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The South Sumatera Songket Motifs for Supporting Students' Proving Process in Learning Reflection

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Abstract

This study addresses the limited use of the South Sumatera Songket motifs for supporting students' proving process in the learning of reflection material. It aims to determine the role of South Sumatera Songket Motifs in students' proving process in reflection learning. The study employed a validation approach within the Design Research framework, involving 30 students from one public junior high school in Palembang. The validation study comprised three stages: preparing for the experiment, the design experiment (including a preliminary teaching experiment and a teaching experiment), and retrospective analysis. Activities in the experiment included: 1) introducing the Songket motifs and having students identify lines of symmetry, 2) guiding students to reflect on the motifs using various axes, 3) engaging students in proving the congruence properties of the reflected motif, and 4) conducting group discussions to refine their reasoning and articulation of geometric concepts. The activities were designed using indicators adapted from Habermas' Construct of Rationality to support the proving process in reflection material. The results from the learning experiment indicate that the provided questions support students' proving process, guiding them from visual recognition to mathematical reasoning through structured activities.

Keywords: Cultural Relevance, Design Research, Proving Process, Reflection, South Sumatera Songket Motif

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INTRODUCTION

Geometric transformations are a fundamental concept in mathematics, involving one-to-one and mapping of points on a plane through motions such as reflection, translation, rotation, and dilation (P & P, 2022). These transformations play a pivotal role in mathematics education across all levels, from primary to university, by bridging geometric principles like similarity and congruence with broader mathematical domains such as algebra, patterns, and reasoning (Akarsu & İler, 2024).

Engaging with transformations fosters critical thinking as students develop justifications and proofs, which are essential for mathematical reasoning (Knuchel, 2004). Hollebrands (2003) emphasized that geometric transformations should be included in the high school curriculum for three key reasons: they provide students with opportunities to think in new ways about critical mathematical concepts (e.g., functions whose domain and range are R2), they offer a context for understanding mathematics as an interconnected discipline, and they engage students in higher-level reasoning activities using a variety of representations. Guven (2012) further highlighted that geometric transformations encourage students to explore geometric ideas through an informal and intuitive approach, fostering conjecturing, sensitivity, and inquisitiveness. It can lead students to explore abstract mathematical concepts such as congruence, symmetry, similarity, and parallelism, thereby enriching their geometrical experience, thought, and imagination while enhancing their spatial abilities.

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Reflection, as the initial congruence transformation taught in primary school, is one of the most essential geometry concepts that children should acquire in their mathematics education at the primary level (Götz & Gasteiger, 2022). Reflection is often described as mapping points in a plane to their mirror image, producing a symmetric figure (Andriyani et al., 2023). Reflection can occur concerning a line (in 2D space), a plane (in 3D space), or even a specific point, depending on the dimension and context (Nurhikmayati et al., 2022). Many traditional and contemporary artworks utilize reflection to create symmetry, patterns, and stunning visual effects (Velichová, 2020). Reflection can be applied across various media, from painting and installation to digital and 3D art, such as scratch-based reflection art that produces different visual effects from various perspectives (Shen et al., 2023). Studying reflection contributes to a deeper understanding of fundamental structures and principles in art and design, including proportion, symmetry, and order (Götz & Gasteiger, 2022).

Students' struggles in solving reflection problems are widely recognized (Yurmalia & Herman, 2021). Students often confuse similarity with congruence, mistakenly viewing reflection as a scaling transformation rather than a congruence-preserving transformation (Haj-Yahya, 2021a). This misunderstanding leads to misconceptions about the preservation of angles and distances during the reflection process, further complicating their ability to understand geometric properties. Furthermore, students struggle to identify the axis of symmetry, particularly in complex figures, and fail to recognize the connection between the symmetry axes in regular polygons and their corresponding sides or diagonals (Aktaş & Ünlü, 2017).

In the proving process, such difficulties manifest in their inability to articulate the invariance properties of reflection, such as equal distances from the pre-image and image to the axis of reflection within a logical framework. Studies have demonstrated that students often draw the reflected image parallel to the original object or mistakenly perform translation instead of reflection (Götz & Gasteiger, 2018). These errors hinder students' ability to engage in the proving process, as they fail to correctly demonstrate the geometric properties of reflection, such as maintaining the distance and orientation between points across the mirror line (Baroa, 2025).

Students also struggle with translating the abstract concept of reflection into practical applications, such as solving real-world mathematical problems, due to a lack of deep conceptual understanding and an overreliance on rote memorization of procedures (Aras et al., 2023). One contributing factor is that instruction tends to be mechanistic, emphasizing conceptual exploration insufficiently. As a result, students often memorize steps without understanding the underlying principles of geometric transformations, which hinders their ability to engage in the proving process or make meaningful connections between theory and application (Kim & Shin, 2023). Additionally, learning materials and teaching approaches often prioritize procedural knowledge over conceptual reasoning, exacerbating this issue (Nurhikmayati et al., 2022).

Additionally, many students experience difficulties visualizing and defining reflection transformations accurately, which hinders their ability to construct and verify proofs of geometric properties (Noto et al., 2019; Indahwati, 2023). Their struggles are often linked to ontogenic obstacles, such as developmental challenges in spatial reasoning; epistemological obstacles, such as misconceptions about the nature of reflection; and didactical obstacles, including teaching methods that inadequately address the conceptual foundations of transformations. Addressing these challenges through improved learning designs that integrate contextual problem-solving tasks and conceptually oriented teaching can help students overcome these obstacles and enhance their proving skills in reflection transformations (Pauji et al., 2023).

In response to the challenges students face in supporting the proving process in reflection material, this study draws on Habermas' Construct of Rationality, which conceptualizes the process of mathematical proof through three types of Rationality: Epistemic, Teleological, and Communicative Rationality (Büscher, 2024). In mathematics education research, Habermas' work has already been integrated into the content of mathematical proof and argumentation (Zhuang & Conner, 2022).

By integrating these types of Rationality into the design of learning activities, the study aims to move students from intuitive visual perception to analytical reasoning grounded in geometric principles. Epistemic Rationality allows students to develop foundational knowledge of geometric transformations. Teleological Rationality focuses on the purposeful learning of geometric properties, and Communicative Rationality emphasizes the importance of articulation and peer collaboration in refining mathematical reasoning. This theoretical framework aligns with the research goal of supporting students' ability to prove geometric concepts by fostering both individual and collaborative reasoning processes, which enhances the conceptual depth of reflection transformations.

To address the challenges students face in understanding and proving reflection transformations, it is essential to shift their focus from intuitive visual perception to analytical reasoning grounded in the defining properties of geometric transformations (Haj-Yahya, 2021b). A practical approach to achieving this shift involves integrating culturally relevant materials, such as the traditional South Sumatera Songket motifs, into the learning process. These motifs enable students to apply epistemic and teleological Rationality to their reasoning while simultaneously engaging in communicative Rationality through collaborative discussions.

The South Sumatera Songket motifs, characterized by their repetitive and symmetrical patterns, provide a meaningful context for exploring geometric transformations, such as reflection (Sari et al., 2024). By analyzing these patterns, students can deconstruct and reconstruct geometric properties, which allows them to identify symmetry axes, construct auxiliary lines, and understand the relationships between geometric figures, critical skills for constructing proofs (Sari et al., 2024).

This approach aligns well with the Indonesian Realistic Mathematics Education, which emphasizes the importance of connecting mathematics to students' lived experiences, including cultural artifacts, to enhance conceptual understanding and reasoning (Nurazizah & Zulkardi, 2022). Incorporating Songket motifs in this context bridges abstract mathematical concepts with familiar patterns, creating a visually stimulating and culturally relevant learning framework. This method helps students transition from rote memorization of procedures to deeper conceptual engagement, thereby reducing misconceptions about reflection as a transformation that preserves distance and orientation.

