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From Parts to Wholes: Investigating Fraction Division through Partitioning Strategies

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Abstract

Fraction division is considered one of the most difficult concepts in learning fractions. This study aims to investigate students' understanding of fraction division through a partition division conceptualization, utilizing their understanding of fractions as parts of a whole. To achieve this, we designed a hypothetical learning trajectory, in which students engaged with incomplete partition tasks. This article reports on a two-week design research intervention involving 21 fifth-grade students. The students' written works, transcripts of teaching experiment recordings, and observational notes were retrospectively analyzed to examine the hypothetical learning trajectory. The study revealed that the students' primary challenge was recognizing the correct whole during the teaching experiment. This study suggests that incorporating a stronger focus on proportional reasoning and varying fraction sizes in instructional approaches may help address this obstacle.

Keywords: Fraction Division, Fair-sharing, Hypothetical Learning Trajectory, Partition Division, Part-whole

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INTRODUCTION

Fraction plays a critical role in the development of students' deep understanding of mathematics (Sidney et al., 2019). However, fraction is considered a difficult subject of elementary mathematics (Sari et al., 2024; Siegler et al., 2013; Wilkins & Norton, 2018) and fraction division is considered the most difficult concept (Rule & Hallagan, 2006; Shin & Bryant, 2015). How to promote students' understanding of the fraction division concept has been a pertinent question. To address this question, it is important to understand how to formulate a proper learning trajectory on fraction division, as such a learning trajectory positively impacts students' understanding (Ivars et al., 2019; Prediger et al., 2022; Setambah et al., 2021).

Adu-Gyamfi et al. (2019) stated that teachers' knowledge of fraction division should extend beyond primary computational competency. This computational competency is regarded as fundamental when teachers employ two basic algorithms: invert-and-multiply and common denominator (Sharp & Adams, 2002). In line with this, Streefland (1991) asserted that teaching fractions in early education should be based on realistic mathematical situations relevant to children, as these situations help them connect fractional concepts while addressing real-world problems. The criteria for these realistic situations should include a common base of experiences drawn from children's real-world knowledge (Sharp & Adams, 2002).

Prior research has summarized fraction division into several conceptualizations: 1) measurement division, 2) partition division, 3) determination of a unit rate, 4) division as the inverse of multiplication, and 5) division as the inverse of Cartesian product (Sinicrope et al., 2002). Meanwhile, other studies

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divided fraction division into two distinct interpretations: measurement division and partitive division (Van de Walle et al., 2008; Zaleta, 2006). Table 1 describes the differences between measurement and partitive interpretations (Bulgar, 2003; Son & Senk, 2010; Adu-Gyamfi et al., 2019) as these two meanings are both essential for the development of various types of fraction problems (Van de Walle et al., 2008).

Table 1. The interpretation of fraction division

Interpretation	Definition	Activity	Example of Problem
Partitive	Sharing a given selection of objects into identical parts to determine the number of objects in each part.	Sharing Activity	$\frac{1}{2}:3$ can be interpreted as $\frac{1}{2}$ pizza shared equally among 3 friends.
Measurement	The number of groups or sets formed from a given total of objects, with an identical quantity of objects in each set or group.	Partitioning Activity	$6:\frac{3}{4}$ can be interpreted as how many $\frac{3}{4}$ kg boxes of candy can be made from 6 kg of candy?

Prior studies agree that the easiest meaning of fraction division to demonstrate is measurement (Ott et al., 1991). This is because partitive fraction division involves transforming the dividend into a fraction whose numerator is divisible by the numerator of the divisor, making it more complex compared to the more intuitive measurement interpretation, which focuses on how many times the divisor fits into the dividend (Li, 2008). However, other researchers also agree that both interpretations are needed to develop different types of fraction problems (Van de Walle et al., 2008).

Van de Walle et al. (2008) stated that the situation involved in partitive fraction division is sharing. When the teacher presents the concept of partitive fraction division during instruction, students may distribute objects into groups to provide an equal number of objects to each group (Purnomo et al., 2021; Roche & Clarke, 2013; Son & Senk, 2010). The partitive fraction division example in Table 1 illustrates a typical problem of partitive fraction division within a fair-sharing context (Wahyu et al., 2020), which has been utilized in various studies to explore students' understanding of partitive fraction division. For instance, Zaleta (2006) highlighted the importance of unitizing in solving partitive problems. He proposed a two-step approach for solving partitive fraction division: 1) students need to divide the objects into the specified equal parts, and 2) obtain the desired number of such parts. Wahyu et al. (2020), Lo and Luo (2012), and Low et al. (2020) emphasized several key design principles to support students' understanding of partitive fraction division: 1) developing students' sense of fair sharing through whole number partitive division, 2) strengthening the concept of the unit in fractions and partitioning, 3) selecting appropriate contexts and graphical representations, and 4) sequencing the fractions used.

