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Learning Trajectory for Teaching the Mean Concept Using Problem-Based Learning and Animated Video

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Abstract

Conceptual understanding is a critical component of effective mathematics learning. However, many students encounter challenges in understanding mathematical concepts, including the concept of the mean. This study aims to design a learning trajectory that enhances students' conceptual understanding of the mean. Employing a design research methodology, the study followed the stages of validation studies in creating and refining a hypothetical learning trajectory (HLT). The research consisted of three stages: preliminary design, design experiment (encompassing pilot experiment and teaching experiment), and retrospective analysis. The participants were fourty eighth-grade students from a public school in Cimahi. Data were collected through interviews, observations, documentation, and field notes, which informed iterative improvement to the HLT. Data analysis involved comparing students' actual learning experiences with the HLT as a reference framework. The findings reveal that integrating technology and problem-based learning enhances students' conceptual understanding of the mean, fostering creativity and real-world problem-solving skills. Based on these results, this study proposes a validated learning trajectory to support students in developing a deeper conceptual understanding of the mean.

Keywords: Animated Video, Design Research, Learning Trajectory, Mean, Problem-Based Learning

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INTRODUCTION

Statistics is an important discipline for students to learn, as it is widely applied in everyday life, including research across various fields, professional tasks, and numerous other activities (Cipta et al., 2023). In Indonesia's curriculum, statistical concepts are introduced from elementary education through higher education. One of the foundational and highly applicable topics in statistics is the concept of measurement of center. This study will focus on average or mean.

In general, there are four types of averages: arithmetic, geometric, harmonic, and least squares averages (Martinez & Bartholomew, 2017). Among these, the arithmetic average, commonly referred to as the mean, is a fundamental concept taught in primary and secondary school curricula in Indonesia. It is defined as the sum of all data points divided by the total number of data points. The mean plays a crucial role in statistics, serving as the basis for various other statistical representations and analyses, including those found in inferential statistics, where it features prominently in formulas and methodologies.

According to Cipta et al. (2023) and Livingston (2004), a strong mathematical understanding of the mean is essential for accurate data analysis and interpretation. This underscores the importance for ensuring that students develop a robust grasp of the concept. Consequently, teachers should prioritize the design and implementation of effective learning strategies to foster a deep understanding of the mean among students. The study of Cooper and Shore (2008) found that students apply the algorithm

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of mean confidently, even though they lack a proper understanding of the underlying data values. However, it is necessary for students to comprehend the mean from the perspective of data distribution, which forms the foundation for calculating it (Martinez & Bartholomew, 2017; Lehrer et al., 2011). This indicates that students' understanding of the mean remains suboptimal, often limited to procedural knowledge. Such limitations can lead to difficulties in solving contextual problems that require higher-oder thinking skills.

To address the challenges mentioned above, the study implements Problem-Based Learning (PBL) in teaching the concept of the mean. PBL has been shown to significantly enhance students' ability to apply mathematical concepts to real-life situations (Nurin et al., 2024). Fundamentally, PBL employs problems as a starting point for acquiring new knowledge. This approach aligns with the findings of Schoen et al. (2025), who demonstrated that inquiry-oriented lessons within a curriculum, supported by targeted learning opportunities for teachers, can improve instructional practices and enhance students learning in statistics. Moreover, the statistical problem-solving process is crucial in statistical learning at the school level (Arnold & Franklin, 2021). Consequently, PBL is expected to serve as an effective strategy for optimizing the learning of statistical concepts, particularly the mean.

Prior study, such as McGatha et al. (2002), sought to shift students' reasoning towards data analysis as inquiry rather than a procedure through a set of problems given to students. However, it appeared that students calculated the mean without considering the appropriateness to the problem situation. Another prior research of Oberlander and Talbert-Johnson (2014) investigated the technology-enhanced PBL in students' learning experiences. They found that the actives included elements of rich environments for active learning. Therefore, the integration of technology is anticipated to optimize PB and enhance students' understanding of the mean concept.