This study uniquely introduces traditional cultural artifacts, specifically South Sumatera Songket motifs, as a tool for exploring geometric transformations. These motifs, characterized by their repetitive and symmetrical patterns, provide a concrete and culturally engaging context, enabling students to connect abstract geometric properties with real-world patterns. By incorporating these motifs into the learning process, students can bridge the gap between intuition and formal reasoning, enhancing their understanding of geometric reflections and other transformations. This study addresses the challenges students face in proving geometric properties by offering a meaningful and contextually relevant framework for learning.

METHODS

Research Procedure

This study employed a validation study design within the Design Research framework, consisting of three primary stages: Preliminary Design, Design Experiment, and Retrospective Analysis (Bakker, 2018). In the preliminary design, the researcher developed a Hypothetical Learning Trajectory (HLT) that outlined the learning activities and anticipated student responses (Putri et al., 2021). The activities were designed to support students' proving process in reflection learning using South Sumatera Songket motifs as the context. The HLT was dynamic and adjusted based on insights gained from Cycle 1 of the pilot experiment. At this stage, learning tools such as lesson plans, worksheets, and ICT media were developed and validated by experts before proceeding to the next phase.

In the design experiment, a pilot experiment was conducted with a small group of students (6 students) to test the activities and observe how well they supported the proving process. Based on the feedback, adjustments were made, and the HLT was refined. After the pilot experiment, a classroom experiment was conducted with a larger group of students (24 students), where the revised HLT was implemented. Discussions between the researcher and teacher took place before and after each activity to assess students' progress and make adjustments as necessary. Following the design experiment in each cycle, data were analyzed to evaluate the effectiveness of the learning trajectory. Based on this analysis, the HLT was refined and tested again in subsequent cycles. The data from observations, interviews, and student worksheets were carefully reviewed to assess how well the learning activities supported the proving process.

Participants and Data Sources

The participants in this study were junior high school students, approximately 14 years old and in the 9th grade, from one public junior high school in Palembang, Indonesia. A total of 30 students participated, with six students involved in Cycle 1 (pilot experiment) and 24 students in Cycle 2 (classroom experiment). The research data were collected using included validation sheets, which were used to gather feedback on the learning tools and activities from experts. Additionally, interviews were conducted with students to gain insights into their understanding and the challenges they faced in learning reflection. Classroom observations were also carried out to document student engagement, behaviors, and interactions during the activities. Lastly, student activity sheets, which contained students' work on the reflection tasks, were analyzed to evaluate their understanding of the proving process and the effectiveness of the designed learning activities.

RESULTS AND DISCUSSION

Preliminary Stage

In this study, the HLT was designed to support students through the process of proving geometric reflections. The trajectory was structured around three key stages of Habermas' Construct of Rationality: Epistemic Rationality, Teleological Rationality, and Communicative Rationality. Each stage aimed at fostering students' understanding of reflection and their ability to justify geometric transformations.

Epistemic Rationality (Mathematical Knowledge Development)

In the first stage, students were introduced to the Ghumah Bhagi Songket motif and tasked with identifying symmetry within the motif. The goal was for students to recognize mirrored parts of the motif and hypothesize where the lines of reflection might be. This task was designed to prompt students to connect their visual observations to the abstract concept of reflection. As students examined the motif, they began to understand how symmetry in the design reflects the properties of geometric transformations. This phase helped students develop the foundational knowledge required for performing reflections and understanding their geometrical properties.

Teleological Rationality (Purposeful Learning Activities)

The next phase of the HLT focused on purposeful learning activities. Students were asked to draw reflection lines that divide the Songket motif into two identical parts and test their hypotheses about symmetry. They were encouraged to perform actual reflections across different axes (e.g., vertical, horizontal, and coordinate axes). Through these tasks, students explored the purpose of geometric reflection, validating the properties of congruence and understanding that reflections preserve shape and size. This stage was crucial for developing both geometric and algebraic reasoning as students began to engage with the mathematical properties of reflections and verified them through physical activities.

Communicative Rationality (Collaboration and Dialogue)

The final stage of the HLT emphasized collaborative learning. As students worked on their reflections, they were asked to justify their reasoning, both verbally and in writing. They used coordinate grids to record the positions of points before and after the reflection, showing how the points were symmetrically distributed across the line of reflection. This activity encouraged communicative rationality as students articulated their mathematical reasoning and communicated their understanding of geometric reflections. By explaining why specific lines served as reflection axes and discussing the properties of the reflected motif, students were able to deepen their understanding of proof construction.

Hypothetical Learning Trajectory (HLT)

In developing the HLT for teaching reflection transformations, it is crucial to structure a learning path that supports students in proving the geometric principles of reflection. This trajectory aims to guide students through the process of justifying and proving the properties of reflections, such as congruence preservation and symmetry. By using the Songket motifs, a culturally rich and visually stimulating context, students are encouraged to apply abstract mathematical concepts to real-world artifacts while working through the proving process.

The focus of this approach is to engage students in active reasoning, where they identify reflection properties, perform mathematical calculations, and justify their findings through collaborative discourse. The HLT is based on Habermas' Construct of Rationality, which guides students through three phases: Epistemic Rationality, Teleological Rationality, and Communicative Rationality. Each phase enables students to test their hypotheses, justify their reflections, and communicate their reasoning, ultimately leading to formal mathematical proofs of reflection properties. The detailed HLT that outlines how students will engage with the reflection material and support the proving process in reflection transformations is presented in the following Table 1.

Table 1. The HLT

Aims	Activity	Conjecture of Students' Thinking
Recognize symmetry and understand reflection	Introduce the Ghumah Bhagi motif and have students identify lines of symmetry, such as vertical, horizontal, and diagonal lines (e.g., $y = x$, $y = -x$).	Students will identify symmetric parts of the motif and hypothesize where the lines of reflection (axes of symmetry) might be located.
Perform reflection over multiple axes	Reflect the motif over the X-axis, Y-axis, and lines like $y = x$, $y = -x, x = h$, and $y = 2$.	Students will explore how reflections occur over different axes and recognize that each axis reflects the motif in a unique way.
Verify reflection properties using coordinate grids	Students perform reflections on coordinate grids, first reflecting over $x = 0$ and $y = 0$, then	Students will calculate the coordinates of reflected points and verify that the reflected points maintain symmetry and congruence.

Aims	Activity	Conjecture of Students' Thinking
	experimenting with more complex lines like $y = 2, x = -3$.	
Justify reflection axes based on geometric principles	Have students explain why lines like $y = 2, x = -3$, and others act as valid axes of reflection based on their observations and calculations.	Students will justify their reasoning, discussing how these axes divide the motif into congruent parts and why they preserve symmetry.
Prove reflection properties and congruence	After reflecting, students will prove that the reflected points are equidistant from the axis, reinforcing the congruence of the reflected motif.	Students will prove that the distance from each point to the line of reflection is the same for the reflected point, thereby establishing the congruence of the reflected figure.

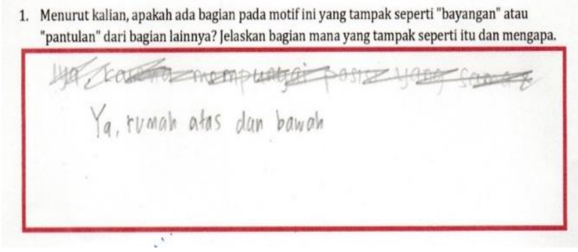
Design Experiment

Pilot Experiment

The design experiment was structured in two cycles: Cycle 1 (Pilot Experiment) and Cycle 2 (Classroom Experiment). In Cycle 1, a small group of students tested the activities developed in the Hypothetical Learning Trajectory (HLT), providing an opportunity for researchers to assess how students engaged with the tasks and whether these activities facilitated their proving process. The insights gained from this phase highlighted several challenges students encountered and prompted refinements to the learning trajectory.

