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From Time-Series Analysis to PCA Clustering: Exploring the Impact of Graphing Quadratic Worksheets on Mathematical Visual Thinking Skills

Ria Noviana Agus, Rina Oktaviyanthi*, Usep Sholahudin

[1]Department of Mathematics Education, Universitas Serang Raya, Serang, Indonesia
*Email: rinaokta@unsera.ac.id

Abstract

The effectiveness of visualization tools in enhancing mathematical visual thinking skills, particularly for quadratic functions, remains underexplored. This study evaluates the impact of quadratic graph visualization worksheets on these skills using Time-Series and Principal Component Analysis (PCA) Clustering approaches. The research involved 60 first-year Calculus students focusing on quadratic functions. A quantitative methodology was applied, with Time-Series analysis tracking score changes over time and PCA Clustering grouping students based on improvement patterns. Results revealed significant variations in score changes after using the worksheets. The highest positive score change reached 19 points, while PCA Clustering identified three student groups: minimal or negative changes (Cluster 0), moderate improvements (Cluster 1), and significant increases (Cluster 2). The findings demonstrate the potential of quadratic graph visualization worksheets to improve mathematical visual thinking skills, though the degree of enhancement varies across individuals. This research highlights the need for instructional tools that accommodate diverse learning trajectories and provides insights into the effectiveness of graph-based methods in mathematics education. It also advocates for refined analytical approaches in evaluating student learning outcomes.

Keywords: Cluster Analysis, Graphing Quadratic Worksheet, Mathematical Visual Thinking Skills, Principal Component Analysis, Time-Series Analysis

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INTRODUCTION

Mathematical visual thinking is a crucial skill in higher education, particularly for students engaged in advanced mathematics such as calculus. The ability to visualize concepts, such as quadratic functions, enhances students' understanding of relationships between variables and demonstrates how changes in inputs affect outputs (Hawes & Ansari, 2020; Odutayo & Fonseca, 2024; Presmeg, 2020; Ziatdinov & Valles, 2022). This ability serves as a bridge between abstract concepts and real-world applications, enabling students to comprehend complex ideas that are often difficult to articulate using symbolic algebra or verbal reasoning alone (Arnheim, 2020; Elsayed & Al-Najrani, 2021; Hamami & Morris, 2020; Staton, 2023). In today's data-driven world, the ability to interpret visual patterns is indispensable across disciplines such as business, engineering, and data science, as it strengthens problem-solving and critical thinking skills. Despite its importance, classroom observations reveal that many students struggle with mathematical visual thinking. For instance, a survey of 120 calculus students found that 65% experienced difficulty graphing quadratic functions, while 72% faced challenges in forming quadratic equations from visual data. These findings underscore a gap in students' visual thinking abilities, which hinder their ability to connect abstract concepts with graphical representations.

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To foster the development of mathematical visual thinking, graphing worksheets play a pivotal role. Worksheets focused on quadratic function visualizations allow students to explore visual representations and identify patterns that traditional algebraic methods might overlook (Göbel, 2021; López & Vivier, 2023; Oktaviyanthi & Agus, 2023). These tools not only deepen students' conceptual understanding of quadratic graphs but also enhance their visual thinking abilities (Kohnle et al., 2020; Martins et al., 2023; Teófilo De Sousa et al., 2022). Despite their widespread use, there has been limited evaluation of their effectiveness in improving mathematical visual thinking. Classroom observations reveal that current worksheets often fail to address students' specific challenges, as evidenced by their inconsistent performance on graphing tasks. These findings raise important questions about the design and implementation of such tools and whether significant improvements are needed to better support student learning.

In educational settings, students’ abilities to engage with visual representations vary widely. While some students excel in interpreting graphs and patterns, others require additional support or tailored approaches to fully grasp the material (Kohen et al., 2022; Mainali, 2021; Tsandilas, 2021). A "one-size-fits-all" approach often falls short in addressing these diverse needs. This highlights the importance of developing nuanced evaluation systems that assess not only the overall effectiveness of visual worksheets but also the differences in students’ abilities. By identifying clusters of students who need extra support or greater challenges, educators can implement more personalized interventions, ultimately improving learning outcomes (Moubayed et al., 2020; Namoun & Alshanqiti, 2020; Wei et al., 2021). These observations underscore the urgency of designing worksheets that are both adaptable to diverse student needs and effective in addressing specific weaknesses in visual thinking.

To address this gap, the combination of time-series analysis and PCA offers a robust framework for evaluating quadratic graph visualization worksheets. Time-series analysis monitors students' mathematical visual thinking skills over time, uncovering trends in their development (Ariens et al., 2020; Xu et al., 2020; Yang & Wang, 2023). This approach enables educators to track progress and assess improvements, particularly in students who initially struggled with quadratic concepts. PCA, on the other hand, clusters students based on their performance, identifying common characteristics and factors influencing the development of visual thinking skills (Iatrellis et al., 2021; Li et al., 2021; Liu et al., 2024), enabling targeted interventions.

Previous research underscores the critical role of visual thinking in understanding quadratic graphs. Studies by Rolfes et al. (2020) and Wilkie (2021, 2024) demonstrate that visual aids, such as graphing worksheets, significantly enhance students' comprehension of variable relationships in quadratic functions. These tools serve as a bridge between abstract equations and real-world applications, ultimately improving performance (Guo et al., 2020; Oktaviyanthi & Agus, 2023). Furthermore, Gatto et al. (2024) also Özsoy and Saribas, 2021 highlight how visual worksheets assist students in solving quadratic equations by illustrating the impact of coefficients on graph shapes.

However, while these studies provide valuable descriptive insights, they often lack quantitative evaluation of how visual thinking develops over time.

