige, 2014). The other students' responses for not applying the Butterfly Method for the subtraction of fractions are summarised in Table 3.

Table 3. Summary of students' responses for not applying the Butterfly Method

Students' Responses	Student ID
Not confident with the Butterfly Method	ST1, ST7 and ST8
Due to the whole number in front of it.	ST12, ST14 and ST15
There is no number (denominator) underneath the whole number	ST17 and ST16
Both of the proper fraction denominators are the same.	ST12 and ST13

From the interviews, the students were not aware that the whole number has a one underneath it and thought that it could never be cross-multiplied by the other number. Shown in Figure 5 below, one student managed to answer it correctly.

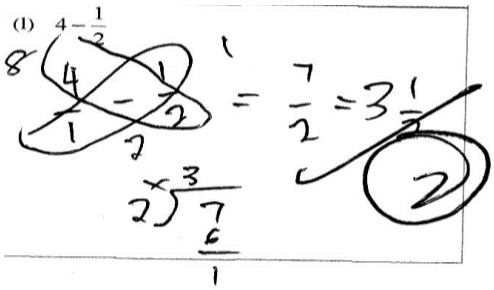

Figure 5. A student who applied the Butterfly Method

Theme 2: Views on Using the Bar Model with the Butterfly Method

The combined strategies were well received, and the students found it beneficial. ST15 explained that the Bar Models could be easily split according to the given denominators. Then, by applying the Butterfly Method to the given fraction, it can help her to check if she answered the question correctly.

All of the girls who were interviewed gave the same views on the combined strategies. One of the girls, ST14, explained that using the Bar Model directly gave her the answer, whereas the Butterfly Method helped her to check if her answer was wrong. She also explained that the Bar Model has a connection with the Butterfly Method. A sample of the ST14 written work is displayed in Figure 6.

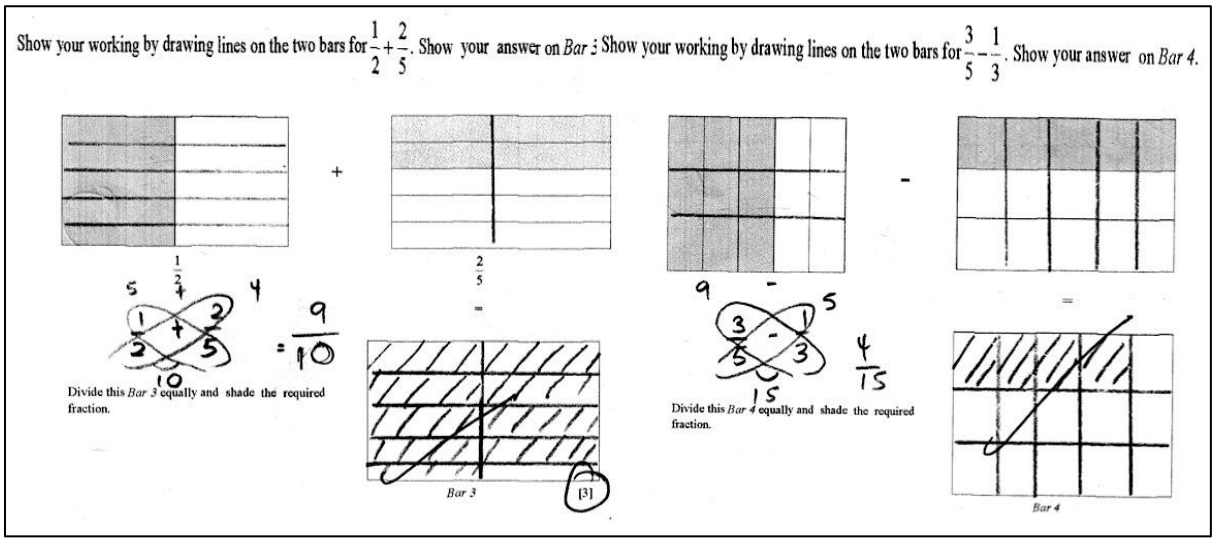

Figure 6. ST14 correctly used the Bar Model to add and subtract the fractions

One of the boys, ST1 who did not apply the Butterfly Method to solve Questions 3 and 4, explained that even though he used the formal method to answer the question, the splitting of the bars can be easily understood by making sure that both of the bars were split equally. After splitting the bars, he could use it to check if his earlier workings on adding the fractions were correct.