Not only did the students' responses during the pilot experiment reveal their initial understanding of reflection and symmetry, they also exposed a significant gap in the students' ability to articulate the underlying mathematical properties of reflection, especially in the context of the proving process. While students successfully identified symmetrical patterns within the Songket motif, their explanations lacked mathematical rigor and depth. For instance, although they recognized parts of the motif as "shadows" or "reflections," they did not address the geometric properties that define this symmetry, such as the equidistant relationship from the line of reflection or the preservation of congruence (see Figure 1).

These findings align with existing literature, which suggests that students frequently struggle to connect visual perception with formal mathematical reasoning, particularly when asked to prove geometric properties (Sari et al., 2024). The design of the HLT in this study was specifically aimed at bridging this gap by guiding students from intuitive visual recognition to more formal mathematical justification, thereby supporting their ability to prove reflection transformations. The pilot experiment emphasized the need to refine the HLT further, incorporating more hands-on activities and targeted prompts to help students establish formal connections between visual patterns and geometric principles. This iterative approach is crucial for fostering both epistemic and communicative rationality, enabling students to transition from informal understanding to structured, formal reasoning and, ultimately, the construction of rigorous mathematical proofs in geometry.

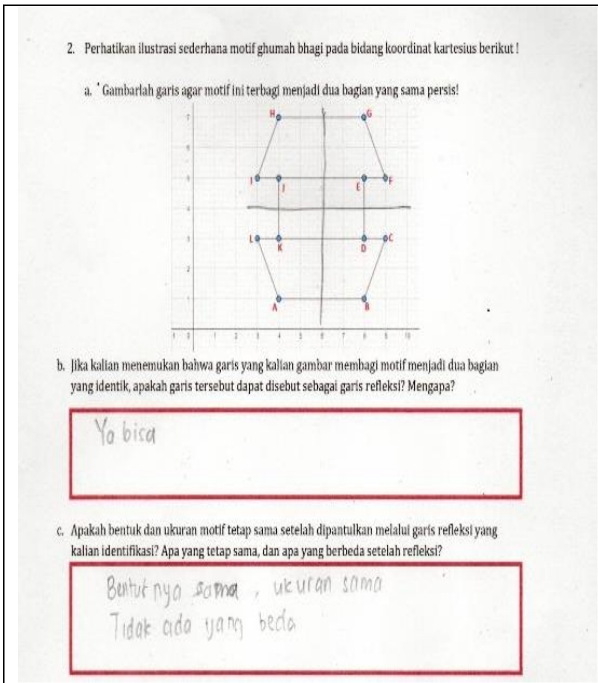

1. Menurut kalian, apakah ada bagian pada motif ini yang tampak seperti "bayangan" atau "pantulan" dari bagian lainnya? Jelaskan bagian mana yang tampak seperti itu dan mengapa.

Ya, rumah atas dan bawah

English version:

In your opinion, are there any parts of this motif that look like a "shadow" or "reflection" of others parts? Explain which parts look like that and why.

Answer:
Yes, upper house and lower


2. Perhatikan ilustrasi sederhana motif ghumah bhagi pada bidang koordinat kartesius berikut !

a. Gambarlah garis agar motif ini terbagi menjadi dua bagian yang sama persis!

b. Jika kalian menemukan bahwa garis yang kalian gambar membagi motif menjadi dua bagian yang identik, apakah garis tersebut dapat disebut sebagai garis refleksi? Mengapa?

Ya bisa

c. Apakah bentuk dan ukuran motif tetap sama setelah dipantulkan melalui garis refleksi yang kalian identifikasi? Apa yang tetap sama, dan apa yang berbeda setelah refleksi?

Bentuknya sama, ukuran sama
Tidak ada yang beda

English version:

Observe the simple illustration of the ghumah baghi motif on the Cartesian coordinate plane below!

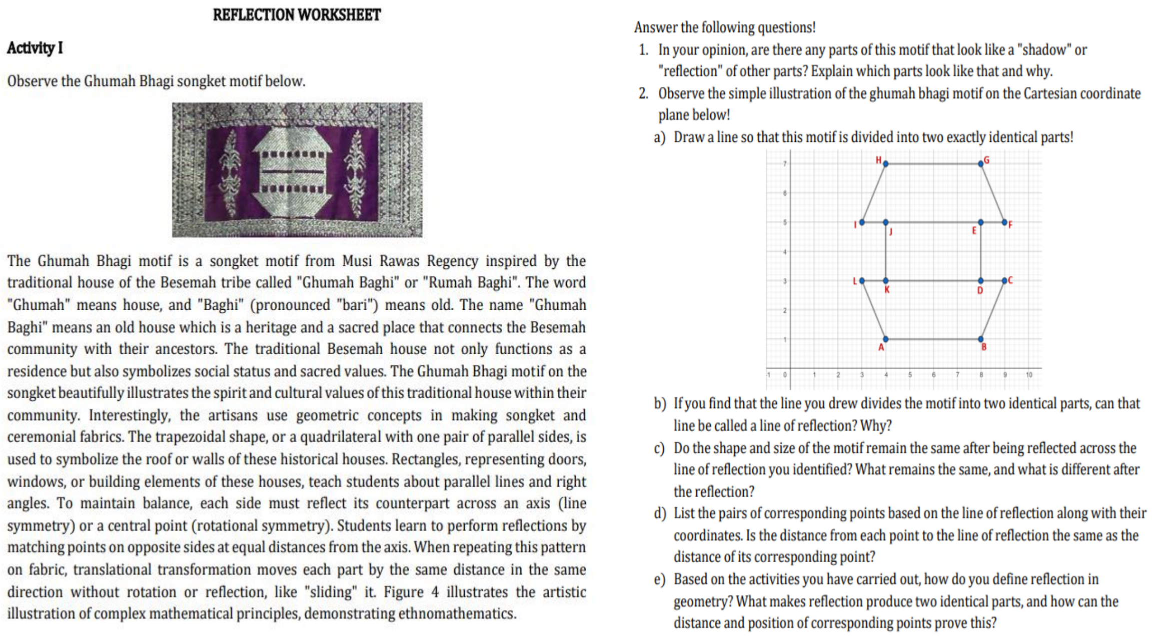
a) Draw a line so that this motif is divided into two exactly identical parts!
b) If you find that the line drew divides the motif into two identical parts, can that line be called a line of reflection? Why?
Answer:
Yes, you can.
c) Do the shape and size of the motif remain the same after being reflected across the line of reflection you identified? What remains the same, and what is different after the reflection?
Answer:
The shape and size are remains the same. Nothing changed

Figure 1. Student's answer on pilot experiment

Teaching Experiment

Figure 2. Teaching experiment classroom situation

The revised HLT was then implemented in Cycle 2, which involved a class of students (Figure 2). Before each activity, the researcher and teacher collaborated to review the goals and expectations for the upcoming tasks. After each activity, they reflected on the outcomes, which provided opportunities for immediate adjustments. Observational data, student interviews, and activity worksheets were collected throughout Cycle 2, and the results were analyzed to determine how effectively the learning trajectory supported students' understanding and proving process in reflection learning



REFLECTION WORKSHEET

Activity I

Observe the Ghumah Bhagi songket motif below.

The Ghumah Bhagi motif is a songket motif from Musi Rawas Regency inspired by the traditional house of the Besemah tribe called "Ghumah Baghi" or "Rumah Baghi". The word "Ghumah" means house, and "Baghi" (pronounced "bari") means old. The name "Ghumah Baghi" means an old house which is a heritage and a sacred place that connects the Besemah community with their ancestors. The traditional Besemah house not only functions as a residence but also symbolizes social status and sacred values. The Ghumah Bhagi motif on the songket beautifully illustrates the spirit and cultural values of this traditional house within their community. Interestingly, the artisans use geometric concepts in making songket and ceremonial fabrics. The trapezoidal shape, or a quadrilateral with one pair of parallel sides, is used to symbolize the roof or walls of these historical houses. Rectangles, representing doors, windows, or building elements of these houses, teach students about parallel lines and right angles. To maintain balance, each side must reflect its counterpart across an axis (line symmetry) or a central point (rotational symmetry). Students learn to perform reflections by matching points on opposite sides at equal distances from the axis. When repeating this pattern on fabric, translational transformation moves each part by the same distance in the same direction without rotation or reflection, like "sliding" it. Figure 4 illustrates the artistic illustration of complex mathematical principles, demonstrating ethnomathematics.