Recently, studies by Chan et al. (2022), Oktaviyanthi and Sholahudin (2023), and Yohannes and Chen (2023) have emphasized the integration of technology in mathematics education. These studies demonstrate how interactive simulations and visual tools can enhance students' comprehension of abstract mathematical concepts, such as quadratic functions. However, much of the existing research lacks advanced data analysis techniques, such as time-series or PCA clustering, which limits the empirical evidence on the impact of visual worksheets on students' visual thinking development. This study aims to address this gap by adopting a data-driven approach that combines time-series analysis and PCA clustering. This methodology provides a more comprehensive evaluation of the effectiveness of visual worksheets in enhancing mathematical visual thinking skills over time.

METHODS

Research Approach

This study adopted a quantitative approach using a quasi-experimental design, specifically the One-Group Pretest-Posttest Design. In this design, pre-test and post-test assessments were administered to the same group of participants to measure changes in mathematical visual thinking skills before and after the intervention. This approach eliminates the need for a control group by enabling a direct comparison of students' performance over time (Morgan et al., 2000; Reichardt et al., 2023). To address the effectiveness of a quadratic graph visualization worksheet in enhancing students' mathematical visual thinking skills, the study employed two analytical methods:

1. Time-Series Analysis: This method tracked changes in students' mathematical visual thinking skills at multiple time points, providing insights into the progression of these skills throughout the intervention period.
2. PCA: PCA clustering analysis was employed to identify underlying patterns and trends in the data, helping to assess the overall impact of the worksheet on students' performance.

The research builds on established sources to support its design, from Alqahtani et al. (2021) and Hirose and Creswell (2023), which offer foundational insights into quantitative research methods and the application of PCA and time-series analysis in educational studies. By integrating these approaches, the study effectively captures dynamic changes over time and identifies significant patterns in the data, providing a comprehensive evaluation of the intervention’s effectiveness.

Research Subject

The population for this study consisted of college students enrolled in a calculus course. The sample was selected through purposive sampling and included 60 first-year students studying calculus and quadratic function concepts. The primary instruments used were a quadratic graph visualization worksheet, pre-test, and post-test, all designed to assess students' mathematical visual thinking skills. Data collection also incorporated observations and questionnaires to capture students' perceptions of the worksheet.

Data Collection Instrument

After validating the quadratic graph visualization worksheet (Oktaviyanthi & Agus, 2023), the study began with a pre-test to evaluate students' baseline mathematical visual thinking skills. The pre-test assessed key indicators, including understanding quadratic graphs, drawing graphs from functions, and interpreting changes based on function modifications. The worksheet was then implemented as an intervention to enhance students' understanding of quadratic graphs. Following the intervention, a post-test was administered to measure improvements in skills such as graphing, analyzing quadratic graphs, and applying quadratic concepts to various problems.

Data Analysis

Time-series analysis was conducted using data collected periodically from the pre-test, worksheet usage, and post-test results. This analysis tracked changes in students' mathematical visual thinking skills over time, with trends visualized through line graphs and regression analysis (Deng et al., 2024; Schlegel & Keim, 2021). Additionally, PCA clustering analysis identified patterns in student performance, grouping students with similar characteristics, and visualizing the results using scatter plots and dendrograms to highlight improvement trends and response variability (Chang et al., 2020; Mohamed Nafuri et al., 2022). Python programming supported the data analysis, utilizing libraries such as pandas for data manipulation, matplotlib and seaborn for visualization, and scikit-learn for PCA clustering. These tools streamlined the handling of large datasets and facilitated precise statistical analysis (Alrammal et al., 2022; Khandare et al., 2023). The study concluded that if time-series analysis showed significant improvement in students' visual thinking skills and PCA clustering revealed consistent patterns of improvement among groups, the worksheet was effective enhancing these skills. Recommendations were provided for refining instructional materials and teaching strategies based on observed student variability and performance trends.

RESULTS AND DISCUSSION

Score Changes

The initial examination of the pre-test and post-test data reveals several important trends in the students' performance after engaging with the quadratic graph visualization worksheet. Across the sample of 60 students, there is a notable overall improvement in post-test scores when compared to pre-test scores, indicating a positive impact of the worksheet intervention on mathematical visual thinking skills. Observe the visualization of the distribution of pre-test and post-test score changes for 60 subjects in Figure 1.

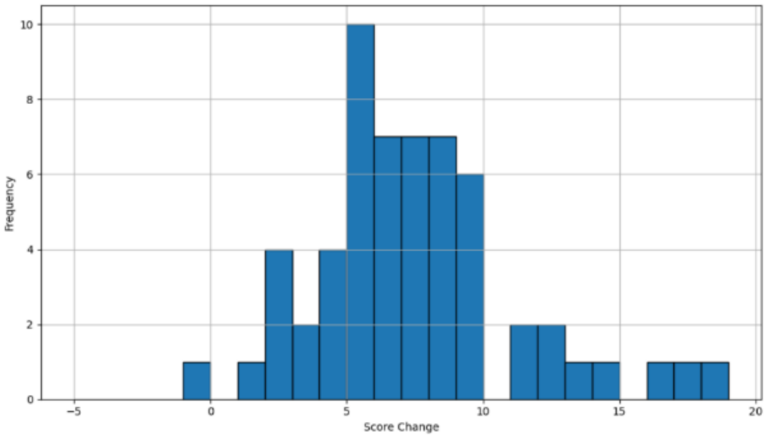


Figure 1. Distribution of score changes between pre-test and post-test

The most striking feature of the dataset is the improvement in students' scores from pre-test to post-test. As shown in Figure 1, it can be observed that the majority of students experienced an increase in their scores, with individual score changes ranging from -1 to $+19$ points. Notably, student 5 exhibited the highest score increase of 19 points, rising from a pre-test score of 35 to a post-test score of 54. On the other hand, only one student (student 3) showed a minor decrease of 1 point.