On the other hand, Pramudiani et al. (2020) conducted research on students' understanding in an incomplete partition situation. They emphasized the significance of incomplete situation problems that address the meaning of fractions as parts of a whole. In addition, students' strong understanding of part-whole relationships forms a foundation for fraction division (Cramer et al., 2010). Building on their works, this study aims to extend their findings regarding students' comprehension of fractions as parts of a whole in the context of partitive fraction division. Therefore, we designed a learning trajectory that highlights the relationship between partitive fraction division and the part-whole subconstruct. Furthermore, the part-whole subconstruct is commonly utilized by both teachers and students (Lamon, 2012). Consequently, the question this study addresses is: How do students utilize their understanding of fractions as parts of a whole to comprehend the concept of partition division?

METHODS

Research Type

This study employed the design research, aimed to formulate a theory on how to teach specific topics and comprised three stages: preparation and design, design experiment, and retrospective analysis (Bakker, 2018). Additionally, design research seeks to identify a trajectory that guides students from their prior knowledge to the learning goals (Putri et al., 2021), which will address this study's research question. This implies that determining the learning goals and understanding students' prior knowledge are crucial in a design research study. The learning goal of this study was to examine how students understand partitive fraction division problems by utilizing their comprehension of fractions as parts of a whole in an incomplete situation. Therefore, this study designed a learning trajectory that includes a sequence of tasks, which will subsequently be tested and retrospectively analyzed to determine whether it aligns with the intended goals.

Research Procedure

The preparation and design stage begins with defining the mathematical learning objectives to explore the fundamental ideas of the domain (Gravemeijer & Cobb, 2006). To investigate these fundamental ideas, a literature review is conducted, and the learning trajectory is designed based on the findings from the review. Following this, the design experiment occurs in two phases: the pilot experiment and the teaching experiment, each lasting sixty to ninety minutes. During the lesson, the tasks guide students from reinforcing their prior knowledge of the part-whole subconstruct to applying it in the context of partitive fraction division. A retrospective analysis was conducted to compare the actual learning outcomes with the learning trajectory. Such an analysis provides an overview that can help identify insights into why particular learning occurs or does not occur (Bakker, 2018).

Research Participants

The participants of this study were 21 fifth-grade students. The pilot experiment involved three fifth graders, comprising low-, middle-, and high-performing students. Before conducting the pilot experiment, we ensured that the students possessed several foundational knowledge areas (Nieveen et al., 2006): an understanding of fractions as parts of a whole and proficiency in partitive division. Three students with heterogeneous mathematical performances participated in a pilot experiment to measure their perceptions of partitive fraction division problems and to identify areas for refinement in the learning trajectory. In the teaching experiment, the participants consisted of eighteen fifth graders in a whole-class setting. The class was diverse, comprising low-, middle-, and high-achieving students, as determined by the teacher.

Data Collection and Analysis

The data sources for this study included students' written work, recordings of the teaching experiment, and observational notes from students' whole-class discussions. The data analysis procedure was retrospective analysis, comparing the learning trajectory with the actual learning outcomes to determine whether the learning goals were addressed. Specifically, the retrospective analysis consisted of: 1) examining the students' written work, 2) transcribing the recordings of the teaching experiment, and 3) describing students' AHA moments during whole-class discussions as noted in the observational records.

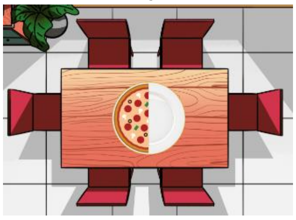
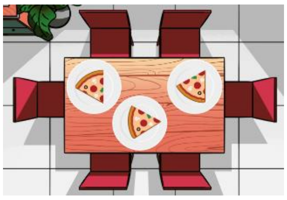
RESULTS AND DISCUSSION