Answer the following questions!

1. In your opinion, are there any parts of this motif that look like a "shadow" or "reflection" of other parts? Explain which parts look like that and why.
2. Observe the simple illustration of the ghumah bhagi motif on the Cartesian coordinate plane below!
   a) Draw a line so that this motif is divided into two exactly identical parts!

   b) If you find that the line you drew divides the motif into two identical parts, can that line be called a line of reflection? Why?
   c) Do the shape and size of the motif remain the same after being reflected across the line of reflection you identified? What remains the same, and what is different after the reflection?
   d) List the pairs of corresponding points based on the line of reflection along with their coordinates. Is the distance from each point to the line of reflection the same as the distance of its corresponding point?
   e) Based on the activities you have carried out, how do you define reflection in geometry? What makes reflection produce two identical parts, and how can the distance and position of corresponding points prove this?

Figure 3. The activity I

Activity I, as seen in Figure 3, introduced students to the proving process by incorporating the Ghumah Bhagi songket motif, a traditional design from the Besemah ethnic group in South Sumatera, as a culturally grounded visual stimulus. This motif embodies mathematical symmetry, particularly reflection, which students are encouraged to explore. The activity began by prompting students to observe and discuss the motif's patterns and hypothesize where reflectional symmetry might occur. The initial question, *In your opinion, which part of the Ghumah Bhaghak motif represents reflection?",* serves as a gateway to activate students' perceptual awareness and cultural familiarity, engaging them in epistemic rationality as they construct initial claims grounded in intuition and informal reasoning. This was immediately followed by the prompt *"Can you explain your reasoning?"*, which transitioned students from recognition to articulation, encouraging them to verbalize the logical structure behind their observation.

To support deeper exploration, a simplified version of the motif was presented within a Cartesian coordinate system, allowing students to draw potential lines of reflection and examine the geometrical relationships involved. When the students were asked whether the points on either side of the line of reflection were congruent and equidistant from the line, the activity invoked formal geometric reasoning. This task promotes epistemic rationality by inviting learners to construct knowledge claims that are empirically observable and logically coherent. Furthermore, communicative rationality becomes central as students engage in structured dialogue to justify their ideas, compare interpretations, and co-construct shared meaning, transforming the act of proving into a dialogic process rather than a solitary demonstration. The discussion prompts served as discursive mechanisms through which learners critically assessed one another's arguments, revising their positions based on reasoned consensus.

Rather than viewing mathematical proof as a fixed, procedural outcome, this activity reframed it as a social and cultural practice rooted in dialogue, shared artefacts, and mutual understanding. In doing so, Activity I not only facilitated conceptual understanding of geometric reflection, but also cultivated a reflective learning environment aligned with Habermas' critical theory where epistemic and communicative rationality intersect to support authentic, culturally responsive mathematical inquiry.

Figure 4. Group discussion

Here is one session of the discussions in one group (Figure 4):

Student H	:	*"Okay, so when I look at the motif, I see that there is a part on the left that looks like it is the mirror image of the part on the right. The pattern on one side repeats exactly on the other side, just flipped. I think we could draw a line right down the middle to split it into two equal parts. This line would be the line of reflection because the distance from each point to the line is the same on both sides."*
Student M	:	*"Yeah, I see that! However, I am not sure why the pattern on the right looks exactly like the one on the left. I thought maybe the pattern was stretched or scaled, but it seems like it is just... flipped over. So, if we draw a line in the middle, we can see both sides are the same size and shape, right?"*
Student H	:	*"Exactly! That is the key part of reflection. The shape and size do not change, only the orientation. When something reflects, it just flips, but it stays congruent. The same thing happens with the Songket motif. If we draw a line down the middle, the left side of the pattern will reflect perfectly onto the right side."*
Student L	:	*"Okay, but I am confused. I do not understand why it flips and still looks the same. How do we know it is the same shape if it looks like it has changed direction? I thought it might get bigger or smaller, but it stayed the same size. How does that happen?"*
Student H	:	*"Great question, L! It does not get bigger or smaller. The key is that reflection preserves the size and shape; the only thing that changes is the orientation. Think of it like looking in a mirror. Your reflection in the mirror is the same size, but it is flipped over, right?"*
Student M	:	*"Yeah, I get that now! So, the line we draw in the middle is like a mirror. It flips everything exactly but keeps it the same size."*

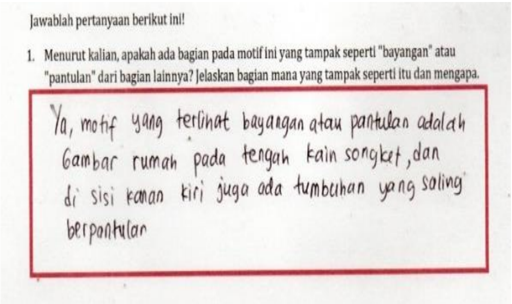
Student H : *"Exactly, M! The line we draw is the axis of reflection. Each point on one side of the line has a matching point on the other side, and they are equidistant from the line, meaning they are the same distance apart. So, if you picked any point on the left, you could find an exact matching point on the right side."*

Student L : *"So, we just need to make sure we draw the line exactly in the middle and that the pattern will be the same on both sides? No stretching or shrinking?"*

Student H : *"Yes, exactly. The line should be exactly in the middle. That is what creates the symmetry. When we reflect on the pattern, it is just like flipping it over like a piece of paper."*

Student M : *"Got it! So now we can try drawing the line and see if we can match the points. Let us do that!"*

Below are some of the students' responses:



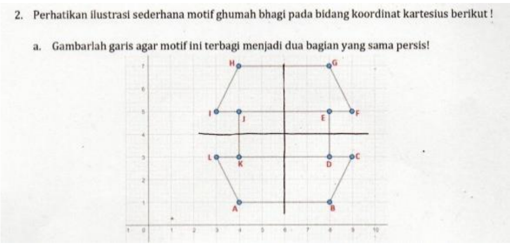
Jawablah pertanyaan berikut ini!

1. Menurut kalian, apakah ada bagian pada motif ini yang tampak seperti "bayangan" atau "pantulan" dari bagian lainnya? Jelaskan bagian mana yang tampak seperti itu dan mengapa.

Ya, motif yang terlihat bayangan atau pantulan adalah Gambar rumah pada tengah kain songket, dan di sisi kanan kiri juga ada tumbuhan yang saling berpantulan

English version:

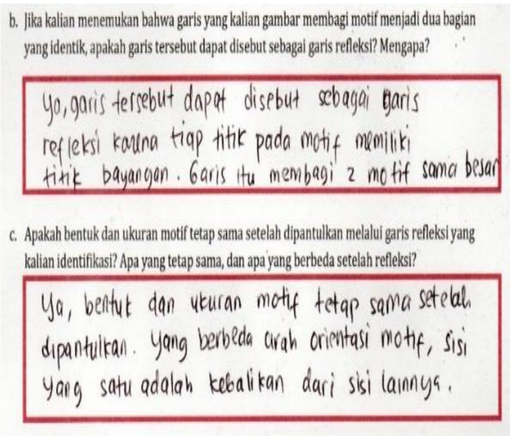
1. In your opinion, are there any parts of this motif that look like a "shadow" or "reflection" of others parts? Explain which parts look like that and why.
   Answer:
   Yes, the motif that looks like a shadow or reflection is the picture of the house in the center of the songket cloth, and on the right and left sides there are also plants that reflect each other.