Mean Score and Variance

The following is an illustration of the mean scores and data variance.

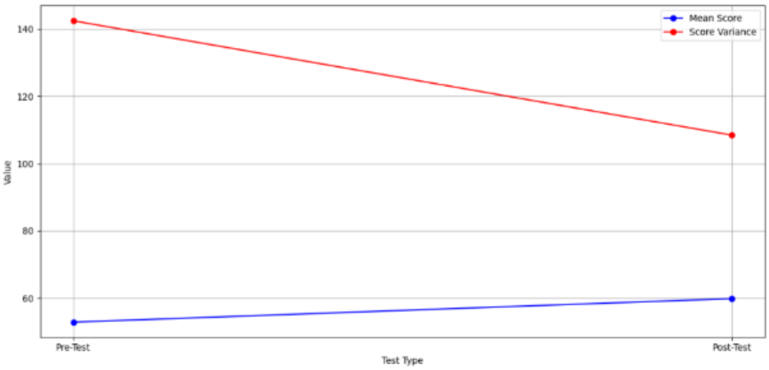


Figure 2. Mean and variance score for pre-test and post-test

Based on Figure 2, it can be explained that the mean score for each student was calculated as the average of their pre-test and post-test scores. While the mean scores for most students tend to cluster around the middle of the scoring range (between 40 and 70), a few outliers can be observed. For instance, student 5, who had a significant improvement, still had a lower mean score (44.5) compared to higher-performing students like student 39, whose mean score was 71.5. The score variance for each student provides insights into the consistency of their performance across the pre-test and post-test. Students with low variance, such as students 3 and 4, exhibited stable performance with minimal fluctuations in their scores (variance $=$ 0.5). In contrast, students with higher variance, such as student 5 (variance $=$ 90.5), experienced significant changes in their performance between the two tests.

General Improvement Trend

The general improvement trend is shown by the representation in Figure 3.

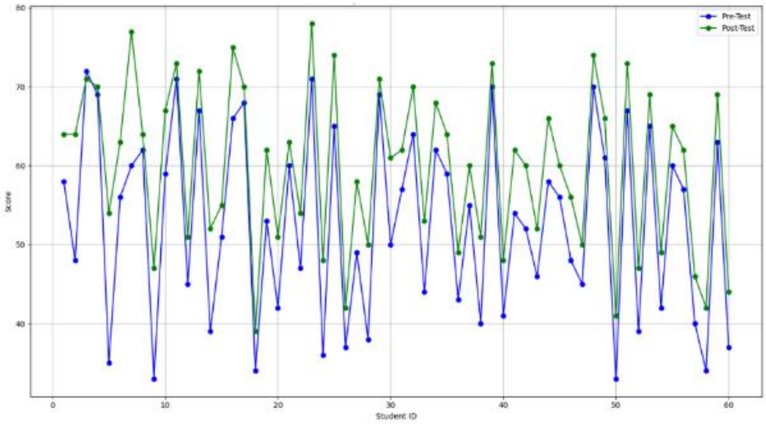


Figure 3. Trend of score improvement from pre-test to post-test

Looking at the data holistically in Figure 3, the majority of students demonstrated an improvement in their scores. More specifically, 58 out of 60 students showed positive changes in their performance, which suggests that the use of the quadratic graph visualization worksheet likely had a beneficial effect on students' mathematical visual thinking abilities. The average score change across the sample was approximately $+7$ points, further supporting the hypothesis that the intervention was effective.

In the next section, a more detailed analysis will be conducted using time-series analysis and PCA clustering to identify underlying patterns and trends in the data. This will provide further insights into how students' visual thinking abilities evolved over time and how the intervention differentially impacted various groups of students.

Time-Series Analysis

This study's time-series analysis evaluates the effectiveness of a quadratic graphing worksheet by examining five aspects: 1) session completion time, measuring how long participants spend on the worksheet to assess efficiency; 2) worksheet usage frequency per session, reflecting participants' engagement; 3) interaction with PhET simulation features, highlighting the impact of interactivity on learning outcomes; 4) scores of mathematical visual thinking skills (Visual Discrimination, Visual Perception, and Visual Analysis), tracking skill progression; and 5) motivation and engagement scores over time, indicating changes in students' involvement. Figure 4 illustrates the worksheet completion duration.

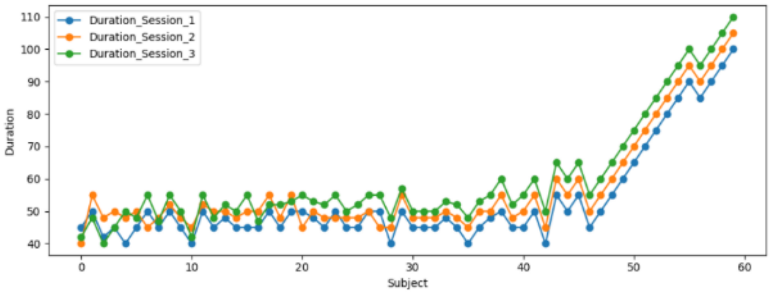


Figure 4. Worksheet completion duration

Figure 4 illustrates that the average completion time gradually increased across sessions, starting from 53.15 minutes in session 1, rising to 56.65 minutes in session 2, and reaching 59.68 minutes in session 3. The range of completion time also shows greater variability, with the minimum duration remaining stable at 40 minutes, while the maximum duration increased from 100 minutes in session 1 to 110 minutes in session 3. The 25th quartile, median, and 75th quartile also exhibited an upward trend, indicating that most participants spent more time completing the worksheets in each subsequent session.