Preparation and Design Stage

We based the learning goals on the Indonesian Mathematics Curriculum established by the Ministry of Education, which expects students to comprehend fraction division with natural numbers. The literature review revealed that prior studies emphasize the importance of foundational concepts such as unitizing and fair sharing in supporting students' understanding of partitive fraction division (Wahyu et al., 2020; Zaleta, 2006). Yeo (2019) observed that, prior to formal instruction, students solving partitive fraction division problems often conceptualize fractions as parts of a whole, from which they derive the interpretation of the quotient. Similarly, Pramudiani et al. (2022), in their study assessing students' understanding of fractions as parts of a whole, utilized incomplete situation problems and concluded that providing an incomplete whole to determine a fraction has the potential to encourage mathematical reasoning about the meaning of fractions. Moreover, Čadež and Kolar (2018) demonstrated that when students identify parts of an equally divided but incomplete whole, they exhibit an understanding of fractions as parts of a whole.

Based on these findings, our study aims to extend students' understanding of the part-whole subconstruct to partitive fraction division through the use of incomplete partition situation problems. The learning goal is to enable students to understand partitive fraction division by reinforcing their understanding of fractions as parts of a whole in incomplete situations. Additionally, fractions can be represented in various contexts. Therefore, we formulated a second question to expose students to different approaches to fractions. Table 2 presents the conceptualization of the tasks used in this study.

Table 2. Task design of partitive fraction division

No.	Question	Context	Content Area
1a	How would you share a half of a pizza to three of you? 	Pizza sharing	Fair-sharing
1b	What fraction of the parts did you get? 		Partitive fraction division
2	There is only half an hour left for the Mathematics End-of-Semester Assessment, and you still have four questions to complete. Can you solve all the problems on time? If so, then how do you think you can do it?	Time	Partitive fraction division

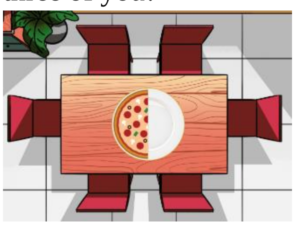
For question 1a, we conjectured that students would face difficulty dividing half of a circular object (pizza) into three equal parts. Research indicates that dividing into an odd number of shares can be particularly challenging due to the inherent discomfort associated with creating non-intuitive partitions (Yeo, 2019). However, in their study, Čadež and Kolar (2018) observed that folding is frequently used to solve such task on continuous models to create equal parts that are congruent. For question 1b, we conjectured that students might answer $\frac{1}{3}$ instead of $\frac{1}{6}$ because, in the study by Čadež and Kolar (2018), one common mistake made by pupils was connecting the idea of fractions only to visually congruent parts of the whole. Last, for question 2, we conjectured that student would be able to connect the time context to the graphical representation, as clock in the classroom usually in round shape, just like pizza.

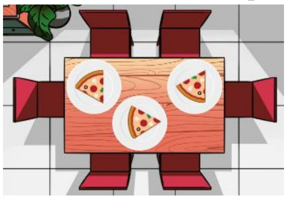
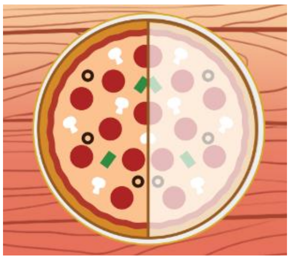
Design Experiment

Pilot Experiment

The pilot experiment indicated that the students could not perceive question 2 as a partitive fraction division problem. Instead, they translated half an hour into 30 minutes and then divided 30 by 4, resulting in 7.5. According to the study conducted by Wahyu et al. (2020) on the partitive fraction division, it was found that using the context of sharing cakes is more effective in helping students comprehend partitive fraction division. Furthermore, it was noted that the utilization of specific fractions can assist students in generating visual representations (Laidin & Tengah, 2021). Regarding this, the current study tried to provide a learning trajectory on partition division using fair-sharing context. For question 1b, the results are aligned with the conjecture we made. According to the study conducted by Wahyu et al. (2020) on partitive fraction division, using the context of sharing cakes is more effective in helping students understand partitive fraction division. Furthermore, it was found that the utilization of specific fractions can assist students in generating visual representations. In this regard, the current study aimed to provide a learning trajectory on partitive fraction division using a fair-sharing context. For question 1b, the results align with the conjecture we made. All students answered 1/3 to represent the fraction of each pizza slice. Hence, we concluded that we need to refine the learning trajectory for students to be able to construct the quotient from part-whole subconstruct. Since we want students to focus on construct the partitive fraction division from part-whole subconstruct, we chose to remove question 2 and elaborate question 1. Therefore, we added question 2a, 3, and 4 and conjectured that through those questions students notice the difference between the whole they refer to (question 2a), did not find it too difficult to determine the parts of the correct whole (question 3), and formulate the formal notation of partitive fraction division problem (question 4). Table 3 serves as the revised version of the learning trajectory. Therefore, we added questions 2a, 3, and 4 and conjectured that through these questions, students would recognize the difference between the whole they refer to in question 2a, find it manageable to determine the parts of the correct whole in question 3, and formulate the formal notation of the partitive fraction division problem in question 4. Table 3 presents the revised version of the learning trajectory.