In this study, technology of audio-visual media was utilized because it engages multiple senses, promotes participation, stimulates interest, serves as a rich source of information, and help students understand learning concepts more effectively (Ashaver & Igyuve, 2013; Alshatri, et al., 2019; Kumar et al.., 2022). According to Kumar (2022), worksheets equipped with audio-visual media lead to better learning outcomes compared to traditional, text-based material. Despite these findings, limited research has specifically addressed the topic of the mean and the use audio-visual media. This study addresses this gap by developing a learning trajectory on the topic of the mean using PBL by and integrating audio-visual media. The aim is to design an innovative learning trajectory that combines PBL with audio-visual media to enhance students' conceptual understanding of the mean. By conducting this research, we aim to provide valuable input for teachers and researchers seeking to implement and develop mathematics education, particularly on the topic of the mean. Furthermore, this study contributes to advancing scientific knowledge in mathematics education by presenting a novel approach to teaching this fundamental concept.

METHODS

This research employed the Design Research methodology, which focuses on developing learning theories and activities through collaborative efforts between researchers and educators to enhance the quality of learning (Gravemeijer & Eerde, 2009). Specifically, this study developed a learning trajectory on the topic of the mean, incorporating PBL with video animation media to support student engagement and understanding.

The study adopts the validation studies approach, which comprises three phases: Preliminary Design, Design Experiment, and Retrospective Analysis (Plomp & Nieveen, 2013; Putri & Zulkardi, 2017). The research was conducted at on public secondary school in West Java, with the following stages outlined:

Preliminary Design

In the preliminary design phase, a Hypothetical Learning Trajectory (HLT) was designed to serve as a framework for guiding the learning process. An HLT generally consists of three components: learning objectives, learning activities, and conjectures about the learning process, which include predictions of students' cognitive engagement when interacting with planned activities (Risdiyanti & Prahmana, 2021). This stage involved two main activities to design the HLT: 1) preliminary study, this activity included observing the characteristics of students at the subject school, analyzing the curriculum to identify learning outcomes, planning instructional approaches and the worksheets, designing the audio-visual media, and preparing evaluations tools; 2) literature study, a comprehensive review of various learning techniques for averages was conducted. This ensured the instructional approach was relevant to the students' characteristics and learning needs. The outcomes of these activities formed the foundation for the HLT, which was collaboratively formulated by researchers and model teachers. The collaboration ensured that the diverse learning needs of students on the topic of the mean were considered. The finalized HLT was then implemented in the classroom during the design experiment stage.

Design Experiment

The developed HLT was implemented in the classroom during the design experiment phase, which consisted of two stages: pilot experiment and teaching experiment. The pilot experiment was conducted in a small group of eight students. The primary aim was to obtain preliminary feedback for refining the learning trajectory before moving to the teaching experiment phase, thereby enhancing the HLT (Sari & Putri, 2021). The revised HLT was then implemented in a larger class of 32 students. At this stage, the mathematics teacher acted as the model teacher, applying PBL supported by audio-visual media. Meanwhile, the researcher assumed the role of an observer to comprehensively observe the learning process. Data collection methods included observation notes, video recordings, and students written. These data were later analyzed retrospectively to evaluate the learning activities and refine the instructional approach.

Retrospective Analysis

The retrospective analysis phase aimed to compare the HLT with the actual learning process (ALT) observed during the teaching experiment (Adelia et al., 2022; Adha et al., 2024). This stage analyzed the learning process, where students engaged in the designed activities. The collected data were analyzed qualitatively to ensure a comprehensive understanding of the learning process. This approach also strengthened the validity and reliability of the developed learning design, providing critical insights for enhancing the instructional framework.