English version:

Observe the simple illustration of the ghumah baghi motif on the Cartesian coordinate plane below!

a) Draw a line so that this motif is divided into two exactly identical parts!



b. Jika kalian menemukan bahwa garis yang kalian gambar membagi motif menjadi dua bagian yang identik, apakah garis tersebut dapat disebut sebagai garis refleksi? Mengapa?

Ya, garis tersebut dapat disebut sebagai garis refleksi karena tiap titik pada motif memiliki titik bayangan. Garis itu membagi 2 motif sama besar

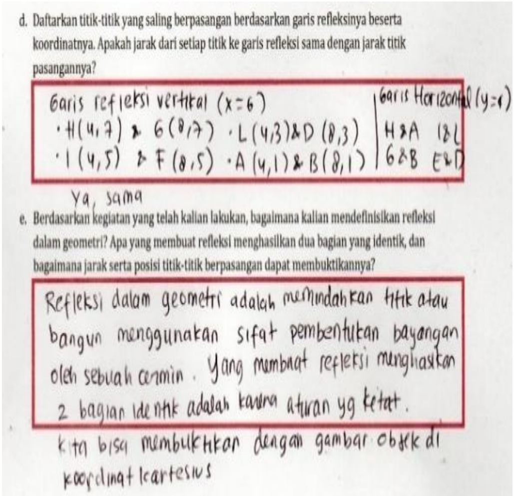
c. Apakah bentuk dan ukuran motif tetap sama setelah dipantulkan melalui garis refleksi yang kalian identifikasi? Apa yang tetap sama, dan apa yang berbeda setelah refleksi?

Ya, bentuk dan ukuran motif tetap sama setelah dipantulkan. yang berbeda arah orientasi motif, sisi yang satu adalah kebalikan dari sisi lainnya.

English version:

b) If you find that the line drew divides the motif into two identical parts, can that line be called a line of reflection? Why?
   Answer:
   Yes, the line can be called a reflection line because each point on the motif has a shadow point. the line divides 2 motifs equally

c) Do the shape and size of the motif remain the same after being reflected across the line of reflection you identified? What remains the same, and what is different after the reflection?
   Answer:
   Yes, the shape and size of the motif remains the same after being reflected. what is different is the orientation direction of the motif, one side is the opposite of the other side.



d. Daftarkan titik-titik yang saling berpasangan berdasarkan garis refleksinya beserta koordinatnya. Apakah jarak dari setiap titik ke garis refleksi sama dengan jarak titik pasangannya?

Garis refleksi vertikal (x=6)
· H(4,7) & G(8,7) · L(4,3) & D(8,3)
· I(4,5) & F(8,5) · A(4,1) & B(8,1)

Garis Horizontal (y=4)
H&A I&L
G&B E&D

Ya, sama

e. Berdasarkan kegiatan yang telah kalian lakukan, bagaimana kalian mendefinisikan refleksi dalam geometri? Apa yang membuat refleksi menghasilkan dua bagian yang identik, dan bagaimana jarak serta posisi titik-titik berpasangan dapat membuktikannya?

Refleksi dalam geometri adalah memindahkan titik atau bangun menggunakan sifat pembentukan bayangan oleh sebuah cermin. Yang membuat refleksi menghasilkan 2 bagian identik adalah karena aturan yg ketat.
kita bisa membuktikan dengan gambar objek di koordinat kartesius

English version:

d) List the pairs of corresponding points based on the line of reflections along with their coordinates. Is the distance from each point to the line of reflections the same as the distance of its corresponding point?
Answer:
Vertical reflection line ($x = 6$)
H(4, 7) & G(8, 7)
I(4, 5) & F(8, 5
L(4, 3) & D(8, 3)
A(4, 1) & B(8, 1)
Horizontal reflection line ($y = 4$)
H&A I&L
G&B E&D

e) Based on the activities you have carried out, how do you define reflection in geometry?
Answer:
What makes reflection procedure two identical parts, and how can the distance and position of corresponding points prove this?
reflection in geometry is moving a point or shape using the properties of the formation of shadows by a mirror. what makes reflection produce 2 identical parts is because of strict rules. we can prove by drawing objects in cartesian coordinates.

Figure 5. Student's answer on activity I

The students' responses in Activity I, see Figure 5, demonstrate varying levels of engagement with the learning design based on Habermas' Construct of Rationality, particularly Epistemic Rationality, Teleological Rationality, and Communicative Rationality. In response to Question 1, students were tasked with identifying symmetrical parts of the Songket motif, and the majority were able to apply Epistemic Rationality by recognizing reflection properties. They demonstrated an understanding of reflection as a geometric transformation, which aligns with the design's goal of bridging visual recognition with formal mathematical concepts. In Question 2a, the Activity encouraged students to translate their understanding into the Cartesian plane, where the reflection of the motif was drawn. This task further supported Epistemic Rationality by requiring students to engage with coordinate geometry, thus reinforcing their ability to apply abstract mathematical concepts. In Question 2b, students identified the reflection axis and justified its role in preserving symmetry, showing an understanding of Teleological Rationality. The task allowed students to comprehend the purpose of the transformation and its geometric implications, specifically that reflections preserve congruence and orientation.

Question 2c involved listing pairs of reflected points, where students applied their understanding of the equidistant property, demonstrating a clear grasp of Epistemic Rationality through calculations. The design's focus on the relationship between visual symmetry and formal calculations effectively guided students to see how reflections preserve geometric properties. In Question 2d, students confirmed that the distance from points to the reflection axis is maintained, further reinforcing their understanding of Epistemic Rationality and communicating the fundamental property of reflections. Finally, in Question 2e, students generalized the concept of reflection, highlighting their ability to reason abstractly and engage in Teleological Rationality as they connected the transformation to broader geometric principles. Overall, the learning design successfully supported the students' proving process by integrating tasks that encouraged critical thinking and the development of formal mathematical reasoning, providing opportunities for students to construct logical proofs, justify geometric transformations, and communicate their understanding. This structured approach allowed students, particularly those with higher ability, to demonstrate a comprehensive understanding of the reflection transformation. In contrast, lower-ability students benefited from the scaffolding provided to facilitate their understanding of geometric reasoning and communication.