Table 3. Revised version of partitive fraction division task design

No.	Question	Content Area
1	How would you share a half of a pizza to three of you? 	Fair-sharing

No.	Question	Content Area
2a	How many slices will each of you get? What fraction of the parts did you get?	Unitizing
2b		
3	How many do each of you will get compared to the whole? 	Part-whole subconstruct
4	What is the result of $\frac{1}{2} : 3$?	Partitive fraction division

Teaching Experiment

Fair Sharing

We first describe the results of the teaching experiment by presenting students' written work, excerpts from the teaching experiment, and whole-class discussions. In general, two main strategies emerged: paper folding and ruler measurement. Paper folding enables students to create and verify three equal parts through a trial-and-error process, ensuring that each edge aligns with the others (see Figure 1).

Figure 1. Students' strategy using paper folding

Another strategy was ruler-measure, where they visually and physically measure parts of a pizza to divide the arc into three equal segments. The students measured the radius of half of the pizza, using the radius to estimate point on the arc that would divide it into three equal parts. They then measured the length between the radius and the dividing point (see Figure 2). Below is the conversation script between the teacher and SY.

Teacher : *How do you know that the pizza slices are of the same size?*
SY : *(the length) It's seven cm! Then we must make sure if the other mark (to the radius) is also 7 cm.*

Figure 2. Students' strategy using measuring ruler

Referring to the Correct-Whole

All students answered 'one slice' for question 2a. Consequently, when they were asked question 2b regarding the fraction of the parts they received, they answered "one-third". The teacher then instructed the students to proceed to question 3. Below is the transcript of the conversation between the teacher and the students.

Teacher : *If you have three slices of this half of pizza (showing half pizza paper), how many slices would you gave from a whole pizza (bring closer another half pizza paper).*
Students : *Six!*
Teacher : *Then, what fraction of this part that you get?*

We heard one-third here and there. However, there is one student that expressed her AHA moment and below is the conversation script between the teacher and DL.

DL : *Ah! I Know! One-sixth! (confidently).*
Teacher : *Could you explain your answer, DL?*
DL : *Because there are six slices of pizza in total, therefore that one slice equals one-sixth.*

To further examine DL's understanding, the teacher asked her what fraction of the part each person would receive if half of the pizza were shared among two people. DL answered correctly: one-fourth, which indicated that she was able to refer to the correct whole; in other words, she determined the denominator by identifying how many parts constitute the whole. Another instance of students' ability to refer to the correct whole is described in the conversation transcript below.

Teacher : *Okay DA, I heard that you answered one-third. Could you explain your answer? Because now we have two different answers. One-sixth and one-third.*
DA : *We only have half of the pizza, and we divided it into three, so it (one part from three parts in total from half of pizza) should be one-third.*
Teacher : *Okay now, If I have this one whole pizza and share it among three people. What*

		fraction each one gets?
DA	:	*One-third.*
Teacher	:	*In your opinion, are these one-thirds the same?*
DA	:	*No (uncertain voice)… one is smaller (than the other one).*

The teacher once again brought up the pizza paper and the following conversation happened.

Teacher	:	*What fraction if this slice (the one half of pizza shared to three people) compared to the whole pizza?*
DA	:	*Oh, that's why DL's answer is one-sixth. (AHA moment).*
Teacher	:	*Why, in your opinion, DL answered one-sixth?*
DA	:	*Because this half contains three slices, and there is another half which means another three slices. So, the total should be six slices.*
Teacher	:	*Therefore…? (what fraction).*
DA	:	*One-sixth is the answer, Miss.*

We also further examined how students perceived the partitive fraction division. Therefore, we asked one student in the conversation below. AI used the term "one-over-two" to represent half, demonstrating her recognition of the whole and her ability to decompose it into equal parts. She made connections between the numerator (the number of parts considered) and the denominator (the number of parts that constitute the whole).