RESULTS AND DISCUSSION

Preliminary Design

In the preliminary design phase, the first activity involved conducting a preliminary study. The researcher interviewed two mathematics teachers who taught grade eight at the subject school to understand the general characteristics of the students and the challenges they faced in the learning process. The teachers reported that students often struggled to comprehend problems, particularly those requiring higher-order analytical skills. These difficulties were compounded when students encountered word problems that demanded an understanding of the underlying concept, rather than rote application of formulas. Another challenge students face was their lack of meaningful understanding, which caused them to become distracted by other formulas As a result, students tended to rely solely on memorization, limiting their ability to apply the concept effectively in problem-solving situations.

Thereafter, the researchers reviewed and analyzed references relevant to students' difficulties, particularly in applying the mean formula, especially in word problems. The findings of this literature review suggest that to PBL could serve as an effective alternative to address these challenges. PBL, rooted in constructivist theory, trains students to solve problems and discover essential concepts related to the mean, with teacher guidance. By presenting problems at the beginning of learning, students are encouraged to engage with contextual problems. Choosing real-life contexts for these problems help students understand mathematical concepts easier and makes the learning experience more meaningful (Putri et al., 2022). In line with Ulfah and colleagues (2020), who argue that students are more likely to feel connected to the material when they see its relevance and application in everyday life, the importance of context is clear. However, students often face difficulties in visualizing the problem and understanding its context during the problem-orientation stage. To addres this, the literature review highlights the need for learning media to aid students in understanding problems given. Animated media created with the Doratoon platform offers a solution by providing a more tangible representation of the context, capturing attention, and simplifying the comprehension of the problem.

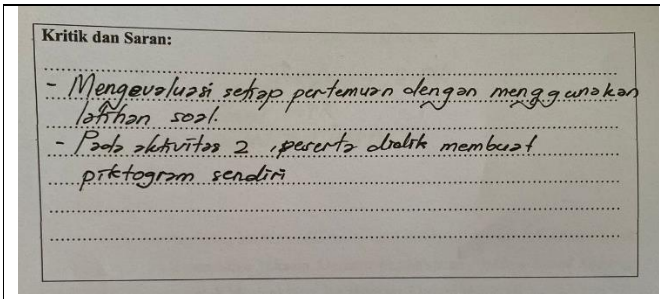
Third, the researchers designed learning tools tailored to students' characteristics and challenges. These tools include teaching modules, student worksheets, observation sheets, assessment instruments and learning media. The design of these instruments was guided by the HLT, which was collaboratively developed with the teacher. In this study, the HLT aimed to help grade eight students understand the concept of the mean through PBL approach supported by animation videos. The details of the developed HLT are presented in Table 1.

Table 1. The HLT of the mean

Activity	Objectives	Conjectures
Identifying problems in the candy-buying activity shown in the animated video	Understand and interpret numerical data	• Students observe the problem related to the average candy purchase presented in the animated video. • Students collect data on the number of candies purchased by each of their friends.
Completing data on a simple pictogram	Organize data	• Students place stickers in the column representing the number of candies based on the data collected.
Balancing data	• Solve problems creatively	• Students derive and understand the formula for the volume of a cylinder.
	• Discober the concept of the mean by balancing data	• Recognize that the balanced data value represents the mean
Finding the mean formula	Derive the mean formula based on the activities they have completed	• Students identify effective calculations methods to balance the data. • Students understand that the total number of candies is the sum of all data divided by the number of data points. • Students summarize the formula for the mean.

Design Experiment

The design experiment phase is divided into two stages: the pilot experiment and the teaching experiment. The pilot experiment was conducted in small classes, while the teaching experiment was carried out in larger classes. During the pilot epxeriment, the researcher tested the initial HLT in a small class setting with eight selected students. The researcher guided the learning process according to the activity stages outlined in the HLT. Meanwhile, the mathematics teacher, acting as the learning observer, recorded observations and provided suggestions using a teacher observation sheet. These notes and suggestions served as the basis for refining the HLT, ensuring it was optimized for implementation in the subsequent teaching experiment (see Figure 1).