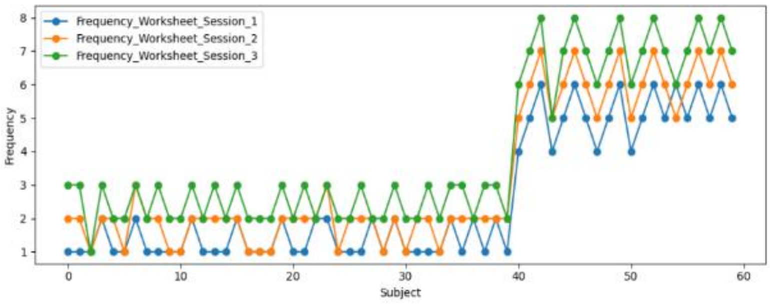


Figure 5. Frequency of worksheet usage

Figure 5 presents the frequency of worksheet usage, which also increased from session 1 to session 3. In session 1, the average usage frequency was 2.58 times, rising to 3.20 times in session 2, and reaching 3.95 times in session 3. The minimum frequency remained at 1, while the maximum frequency increased from 6 times in session 1 to 8 times in session 3. This suggests that students increasingly relied on the worksheets, possibly reflecting a need for more practice or exploration of the material.

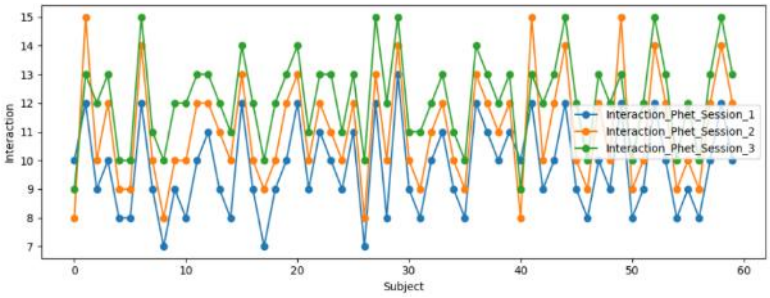
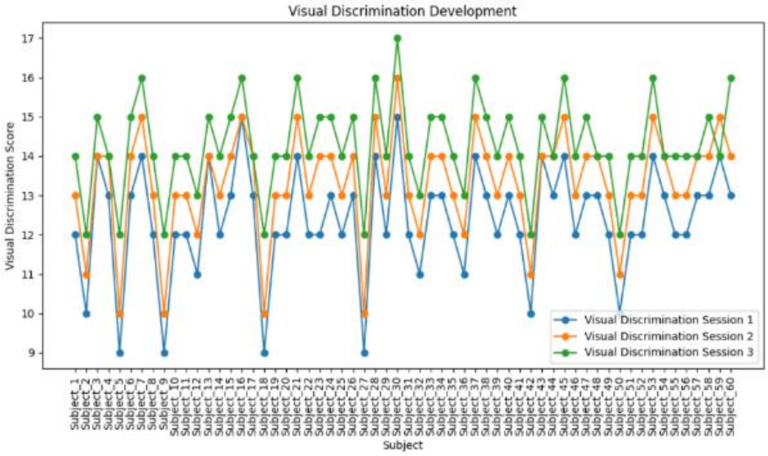


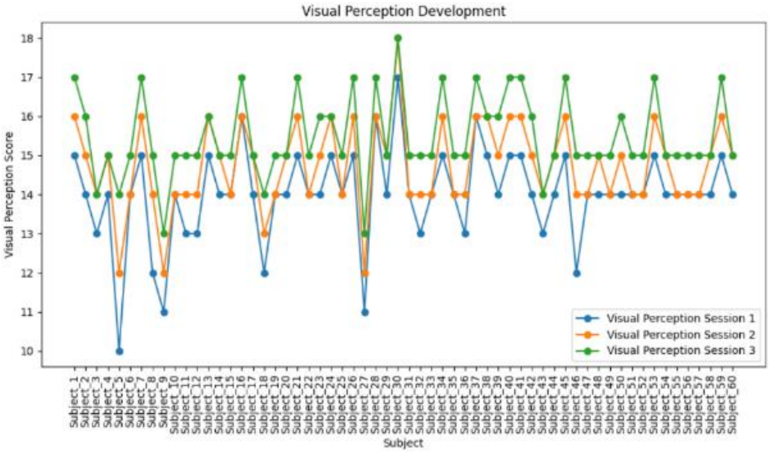
Figure 6. Interaction with PhET

Figure 6 visualizes the students' interaction with the PhET simulation, which similarly increased over time. In session 1, the average number of interactions was 9.75 times, increasing to 11.03 times in session 2, and reaching 12.15 times in session 3. The interaction range also expanded, with a minimum of 7 interactions in session 1 and 9 interactions in session 3, while the maximum number of interactions rose from 13 in session 1 to 15 in sessions 2 and 3. This indicates that students became increasingly engaged with the interactive features of the simulation over time.

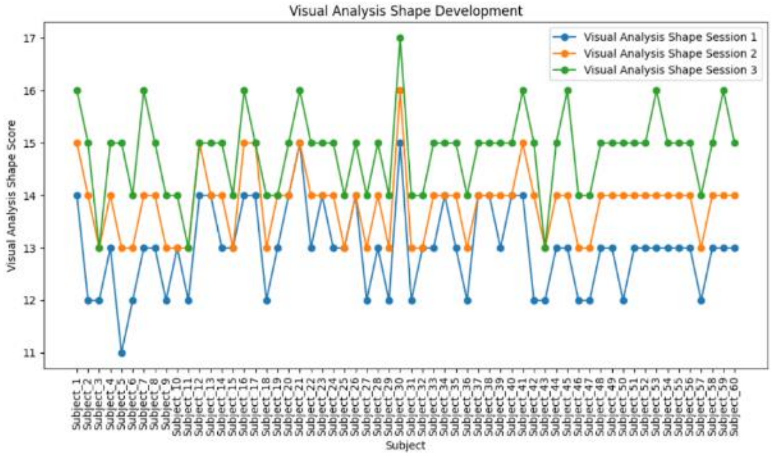
Overall, there was a consistent increase in completion time, worksheet usage frequency, and interaction with the PhET simulation from session 1 to session 3. This trend suggests that the complexity of the worksheets may have increased or that students became more engaged in the learning process. The upward trend indicates the growing significance of worksheets and interactive simulations in enhancing the learning experience. Consider the illustration of the development of mathematical visual thinking skills for each aspect: visual discrimination, visual perception, and visual analysis of shapes.