Teacher	:	*Now, what fraction of the part you get, if this half of pizza was being shared to four of you?*
AI	:	*(drawing something) (see Figure 3)* *One-eighth…one over two divided by four.*
Teacher	:	*Equals to?*
AI	:	*One-eighth.*

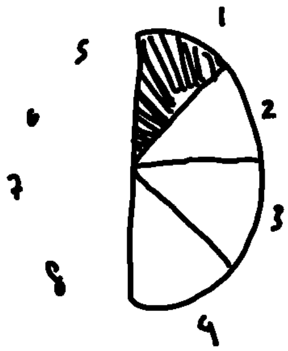


Figure 3. Representation of one-eighth of pizza by student

Retrospective Analysis

This study aimed to bridge students' understanding of partitive fraction division through their understanding of fractions as parts of a whole. For question 1, as in the pilot experiment, the students were able to share half of pizza with three people. There were two strategies emerged in fair-sharing question. The first strategy is paper folding. Folding represents non-counting actions (Čadež & Kolar,

2018) and helps students internalize the process of partitioning a whole into equal parts (Wilkins & Norton, 2018). This strategy requires students to physically divide a whole object into equal segments, reinforcing the idea that each segment is a fractional part of the entire whole. Based on the results of Cramer et al. (2010) study, paper folding strengthens students' understanding of part-to-whole relationships, forming the foundation for constructing strategies to solve fraction division problems. The second strategy is ruler-measure where the students take 7 cm (from the central point to the arc) as the guidance to divide it into three equal segments. Bulgar (2003) explained this strategy as reasoning involving measurement, where students created a unit of measurement and used it to divide an object equally into the required number of parts.

For the solution to be accurate, students needed to provide both the correct answer and sound reasoning. As illustrated in AI's response (Figure 3), she demonstrated her reasoning by modelling the partitioning. Adu-Gyamfi et al. (2019) define this process as modelling the problem with a part-whole diagram and employing a partitive interpretation of fraction division.

However, initially, students may struggle to refer to the correct whole. This finding aligns with our conjecture. For instance, DA's first response indicated that he recognized the whole as the remaining half of the pizza. Therefore, his answer was $\frac{1}{3}$. What he missed out was distributing threes on the other half of pizza, which by Shin and Lee (2018) called as multiplicatively nested unit fraction, because the $\frac{1}{3}$ is nested in the half ($\frac{1}{2}$). Once DA realized that the one-third he meant refer to the remaining pizza not the whole pizza, he noted that each person should get one-third of the remaining pizza, which is $\frac{1}{6}$. Based on their study, Lo and Luo (2012) mentioned that posing appropriate pictorial representation relates to the correct whole in solving partitive fraction division problem and this is also in line with the result of Pramudiani et al. (2022) and Low et al. (2020) study.

The results of the teaching experiment yielded two main findings: 1) the mental action of partitioning not only reinforces the idea that each segment is a fractional part of the whole but also aids students in constructing meaningful strategies for solving partitive fraction division problems; 2) the use of incomplete partition problems supports students' understanding of a fraction as representing a part in relation to a specific whole. Based on these findings, we propose the following design principles to support students' understanding of partitive fraction division by providing students with incomplete partition problems as a context is promising for bridging their understanding from part-whole to partitive fraction division because it 1) encourages students to employ the mental action of partitioning and develop meaningful strategies for solving partitive fraction division problems, and 2) engages students in experiencing situations where the whole is context-dependent, thereby enhancing their ability to reason about fractions in relation to varying wholes. However, further research is necessary to explore the extent to which the part-whole subconstruct aids students' understanding of partitive fraction division. For instance, it would be valuable to investigate which other mental actions within the part-whole subconstruct emerge or potentially interfere with students' reasoning.

CONCLUSION

This study highlights the challenges students face when working with partition division, particularly in identifying the correct whole. The findings suggest that while some instructional strategies and visual aids were helpful, they were not universally effective. This suggests that enhancing partition division understanding requires incorporating a stronger focus on proportional reasoning and varying fraction sizes into instructional approaches. However, the study's small sample size may limit the generalizability of the findings. In addition, the study's focus on specific instructional interventions means other potentially effective methods were not explored. Future research should explore the impact of instructional designs that emphasize proportional reasoning and fraction comparison across diverse student populations.

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