English version:

Critic and suggestion:

- Evaluate each session by administering practice questions.
- In Activity 2, students create ther own pictograms.

Figure 1. HLT revision notes

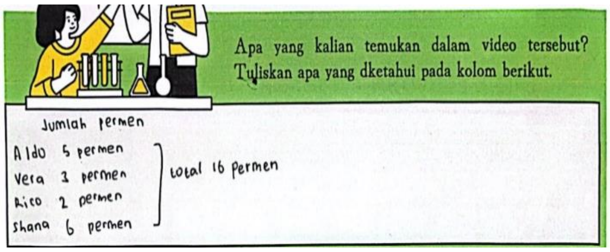
Based on the trial results and observations from the mathematics teacher, the researcher and teacher agreed to revise Activity 2. Instead of completing pre-existing pictograms, students would create their own pictograms according to the data they gathered, fostering greater creativity. Additionally, the mathematics teacher suggested incorporating more practice problems to evaluate of students' understanding. This adjustment aims to better prepare students to solve real-life problems involving the concept of the mean. To implement this, the researcher opted for a quiz an an evaluation tool for the learning process.

Teaching Experiment

Activity 1 begins with students scanning barcodes to access a video that presents a problem about buying candy in a shop. Students then analyze and collect the information presented in the video. At this stage, the teacher guides students to engage with the learning by following the provided instructions.

Figure 2. The first activity

Figure 2 illustrates how the first activity begins with the teacher introducing students to the problem through the animation video. The teacher then organizes the students into groups to investigate the problem. This activity demonstrates that animated media helps students visualize the problem, enabling them to comprehend the story behind it. Moreover, students actively express their opinions and show increased to engage in the investigative learning process. Following this, the teacher guides the students in sharing their initial thoughts on the problem.



English version:

What did you find in the video? Write down what you observed in the column below.

Number of candies

Aldo 5 candies
Vera 3 candies
Rico 2 candies
Shana 6 candies

total 16 candies

Figure 3. Identifying the problem

This activity involves the teacher confirming students' answers by referencing excerpts from interviews with students (FT and MP) about the data in the animated video.

Teacher	:	*"From the video, how many of Wika's friends bought candy?"*
FT	:	*"Four people."*
Teacher	:	*"Okay. Can you tell me the number of candies each of Wika's friends bought?"*
FT	:	*"Aldo bought 5 candies, Vera 3 candies, Rico 2 candies, and Shana 6 candies."*
Teacher	:	*"Okay thank you. MP, can you calculate the total number of candies bought by Wika's friends?"*
MP	:	*"There are 16 candies, Ma'am."*
Teacher	:	*"Yes, that's correct. The total number of candies bought is 16 candies. Thank you, MP."*

Figure 3 illustrates that students are able to identify the number of candies purchased by Wika's friends. Students write down the information they gather and determine the question based on the problem presented. The teacher ensures accuracy by asking clarifying questions. This activit demonstrates that students can understand the problem and extract the correct information. Subsequently, students create pictograms using the data collected in activity 1.