Theme 3: Students' Overall Opinion on the Impact of the Intervention

All the students who were interviewed gave positive responses on using the Butterfly Method to add fractions but found it difficult with subtracting fractions. The students stated that subtracting fractions with the same denominators made it difficult for them to reduce the answer. The majority of the students made careless mistakes, and some forgot to reduce their answer to the lowest term. The students who did not apply the Butterfly Method in the post-test were hoping to use it in the future provided more practice and guidance were to be given by the teacher. A summary of the students' responses is summarised in Table 4.

Table 4. Summary of the students' responses on their views regarding the intervention

Advantages	Students' Opinions
Visual	• Drawing the Butterfly on the fractions visually helped the students to remember the procedure.
Ease of use	• Easy to draw the Butterfly. • Need more practice and guidance with the Butterfly Method to answer different types of adding and subtracting of fractions. • Much easier to be used by adding fractions with unrelated denominators, but difficult to reduce when you apply the Butterfly Method to add fractions with the same denominators.
Effectiveness	• The splitting of the Bar Models is effective. • Helps to check and answer the given fractions when using the Butterfly Method.

Despite acknowledging the advantages of using the Butterfly Method, some of the students remained doubtful towards using it. For example, ST7 and ST8 might want to apply the Butterfly Method in the future, provided they are given more guidance and support from the teacher. These students were lacking in confidence, which hindered them from applying the Butterfly Method. In order to be familiar with and to build the students' confidence by using the Butterfly Method, more practice needs to be given to them. Wong and Evans (2007) stated that the recall of practice routine or procedure would help students to build their confidence and to be familiar with the algorithm.

Findings from the post-test results reflected that the students have the poor basic concept of multiplying and dividing whole numbers, which may also have led to poor performance in the test. The students' poor basic concept of multiplication led to other careless mistakes and errors that as were identified from the students' written works, such as incorrectly changing a mixed number into an improper fraction, carelessly wrote the wrong answer after cross-multiplying with the Butterfly Method and minor computation errors such as incorrectly adding and subtracting the fractions after cross-multiplying the numerator with the denominator. Seah and Booker (2005) discovered that students who demonstrated a limited understanding of the multiplication concept led to the students' knowledge being restricted to procedural rather than conceptual understanding.

The intervention lessons that were carried out for two weeks had helped students to understand the Butterfly concept, which is something new for the students. Nonetheless, teaching the procedure of applying the Butterfly Method to different types of items was a challenging task for the class teacher. Students who tend to rote memorize the procedures and could not connect to the Bar Model concept, such as splitting the bars, found it a bit challenging, especially for fractions with whole and mixed numbers. Cardone (2015) advised teachers to help students not to rote memorize procedures and attempt any shortcutting tricks, but to use Bar Models to make connections with the concepts of understanding fractions. For further improvement in conducting the intervention and prior to implementing the new teaching strategies, teachers should ensure that their students master the multiplication timetables. The concept of equivalent fractions should also be revised in order to avoid students making any careless errors. Teachers should teach with animated visual Manipulatives in order to show the splitting of Bar Models, and this will help students to visualize or to see the relationship of the Bar Model with the Butterfly Method in evaluating the given fractions. Students should also be given ample time to practice more on difficult questions, particularly with the subtraction of fractions. The common mathematical errors and misconceptions (Sarwadi & Shahrill, 2014) will need to be emphasized to the students and discussed throughout the intervention.