(a) Visual discrimination development



(b) Visual perception development

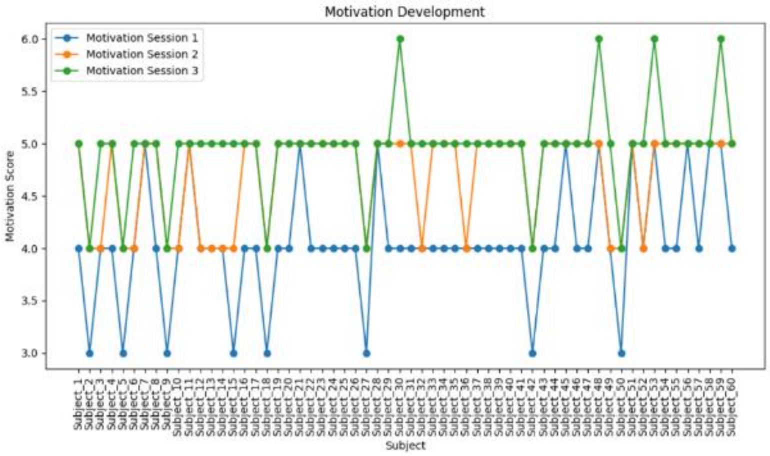


(c) Visual analysis shape development

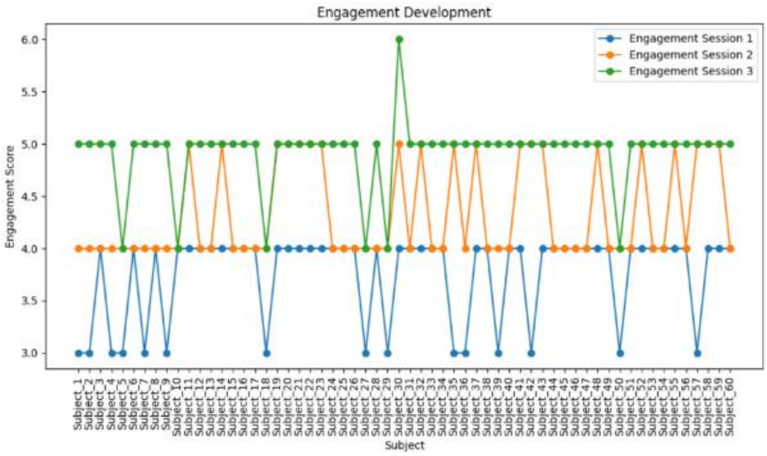
Figure 7. Mathematical visual thinking development

The time-series analysis in Figures 7(a)–7(c) highlights significant improvements in visual skills, motivation, and engagement from session 1 to session 3. Visual Discrimination scores (Figure 7(a)) rose steadily from 12.52 in session 1 to 13.33 in session 2, reaching 14.13 in session 3. Similarly, Visual Perception scores (Figure 7(b)) improved from 14.08 to 15.45, while Visual Analysis of Shapes scores (Figure 7(c)) increased from 13.28 to 14.67, reflecting consistent progress across all aspects.

On the other hand, affective aspects such as participants’ motivation and engagement, visualized in Figures 8(a) and 8(b), also showed improvement. Participants' motivation increased from 4.23 in session 1 to 4.9 in session 3, indicating a significant rise in motivation (Figure 8(a)). Engagement also experienced a steady increase, from 3.77 in session 1 to 4.53 in session 3, reflecting enhanced participation and interest throughout the learning process (Figure 8(b)). In conclusion, the data indicate that the program or intervention successfully enhanced participants’ visual skills, motivation, and engagement consistently across sessions, highlighting the effectiveness of the strategies applied in supporting participant development.



(a) Motivation development



(b) Engagement development

Figure 8. Motivation and engagement development

PCA

PCA was used to identify patterns and key components within the measured variables, reducing dimensionality while retaining primary variability. This technique revealed dominant factors influencing visual skills, motivation, and engagement, as well as relationships between variables. Figure 9 illustrates PCA results, showing subjects' pre-test and post-test scores projected onto two principal components. The scatterplot groups subjects into three distinct clusters, color-coded as yellow (Cluster 0), green (Cluster 1), and purple (Cluster 2).