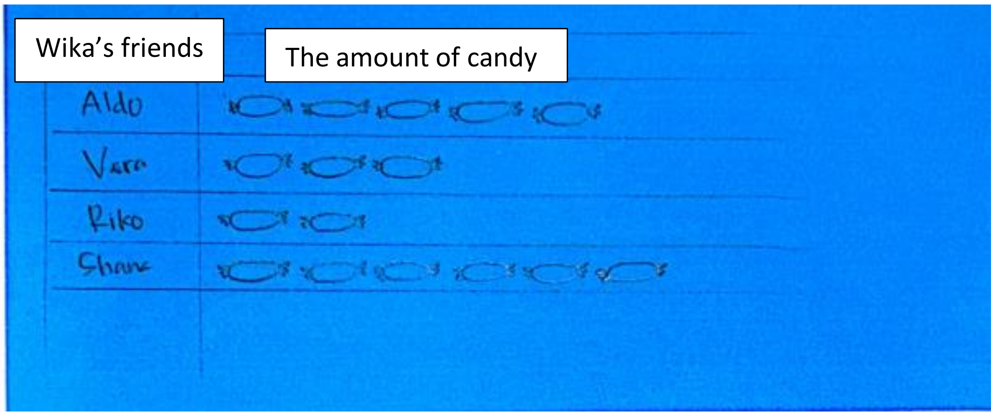


Figure 4. Completing the data on a simple pictogram

Figure 4 depicts the second activity, where students create pictograms based on the data they have collected. This activity enables students to represent the data visually in a simple diagram. Students are encouraged to design their own pictograms. During the process, students actively channel their ideas to create accurate and visually appealing pictograms.

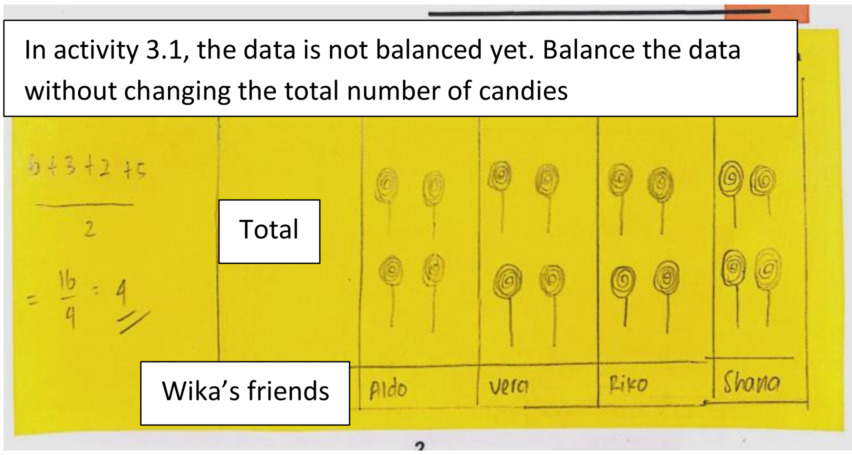


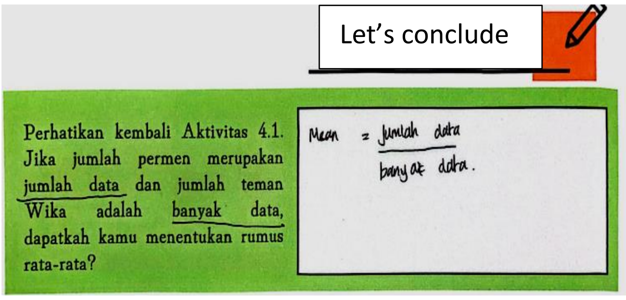
Figure 5. Balancing the data

This activity involves the teacher guiding the investigation to balance the data using a series of questions and answers, encouraging students to discover solutions in their own way.

Teacher	:	*"Look at the pictogram you created. Did Wika's friends buy the same number of candies?"*
All Students	:	*"No, Ma'am."*
Teacher	:	*"Now, try to balance the candy data so that the number of candies bought by Wika's friends becomes equal"*
RH	:	*"How do we do theat?"*
RH	:	*"Ma'am, can we just move the picture? Shana's candies can be given to Rico."*
Teacher	:	*"Yes, that's correct. But who should distribute the candies—those who have more or those who have less?"*
NL	:	*"Those with more, Ma'am"*

NL : *"Would it work if we collected all the candies first and then distributed them one by one?"*

Figure 5 illustrates the balanced pictogram data. In the third activity, students are guided to balance the data displayed in the pictogram. This activity is designed help students grasp the concept of the mean within a data set. Observations during the activity revealed that students began to an emerging understanding of the mean. While students showed progress in developing solutions to the problems, they still need help in balancing the data. The teacher interview highlighted that students often had alternative solution in mind but lacked confidence in their ideas. Consequently, the teacher played a crucial role in guiding students through the process, helping them refine and articulate their solution in alignment with the PBL learning stages.