Subsequently, it is argued by Hunter, Hunter, & Restani (2020) that teachers often carry out rote practice activities rather than implementing collaborative discussions, which can develop students' mathematical understandings. This can be done by providing more opportunities in the practice of asking questions (Shahrill, 2013; Shahrill & Mundia, 2014; Shahrill & Clarke, 2014; Roslan, Panjang, Yusof, & Shahrill, 2018), discussing opinions, justifying their mathematical thoughts should be provided to ensure learning takes place with students' active engagement and creativity in learning (Khalid, Saad, Hamid, Abdullah, Ibrahim, & Shahrill, 2020). This enables them to be confident in expressing their mathematical thoughts and avoid uncertainties that may lead to misconceptions in a new teaching style. As was pointed out by Zakir (2018),"…being a teacher comes with the role to be adaptive towards any change requirements within their professional practice' (p. 50).

CONCLUSION

With reference to the research question, the students in this present study discovered that with the use of the Bar Model during the intervention could visually help them to split the rectangular bars according to the given fractions. From their answers on the Bar Models, the students were able to use it to check their answers with the Butterfly Method. They found that it was quite useful as it helped them to shade the bars for adding the fractions and reducing the bars for subtracting the fractions. From the interviews, the students explained that they could also see the connection of shading the bars with the algorithm of the Butterfly Method. However, the students who did not apply the Butterfly Method during the post-tests also found it quite useful to check their answers after splitting the bars while comparing it with the formal method. They explained that they should also be careful not to make any careless mistakes while shading the bars. The majority of the students who applied the Butterfly Method expressed their satisfaction in learning the new method from the intervention as they found it easier to help them to remember while cross-multiplying the numerator with the denominator. Conversely, students who lack confidence and did not apply the Butterfly Method such as subtracting a proper fraction from a whole number, subtracting proper fractions with the same denominators, and displaying other careless errors needed more practice and support from the teacher.

ACKNOWLEDGMENTS

The authors would like to express our gratitude to Dr Hui-Chuan Li for her initial guidance to this study. Our sincere appreciation also goes to the students, teachers, colleagues and the administrators of the research school sample site.

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Appendix 1: The Pre- and Post-Tests

Student Code No:

Test on Fractions

Section A

1) Divide the bar below equally then shade $\frac{4}{6}$

[1]

2) Write down the fraction for the answer $\frac{7}{8}-\frac{5}{8}=$ ☐

[1]

3) Show your working $\frac{1}{3}+\frac{1}{2}$ below. Use the bars below to help you answer the question.

Working:

$\frac{1}{3}$ + $\frac{1}{2}$

Working: =

Divide this bar equally and shade the required bars.

[3]

4) Show your working for $\frac{1}{2}-\frac{2}{5}$ by using the bar below.

Working:

Working: =

Divide this bar equally and shade the required bars.

[3]

Section B

Show your working for each of the following questions below. **NO** calculators are allowed.

5) Show all the steps for your working.

(a) $\frac{1}{4}+\frac{1}{3}$ Working:	(b) $\frac{3}{4}-\frac{1}{4}$ Working:
(c) $\frac{1}{2}+\frac{1}{6}$	(d) $6\frac{1}{2}-2$
(e) $\frac{2}{3}+\frac{3}{7}$	(f) $7\frac{5}{6}-\frac{1}{6}$
(g) $\frac{1}{3}+\frac{5}{6}$	(h) $5-1\frac{1}{3}$
(i) $\frac{1}{3}+\frac{2}{8}$	(j) $3\frac{1}{7}-\frac{4}{7}$
(k) $\frac{5}{12}+1\frac{1}{3}$	(l) $3-\frac{1}{2}$
(m) $7\frac{3}{4}+\frac{1}{2}$	(n) $1\frac{1}{3}-\frac{1}{4}$
(o) $4\frac{1}{5}+1\frac{1}{4}$	(p) $4\frac{1}{8}-1\frac{7}{8}$

[40]