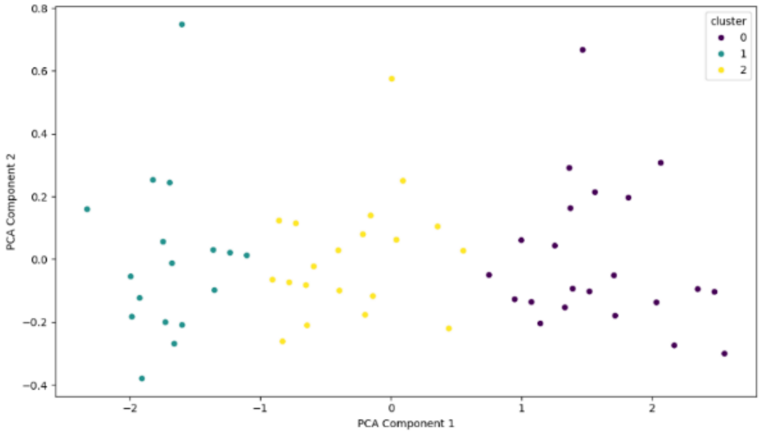


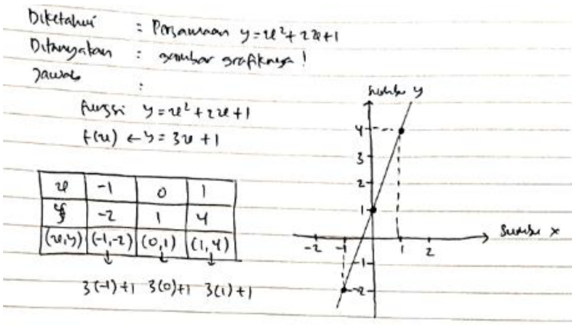
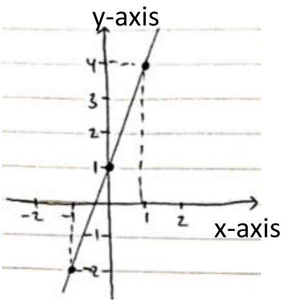
Figure 9. Clustering analysis of student scores

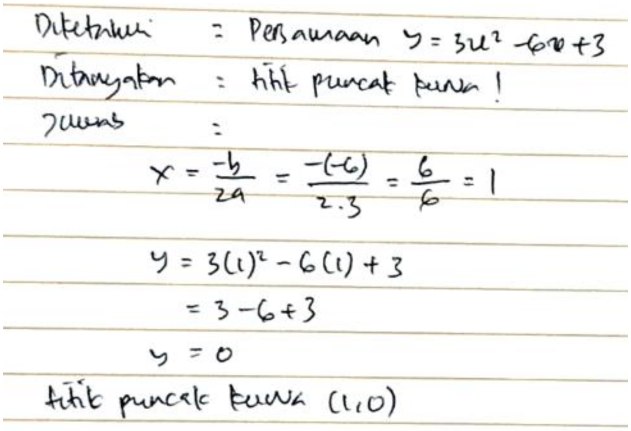
Each cluster represents a group of subjects with distinct performance characteristics. Cluster 0 (yellow) includes subjects with mid-range scores, averaging 54.2 on the pre-test and 58.7 on the post-test, showing consistent but moderate improvement. Cluster 1 (green) displays greater score variation, with an average pre-test score of 59.3 and a post-test score of 64.1, reflecting both significant improvements and declines. Cluster 2 (purple) consists of high-performing subjects with consistent scores, averaging 61.5 on the pre-test and 67.2 on the post-test. These differences highlight opportunities for targeted interventions, such as more intensive strategies for Cluster 1 to address varied performance. This analysis provides valuable insights to refine learning and evaluation processes.

Student Responses and Analysis

This section presents the analysis of student responses to pre-test and post-test questions to complement quantitative findings with qualitative evidence. Student answers are categorized based on their skill levels (low, medium, high) and analyzed to understand patterns of improvement after the intervention using PhET Simulation. This data provides detailed insights into the strengths and weaknesses of students in terms of quadratic graph visualization, visual discrimination, and discriminant interpretation, supporting the key findings of the study. The following table presents student answers that provide concrete evidence of their understanding and skills before and after the intervention, starting from subject 5.

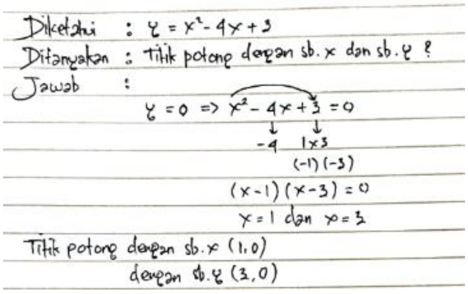
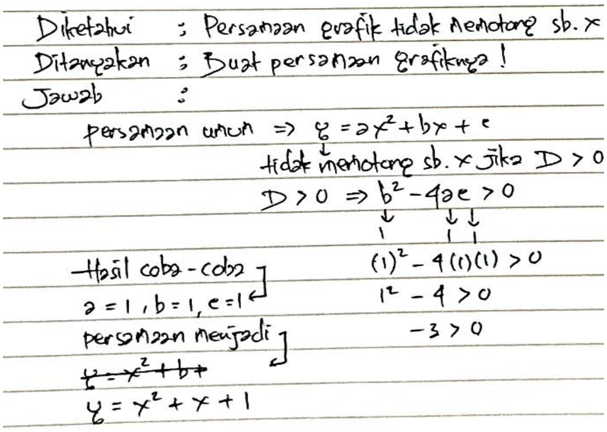
Table 1. Subject 5: pre-test and post-test analysis

Phase and Question	Subject Answer	Subject Argument
Pre-test *Write the equation* $y = x^2 + 2x + 1$ *and graph it.*	 **English version:** Known: equation $y = x^2 + 2x + 1$ Question: illustrate the graph! Answer. Function $y = x^2 + 2x + 1$ $f(x) \leftarrow y = 3x + 1$ 	*"I see* $y = x^2 + 2x + 1$ *there are two* x *variables. I immediately added them together. The graph I made is a linear equation resulting from substituting values of* x *into* y*."*

Phase and Question	Subject Answer	Subject Argument
Post-test *Given* $y = 3x^2 - 6x + 3$. *Determine the vertex*	 Diketahui : Persamaan $y = 3x^2 - 6x + 3$ Ditanyakan : titik puncak kurva ! Jawab : $x = \frac{-b}{2a} = \frac{-(-6)}{2.3} = \frac{6}{6} = 1$ $y = 3(1)^2 - 6(1) + 3$ $= 3 - 6 + 3$ $y = 0$ titik puncak kurva (1,0) **English version:** Known: equation $y = 3x^2 - 6x + 3$ Question: the vertex of the curve! Answer: $x = -\frac{b}{24} = \frac{-(-6)}{2 \times 3} = \frac{6}{6} = 1$ $y = 3\,(1)^2 - 6(1) + 3$ $y = 3 - 6 + 3$ $y = 0$ The vertex of the curve (1,0)	*"I found the vertex (1,0) by using the formula* $x = \frac{-b}{2a}$*. However, I am unsure how this affects the graph."*