English version:

Look at Activity 4.1.

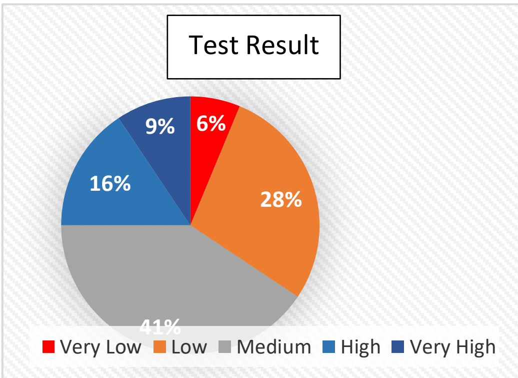
If the number of candies represents the sum of all data and the number of Wika's friends represent the number of data, can you determine the formula of the average?

$$Mean = \frac{the\ sum\ of\ all\ data}{the\ number\ of\ data}$$

Figure 6. Concluding the formula of mean

Figure 6 illustrates the mean formula derived by students from their learning outcomes. In Activity 4, students summarize the process of determining the mean formula based on their previous work. Through discussion with peers and guidance from the teachers, students successfully conclude the mean formula. They were first introduced to the concept of the total number of candies bought by Wika's friends represents the sum of all data, while the number of Wika's friends represents the number of data points.

a



b

Figure 7. Concluding the formula of mean

Activity 5 focuses on solving problems related to the concept of the mean. In this activity, students individually solve five word problems based on everyday situations. Figure 7b shows the number of correct answer for each student's quiz. The results indicate that 25% of students are in the high-performance category, with 9% of them achieving perfect scores. The assessment results reveal that 41% of students fall into the medium category, with scores ranging from 76 to 84. Given the minimum passing grade of 76 at the school, this suggests that the medium category represents students who have met the learning completeness criteria. This indicates that more than half of the students in the subject class are able to solve problems related to the mean quite effectively. In the low category, 28% of students scored between 68 and 70. Although these scores are close to the learning completeness threshold, they suggest that these students are relatively competent in understanding the concept of the mean, though there may be some room for improvement. The very low category consists of 6% of students, who, despite understanding the mean concept, struggle with number operations and calculations. This indicates that these students may need additional support in refining their computational skills. Overall, the results suggest that most students have a solid understanding of the mean concept and are capable of applying it to contextual problems. However, some students still make errors in numerical calculations, highlighting the need for further practice and support in this area.

The research findings from the design experiment phase resulted in a learning trajectory that underwent two experimental stages. The learning trajectory for teaching the concept of the mean developed in this study is presented in the following Figure 8.