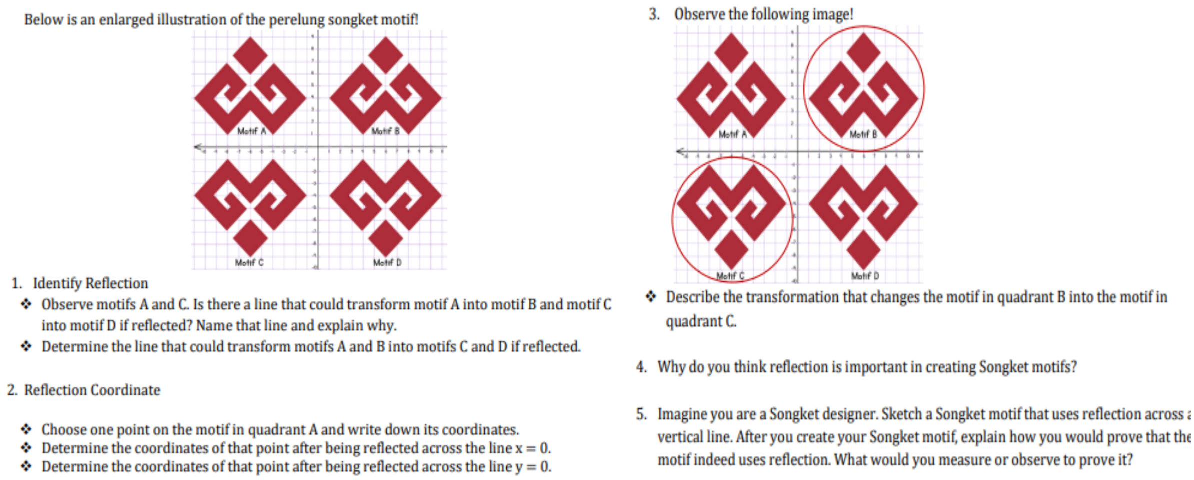


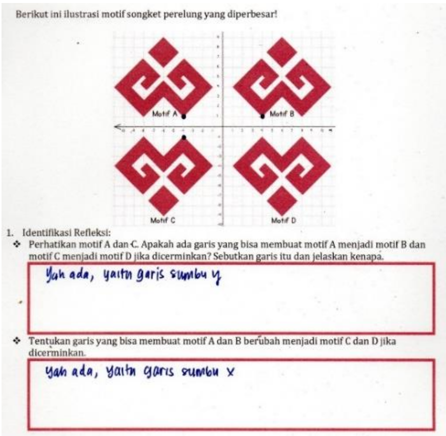
Figure 6. Activity II

Building on Activity I, Activity II (Figure 6) further supported the proving process on students by using a more formal mathematical approach with the Songket Perelung motif. In this task, the students explored reflection transformations in the context of the Cartesian plane, enhancing their ability to apply geometric principles systematically. This task began by introducing the Perelung motif, a design known for its intricate geometric properties, and prompted students to identify potential lines of reflection for the motif. In Question 1, students analyzed the symmetry of motifs A and C, identifying reflection lines that transform motif A into motif B and motif C into motif D. This activity required students to think critically about the geometric properties of symmetry and transformation. Here, students engaged in Epistemic Rationality, where they developed the foundational knowledge necessary to understand reflection as a transformation that preserves distance and orientation. As they analyzed these transformations, they validated their hypotheses by performing actual reflections across axes, such as vertical, horizontal, and coordinate axes, thereby deepening their understanding of Teleological Rationality.

Next, in Question 2, students used coordinate grids to represent the Songket Perelung motif and perform reflections over the x = 0 and y = 0 axes. This step allowed students to apply reflection transformations in a more formal context, reinforcing their understanding of how coordinates change under reflection. In Question 3, students were encouraged to explain the purpose of reflecting the motif along different axes and to justify why these transformations preserve the motif's structure. This step encouraged students to engage in both algebraic reasoning and mathematical justification, reinforcing their understanding of the underlying principles.

In Question 4, students performed consecutive reflections, starting with reflecting the motif across the line $y = 2$ and then performing a second reflection across the line $x = -3$. This allowed students to explore the effects of multiple transformations and understand how reflections combine to create new positions for points in the plane. They were asked to describe how each reflection changes the position of points and the overall symmetry of the motif. This activity encouraged Teleological Rationality, as students thought about how each transformation served a specific purpose in the design process.

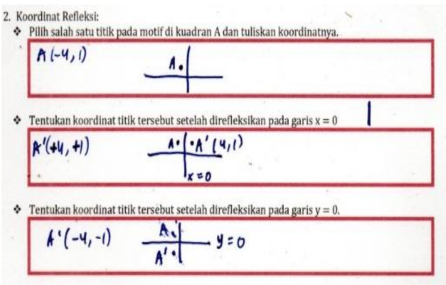
Finally, Question 5 asked students to reflect on the importance of reflection in the creation of Songket motifs. The reflection is important in ensuring symmetry and harmony in the design as the question encouraged students to discuss the cultural and mathematical significance of reflection, encouraging deeper engagement with the material and linking it back to the Communicative Rationality phase. Through group discussions, students justified the role of reflection in the motif's design, thereby enhancing their ability to communicate mathematical reasoning effectively. The student's answer in Activity II can be seen in the following Figure 7.



English version:

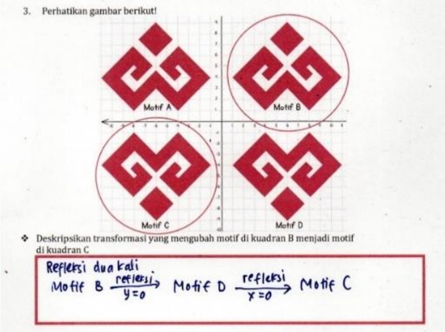
Below is an enlarged illustration of the perelung songket motif!

1. Identify reflection:
   - ❖ Observe motifs A and C, is there a line that could transform motif A into motif B and motif C into motif D if reflected? Name that line and explain why.
     Answer:
     Yes, there is, the y-axis line
   - ❖ Determine the line that could transform motifs A and B into motifs C and D if reflected.
     Yes, there is, namely the x-axis line



English version:

2. Identify reflection:
   - ❖ Choose one point on the motif in quadrant A and write down its coordinates.
   - ❖ Determine the coordinates of that point after being reflected across the line $x = 0$.
   - ❖ Determine the coordinates of that point after being reflected across the line $y = 0$

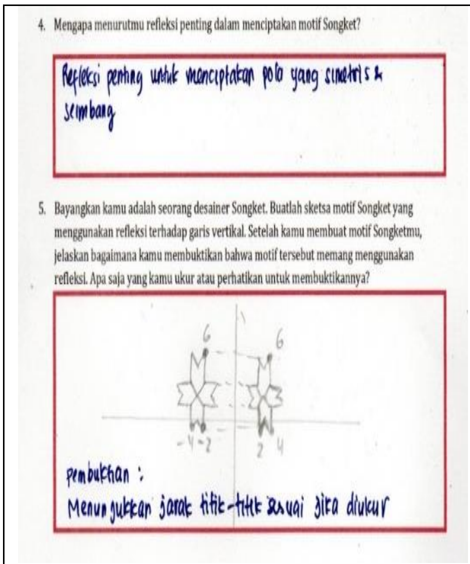


English version:

3. Observe the following image!
   - ❖ Describe the transformation that changes the motif in quadrant B into the motif in quadrant C.

     Answer:

     Two times reflection



English version:

4. Observe the following image!
   - ❖ Why do you think reflection is important in creating Songket motifs?

     Answer:

     Reflection is important to create a symmetrical and balanced pattern.

5. Imagine you are a Songket designer. Sketch a Songket motif that uses reflection across a vertical line. After you create your Songket motif, explain how you would prove that the motif indeed uses reflection. What would you measure or observe to prove it?

   Answer:

   Prove:

   Showing the distance of the points.

Figure 7. Student's answer on Activity II

The students' responses in Activity 2 further illustrate their engagement with the learning design and their ability to apply reflection properties in a more formal geometric context. In Question 1, where students were asked to identify parts of the Songket motif that could be considered as reflections or "shadows," most students demonstrated Epistemic Rationality by identifying the symmetrical parts of the motif. Their understanding of reflection as a geometric transformation was clear, as they recognized the correspondence between the original and reflected parts of the motif. However, the accuracy of their identifications varied, with some students struggling to articulate why these parts were reflections, thus indicating a need for more support in making explicit connections between visual patterns and formal geometric reasoning.

In Question 2, students were tasked with drawing the reflection of the motif on a coordinate grid. This exercise engaged students in Epistemic Rationality, requiring them to translate visual understanding into formal geometric terms. Most students were able to correctly place the reflected motif, demonstrating their ability to apply basic principles of reflection within the coordinate system. However, some students had difficulty translating the visual symmetry of the motif into precise geometric placements on the grid, revealing gaps in their understanding of how geometric transformations are represented algebraically.

Question 3 required students to confirm the coordinates of reflected points, which reinforced the focus on Epistemic Rationality through calculations. High- and moderate-ability students applied their knowledge of reflection to verify that points reflected across the axis retained their equidistant property, demonstrating their ability to prove geometric properties through formal calculation. Low-ability students, on the other hand, found this task more challenging, struggling to perform the necessary calculations to confirm the properties of reflection. This underlines the importance of providing additional scaffolding to support students' computational skills in geometric transformations.