From Table 1, the analysis of Subject 5's rational arguments indicates that, in the pre-test, the subject misunderstood the general form of a quadratic function and failed to recognize the role of x^2 in determining the graph's shape. In the post-test, the subject correctly calculated the vertex using the formula $x = \frac{-b}{2a}$ but did not fully understand the implications of $a > 0$ on the graph's orientation. Initially, the subject's responses showed significant errors, including an incorrect graph representation and miscalculated vertex, which reflected a low initial score. Following the intervention, the subject demonstrated improved comprehension by accurately calculating the vertex and sketching the graph, although their understanding was not yet fully optimal.

Table 2. Subject 3: pre-test and post-test analysis

Phase and Question	Subject Answer	Subject Argument
Pre-test *Given $y = x^2 - 4x + 3$. Determine the intercepts of equation with the x-axis and y-axis.*	 Diketahui : $y = x^2 - 4x + 3$ Ditanyakan : titik potong dengan sb. x dan sb. y ? Jawab : $y = 0 \Rightarrow x^2 - 4x + 3 = 0$ $\downarrow \quad \downarrow$ $-4 \quad 1 \times 3$ $(-1)(-3)$ $(x-1)(x-3) = 0$ $x = 1$ dan $x = 3$ Titik potong dengan sb. x (1,0) dengan sb. y (3,0) **English version:** Known: equation $y = x^2 - 4x + 3$ Question: the intercepts of the equation with the x-axis and y-axis Answer. $Y = 0 \to x^2 - 4x + 3 = 0$ $\downarrow \quad \downarrow$ $-4 \quad 1 \times 3$ $(-1)(-3)$ $x = 1$ and $x = 3$ The intercepts with x-axis (1,0) The intercepts with y-axis (3,0)	*"I found one root, $x = 3$, but I'm not sure if there's another root. I know $y = c$, so $y = 3$."*
Post-test *Create an equation for a graph that does not intersect the x-axis*	 Diketahui : Persamaan grafik tidak memotong sb. x Ditanyakan : Buat persamaan grafiknya ! Jawab : persamaan umum $\Rightarrow y = ax^2 + bx + c$ tidak memotong sb. x jika $D > 0$ $D > 0 \Rightarrow b^2 - 4ac > 0$ $\downarrow \quad \downarrow \downarrow$ $1 \quad 1 \; 1$ Hasil coba-coba $a = 1, b = 1, c = 1$ $(1)^2 - 4(1)(1) > 0$ $1^2 - 4 > 0$ $-3 > 0$ persamaan menjadi ~~$y = x^2 + b +$~~ $y = x^2 + x + 1$ **English version:** Known: the equation does not intersect with x-axis Question: draw the graph! Answer: General equation→ $y = ax^2 + bx + c$ An equation does not intersect with x-axis only if $D > 0$ $D > 0 \to b^2 - 4ac > 0$ Trial-and-error results $a = 1, b = 1, c = 1$ The equation become $y = x^2 + x + 1$	*"The equation $y = x^2 + x + 1$ was chosen because $D = b^2 - 4ac$. The graph does not intersect the x-axis."*

The analysis of Table 2 highlights Subject 3's consistent performance, as reflected by the low variance (0.5). During the pre-test, the subject successfully identified the y-intercept but encountered difficulties in calculating the quadratic roots. By the post-test, the subject demonstrated a solid understanding of the relationship between the discriminant ($D > 0$) and parabolic graphs with no real roots. This stability across assessments indicates a steady grasp of concepts, though minor inaccuracies in solving quadratic equations suggest a need for further skill enhancement. The low variance underscores the reliability of their conceptual understanding, pointing toward targeted improvement rather than foundational reteaching.

The illustrations of student responses from the pre-test and post-test support the research findings, highlighting a significant improvement in mathematical visual thinking skills following the intervention using PhET Simulation-based worksheets. In the Score Changes section, responses from low-performing students reflect fundamental errors, such as miscalculating the vertex or discriminant, consistent with their low pre-test scores. However, improvements in post-test responses, particularly from students like Student 5 with a score increase of $+19$ points, demonstrate the intervention's effectiveness. Meanwhile, high-performing students consistently provided accurate answers, aligned with the low variance observed in Figure 2, indicating stability in their performance throughout the intervention.

The trend of an average score increase of $+7$ points in the General Improvement Trend is mirrored in the enhanced quality of student responses. For instance, students who initially drew straight-line graphs in the pre-test were able to draw parabolas with more accurate vertices and axes of symmetry in the post-test. This data highlights the critical role of student interaction with worksheets and interactive simulations in improving their understanding. In the Time-Series Analysis, increased worksheet completion time, frequency of use, and interaction with the PhET Simulation correlate with better response quality. Mid-performing students, who initially struggled to understand the relationship between coefficients and graph shapes, demonstrated clearer comprehension through improved responses related to the discriminant and vertex. This indicates that active engagement with the learning materials enhances both efficiency and depth of understanding. Finally, the Clustering Analysis confirms varying response patterns among student groups. Students in Cluster 0 showed developing foundational understanding, while those in Cluster 1, characterized by high variance, suggest the need for additional teaching strategies. Conversely, students in Cluster 2, who consistently achieved high scores, reflect the success of the intervention in strengthening their visual understanding. These student responses complement the quantitative data, providing insights into the success of the learning process and identifying areas that require further improvement.