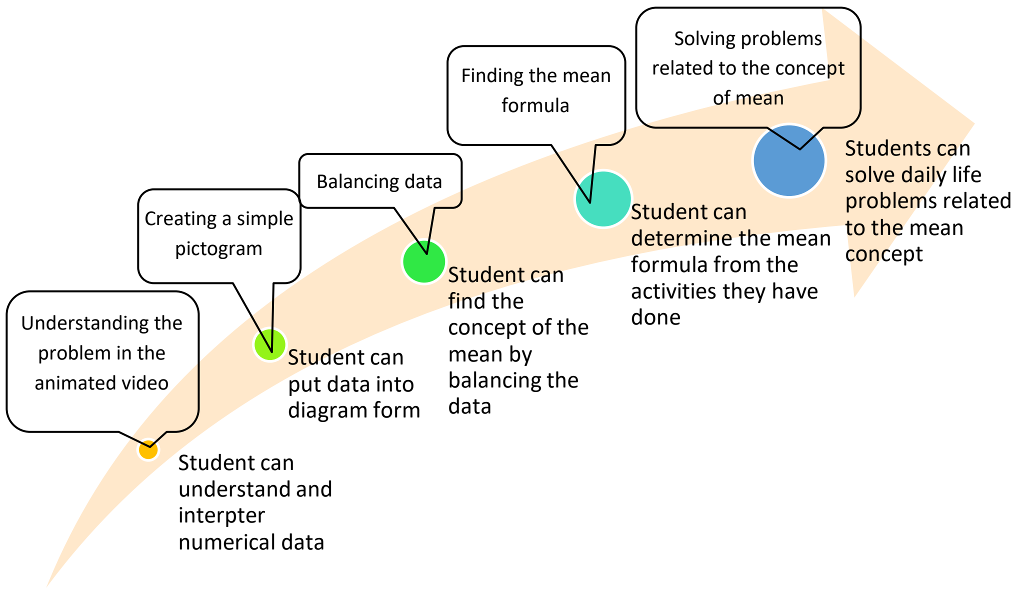


Figure 8. Learning trajectory for the concept of mean

The next step was the retrospective analysis, which compared the HLT with the actual learning process from the teaching experiment. Overall, there were no significant changes in the activities designed to understand the concept of the mean. However, there were unique and noteworthy findings in students’ responses to the learning process. These findings support, complement, and serve as references for comparison between HLT and the actual learning process. Table 2 presents these new findings for each learning activity on the mean concept using PBL with the assistance of animated video.

Table 2. Finding in each activity

Activity	Finding
Activity 1	Students demonstrated interest in the use of technology in learning, which made the process more effective. However, some students were still unfamiliar with and less comfortable using technology in the learning environment.
Activity 2	Students displayed a high level of creativity in presenting data through pictograms; however, they hesitate when selecting the most appropriate visual representation.
Activity 3	Students demonstrated various approaches to balancing the data but remained hesitant about their ideas.
Activity 4	Students demonstrated a good understanding of the mean concept but faced challenges in selecting the correct operations to apply it effectively.
Activity 5	Students demonstrated skills in solving real-life problems related to the concept of mean, but their understanding and application were not consistently thorough.

The findings from the first activity reveal a marked increase in student interest when learning through animated videos. These results align with Li et al. (2021), who noted that students prefer teachers who utilize engaging learning media. Additionally, the use of technology facilitates comprehension of abstract problems by providing visually appealing and accessible representations, as supported by Aripin et al. (2024).

In practice, integrating technology into learning requires careful attention to several key factors, including the readiness of school facilities, infrastructure, and teacher motivation. The findings from Activity 1 indicate that some students remain hesitant and unprepared to use technology, highlighting the need to optimize teaching experiments to ensure students readiness. This aligns with Taleb et. al (2015), who emphasized the importance of training and educating students on the benefits and effective integration of technology in classroom learning. Moreover, teacher readiness plays an important role, As Backfisch (2020) demonstrated, a teacher's confidence and motivation significantly impact the effective use of technology in mathematics education. This underscores the dual need for both students and teachers to be well-prepared and supported in adopting technology-enhanced learning environments.

In the second activity, where students create pictograms, an improvement was implemented from the pilot experiment: instead of completing pre-made pictograms, students were tasked with creating their own. This change was aimed to fostering creativity during the teaching experiment phase. As a results, students demonstrated various ways to represent data through pictogram, showcasing a high level of creativity. However, hesitation in selecting the most appropriate visual representation was also observed, emphasizing the critical role of the teacher as a facilitator in this process. This aligns with Mustaffa et al. (2016), who highlight that in PBL, teachers should act as facilitators rather than information providers, allowing students to engage deeply in the learning process. This approach indirectly supports findings in the third activity, where students explored various ways to balance data but remained uncertain about their ideas. In such cases, the teacher's role in guiding and stimulating students' confidence becomes crucial. Napitupulu et al. (2016) further support this by showing that PBL effectively promotes higher-order thinking skills, particularly among middle- and high-ability students. By facilitating problem-solving activites that students develop themselves, PBL not only strengthens their problem-solving capabilities but also nurtures independence in addressing mathematical problems.