In Question 4, the task of pairing reflected points based on their coordinates reinforced students' understanding of Epistemic Rationality by requiring them to connect abstract mathematical concepts with tangible geometric figures. The majority of students successfully identified pairs of reflected points, reinforcing their understanding of reflection as a distance-preserving transformation. However, a small group of students showed difficulty in accurately determining the pairs, suggesting that the transition from visual understanding to formal calculations needs further refinement.

Overall, the learning design in Activity 2 supported students' development of formal reasoning in geometric transformations by engaging them with both visual and algebraic components of reflection. While high- and moderate-ability students effectively demonstrated epistemic rationality by correctly identifying and calculating reflected points, lower-ability students required additional support to grasp the formal properties of reflection fully. The structured tasks, which integrated hands-on practice with coordinate geometry and reflection properties, provided a scaffold for both high and low-ability students to engage in the proving process and enhance their communication of mathematical reasoning.

Retrospective Analysis

The retrospective analysis in this study involved a critical examination of the entire research process, focusing on the impact on students' proving process in reflection learning. This phase was crucial for consolidating our findings, identifying areas of strength and weakness, and generating insights for future research and pedagogical practice. The integration of South Sumatera's Songket motifs as context proved to be a powerful catalyst for engaging students with the abstract concepts of geometric reflection. The study revealed that the use of these motifs facilitated students' transition from visual pattern recognition to the development of formal mathematical reasoning.

However, the retrospective analysis also highlighted some areas where the learning trajectory could be further refined. While Activity I introduced the basic concepts of reflection through visual exploration, Activity II, which transitioned to a more formal coordinate geometry approach, presented some challenges for students. Some students struggled with making connections between the visual and algebraic representations of reflection, indicating a need for more explicit scaffolding to support this transition.

The Design Research framework proved effective in allowing for iterative refinement of the learning trajectory. The cyclical nature of the research design, with its emphasis on preliminary design, design experiments, and retrospective analysis, allowed us to address emerging challenges and optimize the learning activities systematically. The pilot experiment, in particular, was instrumental in identifying key areas for improvement, leading to a more robust and effective classroom experiment. Furthermore, the study demonstrates the use of Habermas' Construct of Rationality as a theoretical framework for designing learning activities that promote not only conceptual understanding but also support students' proving abilities and communicative competence.

This study demonstrates how integrating culturally relevant materials, such as South Sumatera's Songket motifs, within the PMRI framework enhances students' understanding of reflection transformations in geometry. The findings show that by connecting abstract mathematical concepts with familiar cultural artifacts, students are better able to bridge the gap between visual perception and mathematical reasoning. This aligns with the core principles of PMRI, which emphasize the use of realistic contexts to make mathematics more meaningful and accessible to students (e.g., Nurazizah & Zulkardi, 2022). The designed learning trajectory, guided by Habermas' construct of rationality, supports the development of Epistemic Rationality as students explored symmetry in the Songket motifs and connected their observations to geometric principles of reflection. This approach addresses common difficulties students face in understanding geometric transformations, such as misconceptions about congruence and symmetry, by fostering a deeper conceptual understanding of how reflection preserves shape and size. Previous research (e.g., Götz & Gasteiger, 2018) has highlighted that students often confuse similarity with congruence and fail to recognize reflection as a congruence-preserving transformation. By contextualizing these geometric properties within the culturally familiar Songket motifs, this study provided students with an intuitive and accessible entry point to these concepts, thereby enhancing their conceptual understanding.

In Activity I, students not only engaged with visual patterns but also critically analyzed and articulated the properties of reflection, which helped them transition from intuitive understanding to formal reasoning. This was reinforced in Activity II, where students applied these principles in a more structured mathematical context using Cartesian coordinates. The transition from the concrete analysis of visual symmetry in Activity I to the formal use of coordinate geometry in Activity II exemplifies the progression from Epistemic Rationality to Teleological Rationality, as students tested their hypotheses and justified their reflections using mathematical principles. This aligns with the work of Williford (1972), who emphasized the importance of well-structured learning trajectories in fostering the development of mathematical reasoning. By progressing from informal visual exploration to formal algebraic justification, students experienced a natural progression in their understanding, reinforcing the design's effectiveness in facilitating conceptual development.

The incorporation of Communicative Rationality through group discussions and peer collaboration in both activities allowed students to articulate and refine their understanding. These collaborative discussions are crucial in strengthening students' reasoning and justification skills, which is consistent with Vygotskian sociocultural learning theories that highlight the role of social interaction in knowledge construction (Uygun & Akyuz, 2019). By engaging in peer dialogue, students not only solidified their understanding of reflection but also developed their ability to communicate mathematical concepts clearly and effectively. This collaborative aspect is essential in helping students move from individual, intuitive understanding to shared, formalized reasoning.

The iterative nature of the design research methodology, as outlined by Bakker (2018), played a pivotal role in optimizing the learning trajectory. The design was refined based on student feedback and classroom observations, which led to improved learning outcomes and a more effective learning experience. This iterative process demonstrates the value of dynamic, responsive teaching strategies that can be adapted to meet student's needs and challenges better.

Moreover, integrating cultural elements into the mathematics curriculum not only supported students' understanding of geometric concepts but also aligned with ethnomathematical perspectives that emphasize the importance of cultural relevance in mathematics education (Leonard & Guha, 2002; Sari et al., 2024). The use of South Sumatera's Songket motifs allowed students to engage with mathematical principles in a context that was both culturally significant and geometrically rich, enhancing their motivation and interest in learning.

Finally, this study underscores the importance of addressing the ontogenic, epistemological, and didactical obstacles that students face in learning geometric transformations. The culturally contextualized pathway provided by this learning design helped students navigate these challenges, as evidenced by their increased ability to understand and prove the properties of reflection transformations. Research indicates that addressing these obstacles is critical for effective mathematics education, as they can significantly hinder students' conceptual understanding (Pauji et al., 2023). By offering a comprehensive, culturally grounded approach, this study contributes to the broader effort to reform mathematics education, advocating for contextually enriched, conceptually grounded teaching practices that promote deeper understanding and higher-level reasoning skills.

CONCLUSION

This study demonstrates the potential of integrating South Sumatera's Songket motifs within the PMRI framework to support their proving processes in reflection material. Employing the Design Research methodology, which includes iterative cycles of preparation, design experiments, and retrospective analysis, the study aimed to develop learning activities that align with Habermas' Construct of Rationality. The designed activities facilitated students' engagement with geometric reflections, guiding them from visual observations of symmetry in Songket motifs to formal mathematical reasoning and proof construction. The results from the design experiments indicate that the activities effectively supported students' ability to prove key geometric properties of reflection transformations, particularly in understanding how reflection preserves congruence and orientation.

However, the stages of students' abilities to understand or prove geometric reflection in detail were not fully demonstrated, as the students' answers to each designed activity are not presented in this study. This indicates that further investigation into students' reasoning stages across different abilities is needed to clarify how these students engage with the proving process in reflection learning.

While the learning activities significantly promoted Epistemic Rationality by fostering conceptual understanding, Teleological Rationality by motivating students to understand the purpose of transformations, and Communicative Rationality through the collaborative justification of reflections, the study also identified areas where lower-ability students struggled. These difficulties suggest that more scaffolding is necessary to support students in their transition from visual intuition to formal mathematical reasoning, particularly in computational aspects and justification of reflection properties. This study contributes to the broader discussion on culturally contextualized mathematics education, emphasizing the importance of using cultural artifacts to enrich students' understanding and reasoning. It also highlights the potential for further research in exploring how students' conceptual understanding develops over time, particularly in relation to the use of cultural materials in teaching geometric transformations. Future research should aim to more explicitly detail the stages of students' abilities, investigate the impact of different scaffolding strategies on student learning, and further explore how cultural contexts can enhance not only conceptual understanding but also the development of higher-level reasoning and proving skills in mathematics education.