This study showed a gradual increase in worksheet completion time, usage frequency, and interaction with the PhET simulation. Average completion time rose from 53.15 minutes in session 1 to 59.68 minutes in session 3, while worksheet usage frequency increased from 2.58 to 3.95 times per session. Similarly, student interactions with the PhET simulation grew from an average of 9.75 in session 1 to 12.15 in session 3. These trends indicate heightened student engagement, likely driven by improved understanding or the increasing complexity of the worksheets. The rise in interaction with PhET further highlights the tool's effectiveness in sustaining student interest. Overall, the findings demonstrate that interactive learning technologies like PhET significantly enhance student involvement and learning experiences.

The study findings show that increased worksheet completion time and usage frequency indicate greater student engagement in learning. Spending more time on assignments suggests a deeper understanding of the material or a need for thorough exploration (Howell, 2021; van Dulmen et al., 2023; Wieselmann et al., 2021). Similarly, the rise in interactions with the PhET simulation reflects growing familiarity and engagement with the tool, pointing to enhanced student motivation. Frequent worksheet usage and simulation interaction highlight active participation, likely influenced by the challenging yet engaging worksheet design, which fosters continuous learning and improved outcomes (Barkley & Major, 2020; Chaiyarat, 2024; Ritchhart & Church, 2020). These results emphasize the value of integrating technology into education. Interactive tools like PhET not only boost engagement but also help students develop visual and analytical skills through practical learning experiences. Increased motivation and engagement observed in this study underscore simulations' potential to enhance learning outcomes (Banda & Nzabahimana, 2023; Huang et al., 2022; Koskinen et al., 2023).

This study aligns with student engagement theory, which highlights that active involvement enhances understanding and learning outcomes (Bond et al., 2020; Heilporn et al., 2021; Kahu, 2013; Li & Xue, 2023). The use of interactive simulations like PhET increases study duration, tool usage, and student interactions, fostering deeper engagement. This is supported by previous research demonstrating the positive impact of interactive learning technologies on conceptual understanding and motivation (Cai et al., 2021; Hsiao & Su, 2021; Wang et al., 2023). In mathematics education, PhET enhances engagement and comprehension of abstract concepts such as functions and geometry (Oktaviyanthi & Sholahudin, 2023). However, previous studies focus on general engagement, while this research focuses on developing mathematical visual thinking through interactive exploration of quadratic graphs—an area that remains underexplored. PhET provides an interactive environment where students manipulate coefficients and observe real-time changes (Diab et al., 2024), bridging the gap between algebraic expressions and their graphical representations (Güçler et al., 2022). Additionally, PhET fosters active learning, exploration, and helps students connect abstract concepts with concrete visualizations. This study highlights that increased interaction with PhET simulations positively impacts visual skills, rarely addressed in similar studies. These findings contribute to the argument that interactive simulation-based learning can improve educational quality, offering a foundation for further development of interactive technologies in classrooms.

This study holds a significant position in educational technology and interactive pedagogy by providing empirical evidence of the PhET simulation's effectiveness in enhancing student engagement and visual skills (Chinaka, 2021; Dorji et al., 2024; Salame & Makki, 2021). It demonstrates that interactive simulations not only improve cognitive learning outcomes but also positively influence affective aspects such as motivation and engagement (Lee et al., 2021; Tugtekin & Odabasi, 2022; Yang et al., 2023). While prior research has explored the role of technology in education, this study offers new insights into how interactive simulations specifically enhance visual skills. By bridging student engagement theory with practical classroom applications, it highlights PhET simulations as an effective pedagogical tool that supports both cognitive and motivational development. This research underscores the need for further exploration into integrating interactive technologies into curricula to enhance educational quality.

This study highlights the significant impact of interactive technology in education, showing that increased worksheet completion time, usage frequency, and interaction with PhET simulations enhance student engagement. The findings demonstrate that interactive tools improve visual skills and motivation, contributing to a more effective and engaging learning process. The study recommends integrating PhET simulations more broadly into curricula to boost engagement and understanding of complex concepts. Combining simulations with interactive worksheets is suggested to provide a structured and comprehensive learning experience, enhancing visual and analytical skills. Future research should explore the long-term effects of interactive simulations on learning outcomes across disciplines and develop integrated learning models that combine simulations with other pedagogical methods. These recommendations offer practical insights for educators and promote further innovation in educational technology to enhance learning quality in diverse contexts.

CONCLUSION

This study highlights the impact of graphing quadratic worksheets on students' mathematical visual thinking skills, analyzed through time-series analysis and PCA clustering. The findings reveal progressive improvements in worksheet completion time, usage frequency, and interaction with PhET simulations, reflecting increased student engagement with both the worksheets and interactive tools. Time-series analysis showed an average rise in worksheet completion time from 53.15 minutes in session 1 to 59.68 minutes in session 3, alongside an increase in worksheet usage frequency from 2.58 to 3.95 times and PhET interactions from 9.75 to 12.15 times. These trends indicate growing student commitment and familiarity with the learning materials, promoting deeper engagement.

PCA clustering further identified distinct learner groups based on interaction patterns and performance, demonstrating that students with higher engagement exhibited stronger visual thinking skills. The integration of graphing quadratic worksheets with PhET simulations supported significant improvements in visual perception, analysis, and comprehension of abstract mathematical concepts. These findings align with student engagement theories, underscoring the effectiveness of interactive technologies in enhancing conceptual understanding and skill development.

The clustering analysis also revealed that varied engagement patterns correspond to different skill levels, suggesting that personalized learning approaches could further optimize outcomes. This study confirms the effectiveness of combining graphing quadratic worksheets with PhET simulations in improving mathematical visual thinking. The use of time-series analysis and PCA clustering offers a comprehensive understanding of engagement patterns and their educational impact. Future research should investigate the long-term effects of these interventions and explore their integration into broader pedagogical strategies to maximize learning outcomes.

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