The central focus of learning the mean topic is guiding students to independently discover the concept of mean, with the teacher serving as a facilitator. The findings from Activity 4 reveal that students possess a good conceptual understanding of the mean and can relate it to a set of numbers. However, they face challenges in selecting the appropriate operations to construct a formula for the mean. This indicates that while students grasp the meaning and implementation of the mean, they require support to translate this understanding into the correct mathematical operations. An interesting insight emerged when students were guided through balancing data. One group suggested a method involving the collection of all data and distributing it evenly. Building on this, the teacher prompted students to identify the appropriate mathematical operations: addition for data collection and division for distribution. Through this guided process, students successfully concluded that the formula for calculating the mean is the total sum of data divided by the number of data points. This approach aligns with Martinez and Bartholomew (2017), who describe the mean as the sum of individual values divided by their count. By leveraging students' findings and guiding them with probing questions, the teacher effectively facilitated the discovery process. This underscores the importance of teacher intervention in scaffolding students' understanding and ensuring they develop a concrete grasp of mathematical concepts.

The fifth activity focuses on solving real-world problems related to the concept of mean, following the modeling cycle proposed by Blum and Leiß (2007). This cycle emphasizes the progression from a real situation to a real model, then a mathematical model, mathematical results, and finally back to the real results and the real situation. The learning trajectory designed in this study adheres to this framework, enabling students to learn the concept of mean from real-world contexts and subsequently apply it to solve authentic problems. A key finding from this activity is that students demonstrate proficiency in solving real-life problems related to the mean but not consistently or comprehensively. This indicates that while many students understand the mathematical concept of mean, some struggle to apply it effectively in contextual scenarios. The difficulty seems to stem from insufficient literacy in interpreting and solving contextual problems. By integrating PBL, the study aims to enhance students' contextual literacy, fostering their ability to connect mathematical concepts to real-world situations. Encouraging students to engage with and solve contextual problems repeatedly can build both their mathematical proficiency and their confidence in applying these concepts to practical challenges. This approach aligns with the goals of PBL, which emphasizes higher-order thinking and real-world application, ultimately bridging the gap between abstract mathematical concepts and everyday problem-solving.

CONCLUSION

This study concludes that the designed learning activities contribute to students' understanding of the concept of mean through PBL enhanced by animated videos. The learning process follows a structured sequence: beginning with a problem introduced through animated videos, students then create pictograms based on the information gathered, balance the data, and eventually discover the concept of the mean. Finally, students apply their understanding of the mean to solve contextual problems. This finding provides valuable insight for researchers and teachers on how to effectively apply PBL in teaching the concept of mean using animated video media. However, the study recognizes certain limitations and areas for future development. The use of technology in this study was confined to the initial problem orientation stage, leaving ample room for further innovation in integrating technology throughout the entire learning process. Additionally, the material covered was restricted to the concept of mean, presenting an opportunity for future research to expand this approach to broader statistical topics or other relevant subjects.

Another key finding is the need for further development of students' literacy skills. Effectively applying the concept of mean in contextual problems requires strong literacy, which remains an area for improvement. Future research can focus on how to enhance students' literacy skills through the application of the learning trajectory developed in this study. It could also explore the development of relevant learning media to address challenges faced by students in interpreting and solving contextual problems.

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Author Contribution	:	UA: Conceptualization, Funding acquisition, Investigation, Methodology, Validation, Writing - Original Draft-Review and Editing. TR: Conceptualization, Data curation, Formal Analysis, Investigation, Project administration, and Resources. EER: Conceptualization, Supervision, Validation, and Visualization. WH: Validation, Visualization, and Writing – review and editing.
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