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Author Contribution	:	AS : Conceptualization, Project administration, Data curation, Formal analysis, Writing - Original Draft, Visualization. RIIP : Supervision, Methodology, Conceptualization, Writing – Review & Editing, Validation. Z : Methodology, Validation, Formal analysis, Resources, Writing – Review & Editing. RCIP : Supervision, Conceptualization, Writing – Review & Editing. MARR : Software.
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REFERENCES

Akarsu, M., & İler, K. (2024). A research on transformation geometry: Answered and unanswered questions. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, *25*(3), 1240-1264. https://doi.org/10.17679/inuefd.1453349

Andriyani, R., Rahman, R., Irawati, R., Mutaqin, E. J., & Kamis, N. (2023). Exploration of Batik Jambi on learning transformation geometry. *Prima: Jurnal Pendidikan Matematika*, *7*(1), 89–99. https://doi.org/10.31000/prima.v7i1.7305

Aktaş, G., & Ünlü, M. (2017). Understanding of eighth-grade students about transformation geometry: Perspectives on students' mistakes. *Journal of Education and Training Studies*, *5*(5), 103–119. https://doi.org/10.11114/JETS.V5I5.2254

Aras, A., Prahmana, R. C. I., Buhaerah, B., Busrah, Z., Jumaisa, J., & Setialaksana, W. (2023). Reflection learning innovation in the context of the lipa sabbe bunga caramming motif. *Al-Jabar: Jurnal Pendidikan Matematika*, *14*(1). https://doi.org/10.24042/ajpm.v14i1.15912

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers*. Routledge. https://doi.org/10.4324/9780203701010

Baroa, J. (2025). Students' common errors in geometric proving: Basis for an enhancement program. *International Journal for Multidisciplinary Research*, *7*(2). https://doi.org/10.36948/ijfmr.2025.v07i02.37424

Büscher, C. (2024). Adapting Habermas' construct of communicative rationality into a framework for analyzing students' statistical literacy. *Educational Studies in Mathematics*, *117*, 121-141. https://doi.org/10.1007/s10649-024-10325-5

Götz, D., & Gasteiger, H. (2018). Typical student errors in axis mirroring - An analysis of student responses [in German]. In Fachgruppe Didaktik der Mathematik der Universität Paderborn (Eds.), *Beiträge zum Mathematikunterricht 2018* (pp. 631–634). WTM.

Götz, D., & Gasteiger, H. (2022). Reflecting geometrical shapes: Approaches of primary students to reflection tasks and relations to typical error patterns. *Educational Studies in Mathematics*, *111*(1), 47–71. https://doi.org/10.1007/s10649-022-10145-5

Guven, B. (2012). Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. *Australasian Journal of Educational Technology*, *28*(2), 364-382. https://doi.org/10.14742/ajet.878

Haj-Yahya, A. (2021a). Can a number of diagrams linked to a proof task in 3D geometry improve proving ability? *Mathematics Education Research Journal*, *35*, 215-236. https://doi.org/10.1007/s13394-021-00385-8

Haj-Yahya, A. (2021b). Students' conceptions of the definitions of congruent and similar triangles. *International Journal of Mathematical Education in Science and Technology*, *53*(10), 2703-2727. https://doi.org/10.1080/0020739X.2021.1902008

Hollebrands, K. F. (2003). High school student's understanding of geometric transformation in the context of a technological environment. *The Journal of Mathematical Behavior*, *22*(1), 1-19. https://doi.org/10.1016/S0732-3123(03)00004-X

Indahwati, R. (2023). Students' difficulties in transformation geometry courses viewed from visualizer-verbalizer cognitive style. *Sigma*, *9*(1). https://doi.org/10.53712/sigma.v9i1.1993

Kim, Y., & Shin, B. (2023). An analysis of students' communication in lessons for the geometric similarity using AlgeoMath. *Korean School Mathematics Society*, *26*(2), Article 003. https://doi.org/10.30807/ksms.2023.26.2.003

Knuchel, C. (2004). Teaching symmetry in the elementary curriculum. *The Montana Mathematics Enthusiast*, *1*(1), 3–8. https://doi.org/10.54870/1551-3440.1001

Leonard, J., & Guha, S. (2002). Creating cultural relevance in teaching and learning mathematics. *Teaching Children Mathematics*, *9*(2), 114-118. https://doi.org/10.5951/TCM.9.2.0114

Nurazizah, I., & Zulkardi. (2022). Students' mathematical reasoning ability in solving PISA-like mathematics problem COVID-19 context. *Jurnal Elemen*, *8*(1), 250-262. https://doi.org/10.29408/jel.v8i1.4599

Nurhikmayati, I., Jatisunda, M. G., & Ratnawulan, N. (2022). The practice of reflection based on didactical design research: An analysis of the geometry transformation material. *JTAM (Jurnal Teori dan Aplikasi Matematika)*, *6*(3), 265–277. https://doi.org/10.31764/jtam.v6i3.8441

Noto, M., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of pre-service mathematics teachers on transformation geometry. *Journal on Mathematics Education*, *10*(1), 117–126. https://doi.org/10.22342/JME.10.1.5379.117-126

P, S., & P, S. S. K. (2022). A systematic investigation on geometric transformation. *2022 Third International Conference on Intelligent Computing Instrumentation and Control Technologies (ICICICT)*, 343–350. https://doi.org/10.1109/ICICICT54557.2022.9917968

Pauji, I., Hadi, H., & Juandi, D. (2023). Systematic literature review: Analysis of learning obstacles in didactical design research on geometry material. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *7*(3). https://doi.org/10.31004/cendekia.v7i3.2474

Putri, R. I. I., Setyorini, N. P., Meitrilova, A., Permatasari, R., Saskiyah, S. A., & Nusantara, D. S. (2021). Designing a healthy menu project for indonesian junior high school students. *Journal on Mathematics Education*, *12*(1), 133-146. https://doi.org/10.22342/jme.11.1.9786.135-144

Sari, A., Putri, R. I. I., Zulkardi, & Prahmana, R. C. I. (2024). Ethnomathematics in Indonesian woven fabric: The promising context in learning geometry. *Mathematics Teaching Research Journal*, *16*(5), 157-185. https://files.commons.gc.cuny.edu/wp-content/blogs.dir/34462/files/2024/11/7-Sari-et-al-Ethnomathematics-in-Indonesian-Woven-Fabric.pdf

Shen, P., Li, R., Wang, B., & Liu, L. (2023). Scratch-based reflection art via differentiable rendering. *ACM Transactions on Graphics*, *42*(4), 1–12. https://doi.org/10.1145/3592142

Uygun, T. & Akyuz, D. (2019). Developing subject matter knowledge through argumentation. *Internationa lJournal of Research in Education and Science (IJRES)*, *5*(2), 532–547 https://ijres.net/index.php/ijres/article/view/568

Velichová, D. (2021). The mysterious synergy of geometry and art. In L. Y. Cheng (Ed.), *ICGG 2020 – Proceedings of the 19th International Conference on Geometry and Graphics* (*Advances in Intelligent Systems and Computing*, Vol. 1296). Springer. https://doi.org/10.1007/978-3-030-63403-2_24

Williford, H. J. (1972). A study of transformational geometry instruction in the primary grades. *Journal for Research in Mathematics Education, 3*(4), 260-271. https://doi.org/10.2307/748493

Yurmalia, D., & Herman, T. (2021). Student visualization in solving geometry problems: The case of reflection [Paper presentation]. In *Journal of Physics: Conference Series, 1806*(1). International Conference on Mathematics and Science Education (ICMScE), Jawa Barat, Indonesia. https://doi.org/10.1088/1742-6596/1806/1/012052

Zhuang, Y., & Conner, A. (2022). Teachers' use of rational questioning strategies to promote student participation in collective argumentation. *Educational Studies in Mathematics, 111*(3), 345–365. https://doi.org/10.1007/s10649-022-10160-6